Explicit Solution Of Newton's Law Of Cooling Via Dvt

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Abstract: The Dinesh Verma transformation is a mathematical tool which is used in the solving of differential equations by converting it from one from in to another from. Regularly it is effective in solving linear differential equations either ordinary or partial. The Dinesh Verma transformation is used in solving the time domain function by converting it into frequency domain function. Dinesh Verma transformation makes it easier to solve the differential problem in engineering application and make differential equations simple to solve. The Newton's Law of Cooling stands up in the field of Physics. The Purpose of this paper, Dinesh Verma Transform for solving problems on Newton's Law of Cooling and an example is given in order to prove the success of Dinesh Verma Transform for solving the problems on Newton's Law of Cooling.

[Dr. Dinesh Verma. Dr. Govind Raj Naunya. Dr. Amit Pal Singh. **Explicit Solution Of Newton's Law Of Cooling Via Dvt**. *Researcher* 2023;15(4):37-41]. ISSN 1553-9865 (print); ISSN 2163-8950 (online). http://www.sciencepub.net/researcher.08.doi:10.7537/marsrsj150423.08.

Keywords: Dinesh Verma Transform, Inverse Dinesh Verma Transform , Newton's Law of Cooling, Temperature of environment, Temperature of body.

Introduction: The Dinesh Verma Transform (DVT) has been applied in different areas of science, engineering and technology [1], [2], [7], [9]. The Dinesh Verma Transform (DVT) is applicable in so many fields and effectively solving linear differential equations. Ordinary linear differential equation with constant coefficient and variable coefficient can be easily solved by the Dinesh Verma Transform (DVT) without finding their general solutions [10], [11], [12], [13], [14], [15], [14], [15], [16], [21], Newton's Law of Cooling is called an ordinary differential equation that expects the cooling of a warm body sited in a cold environment [3], [4], [5], [6]. In this law, the rate at which the temperature of the body decreases is proportional to the difference of temperature between the body and its environment [17], [18], [19], [20], [22].

$T' = -k(T - T_e)$ ....(I)

with initial condition as $T(t_0) = T_0$ ....(II)

Where , T is the temperature of the object

T_e is the constant temperature of the environment,

k is the constant of proportionality,

T_0 is the initial temperature of the object at time t_0

The negative sign of RHS in (1), indicate temperature of the body is decreasing with time and so the derivative $\frac{dT}{dt}$ must be negative.

Basic definition: Definition of dinesh verma transform (DVT)

Dr. Dinesh Verma recently introduced a novel transform and named it as Dinesh Verma Transform (DVT). Let f(t) is a well-defined function of real numbers t ≥ 0. The Dinesh Verma Transform (DVT) of f(t), denoted by $D\{$ {f(t)}, is defined as [1]

$$D\{ \{f(t)\} = p^5 \int_0^\infty e^{-pt} f(t) dt = \bar{f}(p)$$

Provided that the integral is convergent, where $p$may be a real or complex parameter and D is the Dinesh Verma Transform (DVT) operator.
Dinesh verma transform of elementary functions:

According to the definition of Dinesh Verma transform (DVT),

$$\mathrm{D}\{t^n\} = p^5 \int_0^\infty e^{-pt}\, t^n dt$$
$$= p^5 \int_0^\infty e^{-z} \left(\frac{z}{p}\right)^n \frac{dz}{p} \quad , z = pt$$
$$= \frac{p^5}{p^{n+1}} \int_0^\infty e^{-z}\, (z)^n dz$$

Applying the definition of gamma function,

$$\mathrm{D}\,\{y^n\} = \frac{p^5}{p^{n+1}} \Gamma(n+1)$$
$$= \frac{1}{p^{n-4}} n!$$
$$= \frac{n!}{p^{n-4}}$$

Hence, $D\{t^n\} = \frac{\boldsymbol{n!}}{\boldsymbol{p^{n-4}}}$

Dinesh Verma Transform (DVT) of some elementary Functions

- $D\{t^n\} = \frac{n!}{p^{n-4}}\ , where\ n = 0,1,2, ..$
- $D\{e^{at}\} = \frac{p^5}{p-a},$
- $D\{sinat\} = \frac{ap^5}{p^2+a^2},$
- $D\{cosat\} = \frac{p^6}{p^2+a^2},$
- $D\{sinhat\} = \frac{ap^5}{p^2-a^2},$
- $D\{coshat\} = \frac{p^6}{p^2-a^2}.$
- $D\{\delta(t)\} = p^5$

The Inverse Dinesh Verma Transform (DVT) of some of the functions are given by

- $D^{-1}\left\{\frac{1}{p^{n-4}}\right\} = \frac{t^n}{n!}\ , where\ n = 0,1,2, ..$
- $D^{-1}\left\{\frac{p^5}{p-a}\right\} = e^{at}\ ,$
- $D^{-1}\left\{\frac{p^5}{p^2+a^2}\right\} = \frac{sinat}{a},$
- $D^{-1}\left\{\frac{p^6}{p^2+a^2}\right\} = cosat\ ,$
- $D^{-1}\left\{\frac{p^5}{p^2-a^2}\right\} = \frac{sinhat}{a},$
- $D^{-1}\left\{\frac{p^6}{p^2-a^2}\right\} = coshat,$
- $D^{-1}\{p^5\} = \delta(t)$

Dinesh verma transform (DVT) of derivatives [1], [2], [10], [21].

$$D\{f'(t)\} = p\bar{f}(p) - p^5 f(0)$$
$$D\{f''(t)\} = p^2\bar{f}(p) - p^6 f(0) - p^5 f'(0)$$
$$D\{f'''(y)\} = p^3\bar{f}(p) - p^7 f(0) - p^6 f'(0) - p^5 f''(0) \text{And so on.}$$
$$D\{tf(t)\} = \frac{5}{p}\bar{f}(p) - \frac{d\bar{f}(p)}{dp},$$
$$D\{tf'(\mathrm{t})\} = \frac{5}{p}\left[p\bar{f}(p) - p^5\mathrm{f}(0)\right] - \frac{d}{dp}\left[p\bar{f}(p) - p^5\mathrm{f}(0)\right] \text{ and}$$

$$D\{tf''(t)\} = \frac{5}{p}[p^2\bar{x}(p) - p^6x(0) - p^5x'(0)] - \frac{d}{dp}[p^2\bar{x}(p) - p^6x(0) - p^5x'(0)]$$ And so on.

Methodology:

From (I), $$T' = -k(T - T_e)$$

Taking Dinesh Verma Transform on both sides,

$$D\{T'\} = -D\{k(T - T_e)\}$$
$$D\{T'\} = -kD\{T(t)\} + kD\{T_e\}$$
$$pD\{T(t)\} - p^5T(0) = -kD\{T(t)\} + kT_eD\{1\}$$

From (II), As $T(t_0) = T_0$

Now, $$pD\{T(t)\} - p^5T_0 = -kD\{T(t)\} + kT_ep^4$$
$$(p+k)D\{T(t)\} = p^5T_0 + kT_ep^4$$
$$D\{T(t)\} = \frac{p^5T_0}{(p+k)} + kT_e\frac{p^4}{(p+k)}$$

Taking inverse DVT, $$\{T(t)\} = T_0e^{-kt} + kT_e - kT_ee^{-kt}$$
$$T(t) = (T_0 - kT_e)e^{-kt} + kT_e$$
$$T(t) = Ce^{-kt} + kT_e \ldots\ldots\ldots\ldots\ldots (III)$$
$$where, C = (T_0 - kT_e)$$

While this function decreases exponentially, it approaches T_e as t→ ∞ instead of zero.

Application:

An apple pie with an initial temperature of 170^0 C is removed from the oven and left to cool with an air temperature 20^0C. Given that the temperature of the pie initially decreases at a rate of 3.0^0 C/min. How long will it take for the pie to cool to a temperature of 30^0 C? . [22]

$$-3 = -k(170 - 20)$$
$$k = 0.02$$

So, the differential equation can be written as

$$T' = -\frac{1}{50}(T - 20)$$

Taking DVT on both sides,

$$D\{T'\} = -\frac{1}{50}D\{(T(t) - 20)\}$$
$$D\{T'\} = -\frac{1}{50}D\{T(t)\} + \frac{2}{5}D\{1\}$$
$$pD\{T(t)\} - p^5T(0) = -\frac{1}{50}D\{T(t)\} + \frac{2}{5}p^4$$
$$pD\{T(t)\} - p^5T(0) = -\frac{1}{50}D\{T(t)\} + \frac{2}{5}p^4$$
$$\left(p + \frac{1}{50}\right)D\{T(t)\} = p^5T(0) + \frac{2}{5}p^4$$
$$\left(p + \frac{1}{50}\right)D\{T(t)\} = 170p^5 + \frac{2}{5}p^4$$
$$D\{T(t)\} = \frac{170p^5}{\left(p + \frac{1}{50}\right)} + \frac{2p^4}{5\left(p + \frac{1}{50}\right)}$$

Suppose the pie is in compliance with newton's cooling law; we have the following information

$$T' = -k(T - 20), T(0) = 170\ , T'(0) = -3.0$$

Where, T is the temperature of the pie in degree Celsius, T' is the time in minutes and k is an unknown constant.

Now, we will find the value of k by putting the given information we know about t = 0 directly into the differential equation:

$$D\{T(t)\} = \frac{170p^5}{\left(p + \frac{1}{50}\right)} - \frac{20p^5}{\left(p + \frac{1}{50}\right)} + 20p^4$$
$$D\{T(t)\} = \frac{150p^5}{\left(p + \frac{1}{50}\right)} + 20p^4$$

Taking inverse DVT on both sides, we get,

$$T(t) = 150e^{-\frac{1}{50}t} + 20\ \ldots\ldots\ldots\ldots\ldots\ldots .(IV)$$

Putting T=30 in (IV), $30 = 150e^{-\frac{1}{50}t} + 20$

$$e^{-\frac{1}{50}t} = \frac{1}{15}$$
$$e^{\frac{1}{50}t} = 15$$
$$\frac{1}{50}t = \ln 15$$
$$t = 50\ln 15$$
$$t = 50 * 2.7080502011$$
$$t = 135.4\ minute$$

Hence, this will require 135.4 minutes for the pie to cool to a temperature of 30^0 C.

Conclusion:

In this paper, we have successfully developed the Dinesh Verma Transform to solve problems related to Newton's Law of Cooling. The applications presented demonstrate effectiveness of Dinesh Verma Transform in the problems of Newton's Law of Cooling. The proposed scheme is widely in various field of Physics, Electrical engineering, Control engineering, Economics, Mathematics, Signal processing and Electronics engineering.

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3/11/2023