Developing a New Mathematical Model for Scheduling Trucks in Cross-Docking Systems

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Abstract: : In this paper, we propose a new mathematical model for Scheduling trucks in cross-docking systems. In the proposed model, the preemption of the trucks is permitted and the outbound organization constraint of them is also existed. It is mentioned that, the present work addresses a scheduling problem of inbound and outbound trailers in a cross-docking in supply chain management. To achieve the best sequence in inbound and outbound trailers and to minimize the operating costs, the proposed model is formulated as a mixed integer programming. In addition, to verify and validate of the model, a test problem is applied and then solved by CPLEX software for small scale instances. Furthermore, computational experiments are carefully designed to illustrate the proposed model and computational results are discussed in detail. Finally, the results show that the effectiveness and good preference of the proposed model and clearly reveal that the model is eligible and confident.
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1. Introduction

In today's distribution environment, with increasing customer demand, logistics service practitioners have tried to meet their requirements and should have the quick response ability in the distribution system. To achieve this purpose, cross-docking has been recognized and also accepted as a powerful tool to speed up the flow of goods and eliminate some major warehouse functions in the supply chain. In order to increase agility and decrease inventory, more logistics companies are turning to cross docking which brings significant benefits including little or no inventory, low handling costs, low space requirement, centralized processing and low transportation costs (Boysen, 2009). The cross docking scheduling model has been first proposed by Chen and Lee from the scheduling points of view (Chen Lee, 2007).

From a detailed examination of the literature, we found that many researchers have proposed some models for cross docking scheduling problem and it has attracted a great attention in both academic research and industrial practice in last decade. The truck scheduling is a major decision in the cross docking problem, since it manages the cross docking operation daily. In the cross docking, the objective of some proposed models for the scheduling problem is minimizing the makespan in order to minimize the operating cost. In the following, the models in the cross docking scheduling problem are reviewed. For example, in the case of scheduling in cross docking system, we can refer to Boysen and Fliedner (2008) which is worked on a particular scheduling problem of trailers in a cross-docking system in food industry in which intermediate storage is forbidden (zero inventory) causing all products have to be unloaded from the incoming trailers and immediately loaded into the outgoing trailers. They modeled this problem with the objective of minimizing flow time, processing time and tardiness of outgoing trailers altogether. In addition, Miao et al. (2007) analyzed an assignment problem with a trailer dock in which the number of trailers and number of docks are unequal. The objective of their work is to minimize two measures simultaneously: shipping cost and the number of shipments remained unfulfilled. They also consider their problem affected by three main factors, arrival and departure times of each truck, shipping time of containers through the docks, and the total obtainable capacity of cross-dock factors, arrival and departure times of each truck, shipping time of containers through the docks, and the total obtainable capacity of cross-dock. Previous literature has provided a number of analytical models with similar structures to that of cross-docking. Yu and Egbelu (2008) developed a scheduling model for single truck in inbound and outbound dock and combine it with the assignment of the products. The model can determine the truck schedule and product allocation simultaneously. The objective of the model is to minimize the total operation time when a temporary storage buffer to hold items temporarily is located at the shipping dock. Their work has been continued by Vahdani and Zandieh (2010) through proposing five meta-heuristic methods to solve and improve the solution obtained from heuristic approach in Yu and Egbelu (2008). Boysen et al. (2010) studied the truck scheduling problem in which one inbound door serves one outbound door. This work is similar with the work of Yu and Egbelu (2008). The problem is modeled as Mixed Integer Programming model and the objective is to minimize the makespan. The model is solved using decomposition approach.

Based on above mentioned literature, some scheduling models of the cross docking have conducted before. However, in the present work, we emphasize a particular type of cross-docking system in which the preemption of the trucks is permitted and the outbound organization constraint of them is also existed. To achieve the best sequence in inbound and outbound doors and to minimize the operating costs, the proposed model is formulated as a mixed integer programming. To verify the model and show the applicably of this model, an example is presented. Then, the CPLEX software is used to sole the model. Finally the results are carefully analyzed. In the following the results are discussed and it revealed that the model is eligible.

The rest of the paper is structured as follows: In section 2, we give some notations for our problem and is dedicated to present a model for Scheduling trucks in cross-docking systems. In Section 3, a test problem is applied to show the superiority and applicably of the proposed model and the computational results are discussed in detail, and finally, section 4 is the conclusions and also some recommendations for future work is presented.

2. Problem definition

In this study, scheduling of the trucks in cross docking system with 2 doors and preemption for inbound trucks to the unloading door, and outbound organization constraint for outbound trucks was presented. Preemption means that the trucks are allowed to preempt each other for unloading. Therefore, each truck refers to the unloading door repeatedly. Also, the purpose of outbound organization constraint is that (some of) outbound trucks shall leave the cross dock within maximum predetermined period of time. It should be mentioned that the unfulfilled products are considered as backorders, delivered to the customer as soon as possible. Hence, it should be noted that this model must be considered only for issues in which the demand of customers can be delayed. Preemption is only considered in the unloading door, and trucks referring to the inbound door leave the system when they either receive the whole of products requested or their predetermined deadline has been reached.

The following assumptions are considered in the proposed model for cross docking problem:

- Trucks come to the unloading door and unload their products;
- Products are transferred to the truck waiting in the loading door through conveyor;
- Trucks leave cross dock after receiving their products;
- In zero time, all trucks are available in the unloading and loading doors;
- Total inbound products to cross dock is equal to the demand of that product;
- The truck changeover time in the unloading door is more than the truck changeover time in the loading door;
- There is only one unloading and loading door in the cross dock, and these doors are separate;
- The time of loading and unloading of each truck has been considered into the changeover time;
- The requirement time for transferring from inbound to the outbound doors are different for products of each truck;
- Products of each truck are classified according to the type of customer, and their replacement is impossible. Therefore, it is unimportant to determine whether each inbound truck to the unloading door includes multi-product or mono-product;
- Penalty considered for each product unfulfilled is different for each type of truck;

The following information is given in the proposed model:

- Amount of product which is exist in the supplier trucks;
- Amount of product that is needed by each customer from suppliers;
- Truck changeover time in the loading and unloading doors;
- Deadline time of trucks by the loading door;

It is assumed that the inbound trucks to the cross dock is not allowed to unload in the cross dock. In other words, the trucks are used as warehouse. It is clear in the supplying chains that transporting corruptible or freezing products and also in the mail industry in which delivery of mails to the customer as soon as possible is very important. Boysen (2009) studied the use of cross docks in the food industry containing freezing products, and analyzed the assumption of failure in unloading in the warehouse as main factor. Since the customers may not demand from all suppliers, the constraints of mathematical model are acceptable only when $S_{ij}>0$, and play role in the model solution. For example, if truck j has no demand from truck i, it is unnecessary for truck i to come to the unloading door.

2.1 Notification

In the IP model, the following notations are used:

n Number of inbound trucks

m Number of outbound trucks

D'	Truck changeover time in outbound doors
D	Truck changeover time in inbound doors
M	Big number
S_{ij}	Number of units of product that outbound truck j needed from inbound truck i
W_{ij}	Number of units of product that transfer from inbound truck i to outbound truck j
T	Moving time of products from the receiving door to the shipping door
tt	Total completion time
RR	Operational cost per unit time
R_i	Penalty cost per unit of unfulfilled product for to truck i
Z	Objective function
T_j	Maximum departure time of truck j, that predetermined
C_{ij}	Time at which inbound truck i enters the receiving dock to serve truck j
F_{ij}	Time at which inbound truck i leaves the receiving dock after serving truck j
D_j	Time at which outbound truck j enters the shipping dock
L_j	Time at which outbound truck j leaves the shipping dock
P_{ijk}	If truck *i* intends to give service to truck *j* and, therefore, preempts truck *k* giving service to truck *l*, it is equal to 1, otherwise it is zero
q_{ij}	If truck *i* comes to the platform after truck *j* in the loading door, it is 1, otherwise it is zero

2.2 Model description

As mentioned before, by regarding the variables, parameters and assumptions that presented above, the mathematical model of problem was obtained as follows:

$$Min\ Z = \left(\left(\sum_{i=1}^{N}\sum_{j=1}^{m}(S_{ij}-W_{ij})\right) * \left(\sum_{i=1}^{n} R_i\right)\right) + (tt * RR)$$

1) $F_i^j \geq C_i^j + \sum_i W_{ij} t_i$

2) $F_i^j \geq d_j + \sum_i W_{ij} t_i$

3) $C_i^j \geq F_k^L + D - M(1 - P_{ijkl}) \quad i \neq j$

4) $C_k^l \geq F_i^j + D' - M\, P_{ijkl} \quad i \neq j$

5) $C_i^j \geq F_k^L - M(1 - P_{ijkl}) \quad i = j, k \neq l$

6) $C_k^l \geq F_i^j - M\, P_{ijkl} \quad i = j, k \neq l$

7) $d_i \geq L_j + D - M(1 - q_{ij}) \quad i \neq j$

8) $d_j \geq L_i + D' - M\, q_{ij} \quad i \neq j$

9) $L_j \geq d_j + \sum_{i=1}^{N} W_{ij}$

10) $L_j \geq F_i^j$

11) $W_{ij} \leq S_{ij}$

12) $S_{ij} \leq M * W_{ij}$

13) $L_j \leq T_j$

14) $tt \geq L_j$

The purpose of this model is to present a scheduling model for succession of inbound and outbound trucks to the cross dock in order to minimize the target function that is including the minimizing of total costs. Total cost includes two parts: 1) operational cost which is related to total time (tt*RR); 2) penalty cost which is related to shipments which must be transferred from truck i to truck j but this transfer didn't occur due to time constraint. In fact, the purpose is that Wij achieves Sij in the shortest time.

Constraints 1 to 7, shows the succession of inbound and outbound trucks to the CD doors. The two first constraints ensure that, upon entering inbound truck to the unloading door, truck receiving goods therein is exist in the loading door, and inbound truck to the unloading door leaves the system when the truck exist in the loading door has been yet received its demand from that truck. Constraints 3 and 4 are related to the succession of two trucks in the loading door in which i preempts k. Thus, if truck i (giving service to truck j) enters unloading door immediately after truck k (giving service to truck l), truck i preempts truck k, and D time is necessary for such displacement.

Two constraints 5 and 6 are the same as 3 and 4, supposing k=1 when there is surely no preemption, and that truck unloading in the loading door and offering service to truck j continues but gives service to truck l. Therefore, there is no need of displacement time (D). Constraints 6 and 7 perform the task of constraints 3 and 4 for unloading door, showing truck i comes to unloading door after truck j, and D' time is necessary for such displacement. Constraint 9 ensures that the inbound trucks to the unloading door don't leave there till they have not received from trucks of loading door their all possible required products. Constraint 10 shows that as long as the truck i giving service to truck j doesn't leave loading door, truck j does not go out from loading door. Constraint 11 means that maximum shipments transferred from truck i to j is equal to the numbers needed by truck j from i.

Constraint 12 ensures that in case the truck j requires product from i, at least 1 unit of this product is transferred to it, and there is indeed no customer whose demand is not supplied. constraint 13 is related to the unloading time constraint, ensuring the unloading trucks leave the unloading door up to predetermined deadline Tj. constraint 14 represents total completion time is more than or equal to unloading time of the last truck from unloading door. In the next section, an example to verify the model is presented.

3. Computational results

To solve the cross docking problem, there are different solution approaches. In this study, to solve the proposed model, the CPLEX software is utilized. The following test problem is also used to test the performance of the mentioned model. We suppose the following information is available for an example in the real world:

n =4 , m=5 , D=2 , D′=1 , M=300 , Ri=[1 2 1 3] , RR=1 , t=[2 3 1 2] , Tj=[20 110 73 185 45].

$$S_{ij} = \begin{bmatrix} 9 & 0 & 4 & 8 & 8 \\ 0 & 3 & 0 & 10 & 0 \\ 8 & 6 & 11 & 0 & 3 \\ 0 & 5 & 4 & 8 & 6 \end{bmatrix}$$

After solving model using CPLEX software, the answerers of the problem are as follows:

$$C_{ij} = \begin{bmatrix} 0 & 0 & 51 & 135 & 24 \\ 0 & 93 & 0 & 103 & 0 \\ 10 & 73 & 61 & 0 & 19 \\ 0 & 81 & 45 & 153 & 40 \end{bmatrix}$$

$$F_{ij} = \begin{bmatrix} 8 & 0 & 59 & 151 & 38 \\ 0 & 102 & 0 & 133 & 0 \\ 18 & 79 & 72 & 0 & 22 \\ 0 & 91 & 49 & 169 & 44 \end{bmatrix}$$

$$W_{ij} = \begin{bmatrix} 4 & 0 & 4 & 8 & 7 \\ 0 & 3 & 0 & 10 & 0 \\ 8 & 6 & 11 & 0 & 3 \\ 0 & 5 & 2 & 8 & 2 \end{bmatrix}$$

D_j = [0 73 45 103 19] ; L_j = [18 102 72 169 44] ; tt = 169 ; Z = 253.

According to the obtained results, succession of inbound and outbound trucks in the cross dock is as follows:

In time of zero, inbound truck 1 enters unloading door and outbound truck 1 enters loading door (C11= 0 and d1= 0). Four units are transferred from truck 1 to 1 (W11= 8). Pay attention that the necessary time for displacement of each unit of inbound product type 1 is 2 time units. Since preemption is allowed in the unloading door, inbound truck 3 preempts truck 1 and enters unloading door (C31 = 10) and truck 1 moves out through unloading door (F31 = 18). In this case, 2 units of time is used for displacement of trucks 1 and 3 (D=2). But there is no change in loading door and outbound truck 1 is yet waiting for receiving service. 8 units of products are loaded from truck 3 to 1 (W31= 8). Outbound truck 1 has no demand from other trucks, receiving its needed demand and moving out of loading door (L1= 18). After passing necessary time for displacement in the loading door, 1 time unit (D'= 1), truck 5 enters loading door (d5= 19) and takes service from inbound truck 3 by the unloading door.

Consider that according to the acquired results, we have F31= 18 and C35= 19: in the unloading door, truck 3 exits in time 18 and enters again in time 19 but it is impossible in practice. In fact, truck 3 never exit the unloading door in time 18 as it remains there till it gives service to the next inbound truck in loading door. The time interval of 18 to 19 is used for displacement in the loading door. 3 units of products are transferred from truck 3 to 5 (W35=3), then truck 3 exits the unloading door (F35=22), and truck 1 enters again the unloading door (C15=24) to give service to truck 5, which transfers 7 units of product to truck 5 by the loading door. So that loading and unloading are performed in this cross dock till time 153 when inbound truck 4 enters unloading door for giving service to outbound truck 4 (C44=153). This truck transports 8 units of product to outbound truck 4 (W44=8), and leaves loading door in time 169 (F44=169). In this time, outbound truck 4 leaves the unloading door too. This time is equal to total time passed in the cross dock (tt= 169).

4. Conclusions and future work

Regarding on the facility and operating requirements that are appropriate, it is possible to develop various cross docking models. However, various models have been developed in the scheduling of the cross docking problems. In this study, a model for scheduling trucks in cross docking systems is presented. Two assumptions including preemption for inbound trucks and outbound organization constraint for outbound trucks are considered, simultaneously. The objective of the proposed model was to minimize the total cost regarding the assumptions and constraints of the problem. Total cost is including the penalty cost per unit of unfulfilled product for truck i and operational cost per unit time. To find the best truck docking sequences for both the inbound and the outbound trucks to minimize total cross-docking operation time of the cross-docking process, the proposed model is formulated as a mixed integer programming. In fact, our main contribution is that we formulate it as mix integer programming model. To solve the model, the CPLEX software is utilized in small-size of the problem. Then, for verification and validation and to test the effectiveness of the model, a test problem is applied and the computational results are discussed in detail. The results show that the proposed model is eligible and confident. For some small-size instances, the proposed model can be solved to optimality by the CPLEX Solver within a reasonable time. It should be noted that, there are some limitations for the considered issue of this paper which could possibly be an opportunity to consider for future research. For future research some tasks can be considered. It should be mentioned that, the model in the medium-size and large size is NP-Hard. Therefore, to solve the problem in the large size the meta-heuristic approaches such as genetic algorithm and simulated annealing can be applied. In addition, the results of this study can be compared and analyzed with the future study.

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