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# Significance Of Delayed Unit Step Function For Discontinuous Functions And Their Laplace Transforms

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**Abstract:** In this paper, we will discuss the significance of delayed unit step function for discontinuous functions. The discontinuous functions are represented in terms of delayed unit step function and their Laplace transform is then found. Laplace transform is a mathematical tool which makes it easier to solve the problems in engineering and science.
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**Key words:** Laplace Transform, Delayed Unit Step Function.

**Sub Area:** Laplace transformation
**Broad Area:** Mathematics

## Introduction

Laplace transformation is a mathematical tool which is used in the solving of differential equations by converting them from one form into another form [1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ]. It is also used to convert the signal system in frequency domain for solving it in a simple and easy way. It has wide applications in different fields of engineering and technology [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 ]. This paper discusses the significance of delayed unit step function for discontinuous functions. The discontinuous functions are represented in terms of delayed unit step functions and their Laplace are found.

## Definition

Let F (t) is a well defined function of t for all t ≥ 0. The Laplace transformation [6, 7] of F (t), denoted by f ( $p$ ) or L {F (t)}, is defined asL {F (t)} $=\int_0^\infty e^{-pt}F(t)dt = f(p)$, provided that the integral exists, i.e. convergent. If the integral is convergent for some value of $p$, then the Laplace transformation of F (t) exists otherwise not. Where $p$ the parameter which may be real or complex number and L is the Laplace transformation operator [8-15]. The Laplace transformation of F (t) i.e. $\int_0^\infty e^{-pt}F(t)dt$ exists for $p$>a, i.e. F (t) is continuous and$\lim_{n\to\infty}\{e^{-at}F(t)\}$ is finite. It should however, be keep in mind that above condition are sufficient and not necessary [8, 9].

**Laplace transformation of some elementary function** [12-14]:

$$1. L\{1\} = \frac{1}{p}, p > 0$$

$$2. L\{t^n\} = \frac{n!}{p^{n+1}},$$

$$where\ n = 0,1,2,3 \ldots\ldots.$$

$$3. L\{e^{at}\} = \frac{1}{p-a}, p > a$$

$$4. L\{sinat\} = \frac{a}{p^2+a^2}, p > 0$$

$$5. L\{sinhat\} = \frac{a}{p^2-a^2}, p > |a|$$

$$6. L\{cosat\} = \frac{p}{p^2+a^2}, p > 0$$

$$7. L\{coshat\} = \frac{p}{p^2-a^2}, p > |a|$$

$$8. L\{u_a(t)\} = \frac{e^{-ap}}{p}$$

$$9. L\{u_a(t)\,f(t-a)\} = e^{-ap}\bar{f}(p)$$

**Unit Step Function:**

$$If\ L\{f(t)\} = \bar{f}(p) and\ u_a(t) is\ unit\ step\ function\ [10, 11]\ i.e.$$

$$u_a(t) = \begin{cases} 1, t > a \\ 0, 0 < t < a \end{cases}$$

Then, $L\{u_a(t)\,f(t-a)\} = e^{-ap}\bar{f}(p)$

$$or$$

$$L^{-1}\{e^{-ap}\bar{f}(p)\} = u_a(t)\, f(t-a)$$

**Application I**

**Express the following function in term of unit step function and find the Laplace Transform**

$$\boldsymbol{f(t)} = \begin{cases} \mathbf{1}, \boldsymbol{t} > \pi \\ \mathbf{2}\boldsymbol{t}, \mathbf{0} < t < \pi \end{cases}$$

***Solution***:

$$L\{2tu(t)\} - L\{2tu(t-\pi)\} + L\{u(t-\pi)\}$$

$$= 2L\{(t-0)u(t-0)\} - 2L\{(t-\pi+\pi)u(t-\pi)\} + L\{u(t-\pi)\}$$

$$= 2e^{-0p}L\{f(t)\} - 2L\{(t-\pi)u(t-\pi)\} - 2\pi L\{(t-\pi)\} + L\{u(t-\pi)\}$$

After solving, we get,

$$\frac{2}{p^2} + e^{-\pi p}\left[\frac{1}{p} - \frac{2}{p^2} - \frac{2\pi}{p}\right]$$

Or,

$$\frac{2}{p^2} + e^{-\pi p}\left[-\frac{2}{p^2} + \frac{1-2\pi}{p}\right]$$

**Application II**

**Express the following function in term of unit step function and find the Laplace Transform**

$$\boldsymbol{f(t)} = \begin{cases} \boldsymbol{t}, \mathbf{0} < \ \boldsymbol{t} \leq \mathbf{1} \\ \mathbf{2} - \boldsymbol{t}, \mathbf{1} < t < \mathbf{2} \end{cases}$$

***Solution***:

$$L[t\{u(t-0) - u(t-1)\}] + (2-t)[\{u(t-1) - u(t-2)\}]$$

$$= [L\{(t-0)u(t-0)\} - L\{tu(t-1)\}] + 2\{u(t-1)\} - 2L\{u(t-2)\} - L\{tu(t-1)\} + L\{tu(t-2)\}$$

After solving, we get,

$$\frac{1}{p^2} + [1 - 2e^{-p} + (e^{-p})^2]$$

Or,

$$\frac{(1-e^{-p})^2}{1-e^{-2p}}$$

**Application III**

**Express the following function in term of unit step function and find the Laplace Transform**

$$\boldsymbol{f(t)} = \begin{cases} \mathbf{0}, \boldsymbol{t} > \pi \\ \boldsymbol{ksint}, \mathbf{0} < t < \pi \end{cases}$$

***Solution***:

$ksint[\, u(t-0) - u(t-\pi)]$

$$= kL\{sin(t-0)u(t-0)\} - kL\{sin(t-\pi+\pi)u(t-\pi)\}$$

$$= \frac{k}{p^2+1} - kcos\pi\ L\{sin(t-\pi)u(t-\pi)ksin\pi L\{cos(t-\pi)u(t-\pi)\}$$

$$= \frac{k}{p^2+1} - \left[\frac{kcos\pi}{p^2+1} - \frac{kpsin\pi}{p^2+1}\right]e^{-p\pi}$$

$$= \frac{k}{p^2+1}[1 + e^{-p\pi}]$$

**Application IV**

**Express the following square wave function in term of unit step function and find the Laplace Transform**

$$f(t) = \begin{cases} b, 0 \le t < b \\ -b, b < t < 2b \end{cases}$$

***Solution***:

$b[u(t-0) - u(t-b)] - b[u(t-b) - u(t-2b)$

$$= bL\{u(t)\} - 2bL\{u(t-b)\} + bL\{u(t-2b)\}$$

After solving, we get,

$$\frac{b}{p} - 2be^{-bp}\frac{1}{p} + \frac{1}{p}be^{-2bp}$$

Or,

$$\frac{b}{p} - 2be^{-bp}\frac{1}{p} + \frac{1}{p}be^{-2bp}$$

Or,

$$\frac{b}{p}[1 - 2e^{-bp} + e^{-2bp}]$$

**Application V**

**Express the following full wave rectifier function in term of unit step function and find the Laplace Transform**

$$f(t) = \begin{cases} sint, 0 < t < \pi \\ -sint, -\pi < t < 0 \end{cases}$$

***Solution***:

$$sint[u(t-0) - u(t-\pi)] - sint[u(t+\pi) - u(t-0)]$$
$$= 2sintu(t) - sintu(t-\pi) - sintu(t+\pi)$$
$$= 2L\{sin(t-0)u(t-0)\} - L\{sin(t-\pi+\pi)u(t-\pi)\} - L\{sin(t+\pi-\pi)u(t+\pi)\}$$

After solving, we get,

$$\frac{2}{p^2+1} - \frac{e^{-p\pi}cos\pi}{p^2+1} - \frac{e^{-p\pi}sin\pi}{p^2+1} - \frac{e^{p\pi}cos\pi}{p^2+1} + \frac{e^{p\pi}sin\pi}{p^2+1}$$
$$= \frac{2}{p^2+1}[1 - cos\pi cosp\pi + sin\pi sinp\pi]$$
$$= \frac{2}{p^2+1}[1 - cos\pi(1+p)]$$

**Application VI**

**Express the following function in term of unit step function and find the Laplace Transform**

$$f(t) = \begin{cases} 15, t < 3 \\ 12, t \ge 3 \end{cases}$$

**Solution:**

$$f(t) = \begin{cases} 15+0, t < 3 \\ 15-3, t \ge 3 \end{cases}$$

Or,

$$f(t) = 15 + \begin{cases} 0, t < 3 \\ -3, t \ge 3 \end{cases}$$

Or,

$$f(t) = 15 + (-3)\begin{cases} 0, t < 3 \\ 1, t \ge 3 \end{cases}$$

Or,

$$f(t) = 15 - 3u(t-3)$$

Or,

$L\{f(t)\} = 15L\{1\} - 3L\{u(t-3)\}$

$$L\{f(t)\} = \frac{15}{p} - \frac{3e^{-3p}}{p}$$

**Conclusion:**

This Paper discussed the significance of delayed unit step function for discontinuous functions. We have found the Laplace transform of the discontinuous functions represented in terms of delayed unit step function.

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