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The New Prime theorems (1541)—（1590）

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Abstract

In this paper using Jiang function $J_2(\omega)$ we prove that the new prime theorems (1541)-(1590) contain infinitely many prime solutions and no prime solutions. From (6) we are able to find the smallest solution. $\pi_k(N_0,2)\geq 1$. This is the Book theorem。It is the greatest mathematical discovery that was ever made.

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It will be another million years at least, before we understand the primes.

Paul Erdos (1913-1996)

The New Prime Theorem (1541)

$$p, jp^{3022}+k-j(j=1,2,...,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3022}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3022}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3022}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3022}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3022}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3022)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N},\quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,3023$

From (2) and (3) we have

$$J_2(\omega)=0.\quad (7)$$

We prove that for $k=3,3023$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,3023$

From (2) and (3) we have

$$J_2(\omega)\ne 0.\quad (8)$$

We prove that for $k\ne 3,3023$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1542)

$$p, jp^{3024}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3024}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3024}+k-j(j=1,\cdots,k-1)\quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)].\quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3024}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1.\quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0\quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3024}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0\quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3024}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3024)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N},\quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,17,19,29,37,43,73,109,127,433,757$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,7,13,17,19,29,37,43,73,109,127,433,757$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,7,13,17,19,29,37,43,73,109,127,433,757$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,5,7,13,17,19,29,37,43,73,109,127,433,757$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1543)

$$p, jp^{3026}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3026}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3026}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3026}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3026}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3026}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3026)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,179$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,179$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,179$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,179$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1544)

$$p, jp^{3028}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3028}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3028}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3028}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3028}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\leq N: jp^{3028}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3028)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,5$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k \neq 3,5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1545)

$$p, jp^{3030}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3030}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3030}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3030}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3030}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\leq N: jp^{3030}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3030)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,7,11,31,607$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,7,11,31,607$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,7,11,31,607$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,7,11,31,607$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1546)

$$p, jp^{3032}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3032}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3032}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3032}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3032}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3032}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3032)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1547)

$$p, jp^{3034}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3034}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3034}+k-j(j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3034}+k-j)\equiv 0 \pmod p, q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3034}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3034}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3034)^{k-1}\varphi^k(\omega)}\frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,83$

From (2) and (3) we have

$$J_2(\omega)=0. \qquad (7)$$

We prove that for $k=3,83$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,83$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \qquad (8)$$

We prove that for $k\ne 3,83$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1548)

$$p, jp^{3036}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3036}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3036}+k-j(j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3036}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3036}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3036}+k-j=\text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3036)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,23,47,67,139,277,3037$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,7,13,23,47,67,139,277,3037$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,7,13,23,47,67,139,277,3037$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,5,7,13,23,47,67,139,277,3037$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1549)

$$p, jp^{3038}+k-j (j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3038}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3038}+k-j (j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3038}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3038}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\leq N: jp^{3038}+k-j=\text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3038)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega)=\prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3$

From (2) and (3) we have

$$J_2(\omega)=0. \qquad (7)$$

We prove that for $k=3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \qquad (8)$$

We prove that for $k\neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1550)

$$p, jp^{3040}+k-j (j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang

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Abstract

Using Jiang function we prove that $jp^{3040}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3040}+k-j (j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3040}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3040}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jp^{3040}+k-j=\text{prime}\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3040)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,5,11,17,41,191,761,3041$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,11,17,41,191,761,3041$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,11,17,41,191,761,3041$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,5,11,17,41,191,761,3041$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1551)

$$p, jp^{3042}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3042}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3042}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3042}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3042}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \le N : jp^{3042} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3042)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 7, 19, 79$

From (2) and (3) we have

$$J_2(\omega) = 0. \quad (7)$$

We prove that for $k = 3, 7, 19, 79$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 7, 19, 79$

From (2) and (3) we have

$$J_2(\omega) \ne 0. \quad (8)$$

We prove that for $k \ne 3, 7, 19, 79$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1552)

$$p, jp^{3044} + k - j (j = 1, 2, \cdots, k-1), p > k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3044} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3044} + k - j (j = 1, \cdots, k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{3044} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \quad (3)$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3044} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \le N : jp^{3044} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3044)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,5,1523$

From (2) and (3) we have

$$J_2(\omega)=0. \qquad (7)$$

We prove that for $k=3,5,1523$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,1523$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \qquad (8)$$

We prove that for $k\neq 3,5,1523$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1553)

$$p, jp^{3046}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3046}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3046}+k-j(j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3046}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3046}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\leq N: jp^{3046}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3046)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3$

From (2) and (3) we have

$$J_2(\omega)=0. \qquad (7)$$

We prove that for $k=3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \quad (8)$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1554)

$$p, jp^{3048}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3048}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3048}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3048}+k-j) \equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P) \leq P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3048}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P \leq N: jp^{3048}+k-j=\text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3048)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \geq 1$.

Example 1. Let $k=3,5,7,13,509,3049$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,7,13,509,3049$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,5,7,13,509,3049$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \quad (8)$$

We prove that for $k \neq 3,5,7,13,509,3049$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1555)

$$p, jp^{3050}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3050}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3050}+k-j(j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3050}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3050}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\leq N: jp^{3050}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3050)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,11$

From (2) and (3) we have

$$J_2(\omega)=0. \qquad (7)$$

We prove that for $k=3,11$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,11$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \qquad (8)$$

We prove that for $k\neq 3,11$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1556)

$$p, jp^{3052}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang

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Abstract

Using Jiang function we prove that $jp^{3052}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3052}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3052}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3052}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3052}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3052)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,29$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,29$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,29$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,5,29$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1557)

$p, jp^{3054}+k-j(j=1,2,\cdots,k-1), p>k$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3054}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3054}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3054}+k-j)\equiv 0 \pmod p, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3054}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3054}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3054)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,1019$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,7,1019$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,7,1019$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,7,1019$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1558)

$$p, jp^{3056}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3056}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3056}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3056}+k-j)\equiv 0 \pmod p, q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3056}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3056}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3056)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,17,383$

From (2) and (3) we have

$$J_2(\omega)=0. \qquad (7)$$

We prove that for $k=3,5,17,383$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,17,383$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \qquad (8)$$

We prove that for $k\neq 3,5,17,383$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1559)

$$p, jp^{3058}+k-j (j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3058}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3058}+k-j (j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3058}+k-j)\equiv 0 \pmod p, q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3058} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \leq N : jp^{3058} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3058)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 23$

From (2) and (3) we have

$$J_2(\omega) = 0. \quad (7)$$

We prove that for $k = 3, 23$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 23$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \quad (8)$$

We prove that for $k \neq 3, 23$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1560)

$$p, jp^{3060} + k - j (j = 1,2,\cdots,k-1), p > k$$

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Abstract

Using Jiang function we prove that $jp^{3060} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3060} + k - j (j = 1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{3060} + k - j) \equiv 0 \pmod{p}, q = 1,\cdots,p-1. \quad (3)$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3060} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3060}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3060)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,11,13,19,31,37,61,103,181,307,613,1531,3061$

From (2) and (3) we have

$$J_2(\omega)=0. \qquad (7)$$

We prove that for $k=3,5,7,11,13,19,31,37,61,103,181,307,613,1531,3061$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,7,11,13,19,31,37,61,103,181,307,613,1531,3061$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \qquad (8)$$

We prove that for $k\neq 3,5,7,11,13,19,31,37,61,103,181,307,613,1531,3061$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1561)

$p, jp^{3062}+k-j(j=1,2,\cdots,k-1), p>k$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3062}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3062}+k-j(j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3062}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3062}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \le N : jp^{3062} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3062)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \quad (7)$$

We prove that for $k = 3$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3$

From (2) and (3) we have

$$J_2(\omega) \ne 0. \quad (8)$$

We prove that for $k \ne 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1562)

$$p, jp^{3064} + k - j (j = 1,2,\cdots,k-1), p > k$$

Chun-Xuan Jiang

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Abstract

Using Jiang function we prove that $jp^{3064} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3064} + k - j (j = 1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P - 1 - \chi(P)]. \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3064} + k - j) \equiv 0 \pmod{p}, q = 1,\cdots,p-1. \quad (3)$$

If $\chi(P) \le P - 2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3064} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \le N : jp^{3064} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3064)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k=3,5,7,13$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,7,13$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,5,7,13$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k \neq 3,5,7,13$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1563)

$$p, jp^{3066}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3066}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3066}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3066}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3066}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\leq N: jp^{3066}+k-j=\text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3066)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,7,43,439,3067$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,7,43,439,3067$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,7,43,439,3067$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \qquad (8)$$

We prove that for $k \neq 3,7,43,439,3067$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1564)

$$p, jp^{3068} + k - j (j = 1,2,\cdots,k-1), p > k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3068} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3068} + k - j (j = 1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P - 1 - \chi(P)]. \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3068} + k - j) \equiv 0 \pmod{p}, q = 1,\cdots,p-1. \qquad (3)$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3068} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \leq N : jp^{3068} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3068)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \geq 1$.

Example 1. Let $k = 3,5,53$

From (2) and (3) we have

$$J_2(\omega) = 0. \qquad (7)$$

We prove that for $k = 3,5,53$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,5,53$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \qquad (8)$$

We prove that for $k \neq 3,5,53$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1565)

$$p, jp^{3070}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract
Using Jiang function we prove that $jp^{3070}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3070}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3070}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3070}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3070}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3070)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,11$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,11$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,11$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,11$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1566)

$$p, jp^{3072}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3072}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3072}+k-j (j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3072}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3072}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3072}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3072)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,17,97,193,257,769$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,7,13,17,97,193,257,769$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,7,13,17,97,193,257,769$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,5,7,13,17,97,193,257,769$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1567)

$$p, jp^{3074}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3074}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3074}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3074}+k-j) \equiv 0 \pmod{p}, q = 1,\cdots,p-1. \qquad (3)$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3074}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \le N : jp^{3074}+k-j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3074)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \ge 1$.

Example 1. Let $k = 3,59,107$

From (2) and (3) we have

$$J_2(\omega) = 0. \qquad (7)$$

We prove that for $k = 3,59,107$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3,59,107$

From (2) and (3) we have

$$J_2(\omega) \ne 0. \qquad (8)$$

We prove that for $k \ne 3,59,107$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1568)

$$p, jp^{3076}+k-j (j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3076}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3076}+k-j (j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3076}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3076}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3076}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3076)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1569)

$$p, jp^{3078}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3078}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3078}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3078}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3078}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3078}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3078)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,19,163,487,3079$

From (2) and (3) we have

$$J_2(\omega)=0. \qquad (7)$$

We prove that for $k=3,7,19,163,487,3079$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,7,19,163,487,3079$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \qquad (8)$$

We prove that for $k\neq 3,7,19,163,487,3079$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1570)

$$p, jp^{3080}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3080}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3080}+k-j(j=1,\cdots,k-1) \qquad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3080}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3080}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3080}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3080)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,11,29,41,71,89,617$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,11,29,41,71,89,617$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,11,29,41,71,89,617$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,5,11,29,41,71,89,617$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1571)

$$p, jp^{3082}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3082}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3082}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3082}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3082}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3082}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3082)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_{P}(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,47,3083$

From (2) and (3) we have

$$J_2(\omega)=0. \tag{7}$$

We prove that for $k=3,47,3083$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,47,3083$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \tag{8}$$

We prove that for $k\ne 3,47,3083$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1572)

$$p, jp^{3084}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3084}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3084}+k-j(j=1,\cdots,k-1) \tag{1}$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \tag{2}$$

where $\omega=\prod_{P}P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3084}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3084}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3084}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3084)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \tag{6}$$

where $\phi(\omega)=\prod_{P}(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,1543$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,7,13,1543$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,7,13,1543$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,5,7,13,1543$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1573)

$$p, jp^{3086}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3086}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3086}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3086}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3086}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\leq N: jp^{3086}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3086)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \quad (8)$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1574)

$$p, jp^{3088} + k - j (j = 1, 2, \cdots, k-1), p > k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3088} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3088} + k - j (j = 1, \cdots, k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{3088} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \quad (3)$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3088} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N, 2) = \left| \{ P \leq N : jp^{3088} + k - j = \text{prime} \} \right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3088)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3, 5, 17, 773, 3089$

From (2) and (3) we have

$$J_2(\omega) = 0. \quad (7)$$

We prove that for $k = 3, 5, 17, 773, 3089$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3, 5, 17, 773, 3089$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \quad (8)$$

We prove that for $k \neq 3, 5, 17, 773, 3089$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1575)

$$p, jp^{3090} + k - j (j = 1, 2, \cdots, k-1), p > k$$

Chun-Xuan Jiang
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Abstract
Using Jiang function we prove that $jp^{3090}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3090}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3090}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3090}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3090}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3090)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,11,31,619$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,7,11,31,619$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,7,11,31,619$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,7,11,31,619$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1576)

$$p, jp^{3092}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract
Using Jiang function we prove that $jp^{3092}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3092}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3092}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3092}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jp^{3092}+k-j=\text{prime}\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3092)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1577)

$$p, jp^{3094}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3094}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3094}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3094}+k-j)\equiv 0 \pmod p, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3094}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3094}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3094)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,239,443$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,239,443$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,239,443$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,239,443$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1578)

$$p, jp^{3096}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3096}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3096}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3096}+k-j)\equiv 0 \pmod p, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3096}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \le N : jp^{3096}+k-j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3096)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \ge 1$.

Example 1. Let $k = 3,5,7,13,19,37,73,173,1033,1549$

From (2) and (3) we have

$$J_2(\omega) = 0. \quad (7)$$

We prove that for $k = 3,5,7,13,19,37,73,173,1033,1549$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3,5,7,13,19,37,73,173,1033,1549$

From (2) and (3) we have

$$J_2(\omega) \ne 0. \quad (8)$$

We prove that for $k \ne 3,5,7,13,19,37,73,173,1033,1549$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1579)

$$p, jp^{3098}+k-j (j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3098}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3098}+k-j (j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3098}+k-j) \equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3098}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \le N : jp^{3098} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3098)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0. \quad (7)$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \quad (8)$$

We prove that for $k \neq 3$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1580)

$$p, jp^{3100} + k - j (j = 1, 2, \cdots, k-1), p > k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3100} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3100} + k - j (j = 1, \cdots, k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2} [P - 1 - \chi(P)]. \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq^{3100} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \quad (3)$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3100} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3100}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3100)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N},\quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,11,101,311$

From (2) and (3) we have

$$J_2(\omega)=0.\quad (7)$$

We prove that for $k=3,5,11,101,311$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,11,101,311$

From (2) and (3) we have

$$J_2(\omega)\ne 0.\quad (8)$$

We prove that for $k\ne 3,5,11,101,311$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1581)

$$p, jp^{3102}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3102}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3102}+k-j(j=1,\cdots,k-1)\quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)].\quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3102}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1.\quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0\quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3102}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0\quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3102}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3102)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N},\quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,23,67,283$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,7,23,67,283$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,7,23,67,283$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,7,23,67,283$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1582)

$$p, jp^{3104}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3104}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3104}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3104}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3104}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3104}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3104)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,17,289,1553$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k = 3,5,17,289,1553$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,5,17,289,1553$

From (2) and (3) we have

$$J_2(\omega) \neq 0 . \quad (8)$$

We prove that for $k \neq 3,5,17,289,1553$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1583)

$$p, jp^{3106} + k - j(j = 1,2,\cdots,k-1), p > k$$

Chun-Xuan Jiang

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Abstract

Using Jiang function we prove that $jp^{3106} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3106} + k - j(j = 1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P - 1 - \chi(P)] . \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3106} + k - j) \equiv 0 \pmod{p}, q = 1,\cdots,p-1 . \quad (3)$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3106} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\left\{P \leq N : jp^{3106} + k - j = \text{prime}\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3106)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P - 1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3$

From (2) and (3) we have

$$J_2(\omega) = 0 . \quad (7)$$

We prove that for $k = 3$

（1）contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$

From (2) and (3) we have

$$J_2(\omega) \neq 0 . \quad (8)$$

We prove that for $k \neq 3$
(1) contain infinitely many prime solutions.

The New Prime Theorem (1584)

$$p, jp^{3108} + k - j (j = 1,2,\cdots,k-1), p > k$$

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Abstract

Using Jiang function we prove that $jp^{3108} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3108} + k - j (j = 1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P - 1 - \chi(P)]. \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3108} + k - j) \equiv 0 \pmod{p}, q = 1,\cdots,p-1. \quad (3)$$

If $\chi(P) \leq P - 2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3108} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P - 1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \leq N : jp^{3108} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3108)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_P (P - 1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \geq 1$.

Example 1. Let $k = 3,5,7,13,29,149,223,3109$

From (2) and (3) we have

$$J_2(\omega) = 0. \quad (7)$$

We prove that for $k = 3,5,7,13,29,149,223,3109$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,5,7,13,29,149,223,3109$

From (2) and (3) we have

$$J_2(\omega) \neq 0. \quad (8)$$

We prove that for $k \neq 3,5,7,13,29,149,223,3109$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1585)

$$p, jp^{3110} + k - j (j = 1,2,\cdots,k-1), p > k$$

Chun-Xuan Jiang

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Abstract

Using Jiang function we prove that $jp^{3110}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3110}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3110}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3110}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3110}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3110)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,11$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,11$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,11$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,11$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1586)

$$p, jp^{3112}+k-j(j=1,2,\cdots,k-1), p>k$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jp^{3112}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3112}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3112}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3112}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3112}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3112)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,5$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1587)

$p, jp^{3114}+k-j(j=1,2,\cdots,k-1), p>k$

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Abstract

Using Jiang function we prove that $jp^{3114}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3114}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega = \prod_{P} P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3114} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \quad (3)$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3114} + k - j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\{P \le N : jp^{3114} + k - j = \text{prime}\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3114)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega) = \prod_{P}(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0, 2) \ge 1$.

Example 1. Let $k = 3, 7, 19, 347, 1039$

From (2) and (3) we have

$$J_2(\omega) = 0. \quad (7)$$

We prove that for $k = 3, 7, 19, 347, 1039$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3, 7, 19, 347, 1039$

From (2) and (3) we have

$$J_2(\omega) \ne 0. \quad (8)$$

We prove that for $k \ne 3, 7, 19, 347, 1039$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1588)

$$p, jp^{3116} + k - j (j = 1, 2, \cdots, k-1), p > k$$

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Abstract

Using Jiang function we prove that $jp^{3116} + k - j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3116} + k - j (j = 1, \cdots, k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P - 1 - \chi(P)]. \quad (2)$$

where $\omega = \prod_{P} P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3116} + k - j) \equiv 0 \pmod{p}, q = 1, \cdots, p-1. \quad (3)$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3116}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jp^{3116}+k-j=\text{prime}\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(3116)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,5,83,1559$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,83,1559$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,83,1559$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,5,83,1559$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1589)

$$p, jp^{3118}+k-j(j=1,2,\cdots,k-1), p>k$$

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Abstract

Using Jiang function we prove that $jp^{3118}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3118}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3188}+k-j)\equiv 0 \pmod{p}, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3188}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jp^{3188}+k-j=\text{prime}\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3188)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,3119$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,3119$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,3119$

From (2) and (3) we have

$$J_2(\omega)\neq 0. \quad (8)$$

We prove that for $k\neq 3,3119$

(1) contain infinitely many prime solutions.

The New Prime Theorem (1590)

$p, jp^{3120}+k-j(j=1,2,\cdots,k-1), p>k$

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Abstract

Using Jiang function we prove that $jp^{3120}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$p, jp^{3120}+k-j(j=1,\cdots,k-1) \quad (1)$$

Contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]. \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}(jq^{3120}+k-j)\equiv 0 \pmod p, q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{3120}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jp^{3120}+k-j=\text{prime}\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(3120)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}, \quad (6)$$

where $\phi(\omega)=\prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,11,13,17,31,41,53,61,79,131,157,313,521,1249,3121$

From (2) and (3) we have

$$J_2(\omega)=0. \quad (7)$$

We prove that for $k=3,5,7,11,13,17,31,41,53,61,79,131,157,313,521,1249,3121$ (1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,7,11,13,17,31,41,53,61,79,131,157,313,521,1249,3121$

From (2) and (3) we have

$$J_2(\omega)\ne 0. \quad (8)$$

We prove that for $k\ne 3,5,7,11,13,17,31,41,53,61,79,131,157,313,521,1249,3121$ (1) contain infinitely many prime solutions.

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Szemerédi's theorem does not directly to the primes, because it cannot count the number of primes. Cramér's random model cannot prove any prime problems. The probability of $1/\log N$ of being prime is false. Assuming that the events "P is prime", "$P+2$ is prime" and "$P+4$ is prime" are independent, we conclude that P,

$P+2$, $P+4$ are simultaneously prime with probability about $1/\log^3 N$. There are about $N/\log^3 N$ primes less than N. Letting $N \to \infty$ we obtain the prime conjecture, which is false. The tool of additive prime number theory is basically the Hardy-Littlewood prime tuples conjecture, but cannot prove and count any prime problems[6].

Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Leonhard Euler(1707-1783)

It will be another million years, at least, before we understand the primes.

Paul Erdos(1913-1996)

Jiang's function $J_{n+1}(\omega)$ in prime distribution

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Dedicated to the 30-th anniversary of hadronic mechanics

Abstract

We define that prime equations

$$f_1(P_1,\cdots,P_n),\cdots,f_k(P_1,\cdots P_n) \quad (5)$$

are polynomials (with integer coefficients) irreducible over integers, where $P_1,\cdots,P_n$ are all prime. If Jiang's function $J_{n+1}(\omega)=0$ then (5) has finite prime solutions. If $J_{n+1}(\omega)\neq 0$ then there are infinitely many primes $P_1,\cdots,P_n$ such that $f_1,\cdots f_k$ are primes. We obtain a unite prime formula in prime distribution

$$\pi_{k+1}(N,n+1)=\left|\{P_1,\cdots,P_n \le N : f_1,\cdots,f_k \text{ are } k \text{ primes}\}\right|$$

$$=\prod_{i=1}^{k}(\deg f_i)^{-1}\times\frac{J_{n+1}(\omega)\omega^k}{n!\phi^{k+n}(\omega)}\frac{N^n}{\log^{k+n}N}(1+o(1)). \quad (8)$$

Jiang's function is accurate sieve function. Using Jiang's function we prove about 600 prime theorems [6]. Jiang's function provides proofs of the prime theorems which are simple enough to understand and accurate enough to be useful.

Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Leonhard Euler

It will be another million years, at least, before we understand the primes.

Paul Erdös

Suppose that Euler totient function

$$\phi(\omega)=\prod_{2\le P}(P-1)=\infty \quad \text{as} \quad \omega\to\infty, \quad (1)$$

where $\omega=\prod_{2\le P}P$ is called primorial.

Suppose that $(\omega,h_i)=1$, where $i=1,\cdots,\phi(\omega)$. We have prime equations

$$P_1=\omega n+1,\cdots,P_{\phi(\omega)}=\omega n+h_{\phi(\omega)} \quad (2)$$

where $n=0,1,2,\cdots$.

(2) is called infinitely many prime equations (IMPE). Every equation has infinitely many prime solutions. We have

$$\pi_{h_i} = \sum_{\substack{P_i \le N \\ P_i \equiv h_i \pmod{\omega}}} 1 = \frac{\pi(N)}{\phi(\omega)}(1+o(1)).,\qquad (3)$$

where π_{h_i} denotes the number of primes $P_i \le N$ in $P_i = \omega n + h_i$ $n = 0,1,2,\cdots$, $\pi(N)$ the number of primes less than or equal to N.

We replace sets of prime numbers by IMPE. (2) is the fundamental tool for proving the prime theorems in prime distribution.

Let $\omega = 30$ and $\phi(30) = 8$. From (2) we have eight prime equations

$$P_1 = 30n+1,\ P_2 = 30n+7,\ P_3 = 30n+11,\ P_4 = 30n+13,\ P_5 = 30n+17,$$
$$P_6 = 30n+19,\ P_7 = 30n+23,\ P_8 = 30n+29,\ n = 0,1,2,\cdots \qquad (4)$$

Every equation has infinitely many prime solutions.

THEOREM. We define that prime equations

$$f_1(P_1,\cdots,P_n),\cdots,f_k(P_1,\cdots,P_n) \qquad (5)$$

are polynomials (with integer coefficients) irreducible over integers, where $P_1,\cdots,P_n$ are primes. If Jiang's function $J_{n+1}(\omega) = 0$ then (5) has finite prime solutions. If $J_{n+1}(\omega) \ne 0$ then there exist infinitely many primes $P_1,\cdots,P_n$ such that each f_k is a prime.

PROOF. Firstly, we have Jiang's function [1-11]

$$J_{n+1}(\omega) = \prod_{3 \le P}[(P-1)^n - \chi(P)], \qquad (6)$$

where $\chi(P)$ is called sieve constant and denotes the number of solutions for the following congruence

$$\prod_{i=1}^{k} f_i(q_1,\cdots,q_n) \equiv 0 \pmod{P}, \qquad (7)$$

where $q_1 = 1,\cdots,P-1,\cdots,q_n = 1,\cdots,P-1$.

$J_{n+1}(\omega)$ denotes the number of sets of $P_1,\cdots,P_n$ prime equations such that $f_1(P_1,\cdots,P_n),\cdots,f_k(P_1,\cdots,P_n)$ are prime equations. If $J_{n+1}(\omega) = 0$ then (5) has finite prime solutions. If $J_{n+1}(\omega) \ne 0$ using $\chi(P)$ we sift out from (2) prime equations which can not be represented $P_1,\cdots,P_n$, then residual prime equations of (2) are $P_1,\cdots,P_n$ prime equations such that $f_1(P_1,\cdots,P_n),\cdots,$ $f_k(P_1,\cdots,P_n)$ are prime equations. Therefore we prove that there exist infinitely many primes $P_1,\cdots,P_n$ such that $f_1(P_1,\cdots,P_n),\cdots,$ $f_k(P_1,\cdots,P_n)$ are primes.

Secondly, we have the best asymptotic formula [2,3,4,6]

$$\pi_{k+1}(N,n+1) = \left|\{P_1,\cdots,P_n \le N : f_1,\cdots,f_k \text{ are } k \text{ primes}\}\right|$$
$$= \prod_{i=1}^{k} (\deg f_i)^{-1} \times \frac{J_{n+1}(\omega)\omega^k}{n!\phi^{k+n}(\omega)} \frac{N^n}{\log^{k+n} N}(1+o(1)). \qquad (8)$$

(8) is called a unite prime formula in prime distribution. Let $n = 1, k = 0$, $J_2(\omega) = \phi(\omega)$. From (8) we have prime number theorem

$$\pi_1(N,2) = \left|\{P_1 \le N : P_1 \text{ is prime}\}\right| = \frac{N}{\log N}(1+o(1)).. \qquad (9)$$

Number theorists believe that there are infinitely many twin primes, but they do not have rigorous proof of this old conjecture by any method. All the prime theorems are conjectures except the prime number theorem, because they do not prove that prime equations have infinitely many prime solutions. We prove the following conjectures by this theorem.

Example 1. Twin primes $P, P+2$ (300BC).
From (6) and (7) we have Jiang's function

$$J_2(\omega)=\prod_{3\le P}(P-2)\neq 0.$$

Since $J_2(\omega)\neq 0$ in (2) exist infinitely many P prime equations such that $P+2$ is a prime equation. Therefore we prove that there are infinitely many primes P such that $P+2$ is a prime.
Let $\omega=30$ and $J_2(30)=3$. From (4) we have three P prime equations

$$P_3=30n+11,\quad P_5=30n+17,\quad P_8=30n+29.$$

From (8) we have the best asymptotic formula

$$\pi_2(N,2)=\left|\{P\le N: P+2 \text{ prime}\}\right|=\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}(1+o(1))$$
$$=2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N}(1+o(1)).$$

In 1996 we proved twin primes conjecture [1]

Remark. $J_2(\omega)$ denotes the number of P prime equations, $\frac{\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}(1+o(1))$ the number of solutions of primes for every P prime equation.

Example 2. Even Goldbach's conjecture $N=P_1+P_2$**.** Every even number $N\ge 6$ is the sum of two primes**.**
From (6) and (7) we have Jiang's function

$$J_2(\omega)=\prod_{3\le P}(P-2)\prod_{P|N}\frac{P-1}{P-2}\neq 0.$$

Since $J_2(\omega)\neq 0$ as $N\to\infty$ in (2) exist infinitely many P_1 prime equations such that $N-P_1$ is a prime equation. Therefore we prove that every even number $N\ge 6$ is the sum of two primes.
From (8) we have the best asymptotic formula

$$\pi_2(N,2)=\left|\{P_1\le N, N-P_1 \text{ prime}\}\right|=\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}(1+o(1)).$$
$$=2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\prod_{P|N}\frac{P-1}{P-2}\frac{N}{\log^2 N}(1+o(1)).$$

In 1996 we proved even Goldbach's conjecture [1]

Example 3. Prime equations $P, P+2, P+6$**.**
From (6) and (7) we have Jiang's function

$$J_2(\omega)=\prod_{5\le P}(P-3)\neq 0,$$

$J_2(\omega)$ is denotes the number of P prime equations such that $P+2$ and $P+6$ are prime equations. Since $J_2(\omega)\neq 0$ in (2) exist infinitely many P prime equations such that $P+2$ and $P+6$ are prime equations. Therefore we prove that there are infinitely many primes P such that $P+2$ and $P+6$ are primes.
Let $\omega=30$, $J_2(30)=2$. From (4) we have two P prime equations

$$P_3=30n+11,\quad P_5=30n+17.$$

From (8) we have the best asymptotic formula

$$\pi_3(N,2)=\left|\{P\le N: P+2, P+6 \text{ are primes}\}\right|=\frac{J_2(\omega)\omega^2}{\phi^3(\omega)}\frac{N}{\log^3 N}(1+o(1)).$$

Example 4. Odd Goldbach's conjecture $N=P_1+P_2+P_3$. Every odd number $N\ge 9$ is the sum of three

primes.
From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}\left(P^2-3P+3)\right)\prod_{P|N}\left(1-\frac{1}{P^2-3P+3}\right)\neq 0 .$$

Since $J_3(\omega)\neq 0$ as $N\to\infty$ in (2) exist infinitely many pairs of P_1 and P_2 prime equations such that $N-P_1-P_2$ is a prime equation. Therefore we prove that every odd number $N\ge 9$ is the sum of three primes.
From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\le N:N-P_1-P_2\text{ prime}\}\right|=\frac{J_3(\omega)\omega}{2\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)) .$$

$$=\prod_{3\le P}\left(1+\frac{1}{(P-1)^3}\right)\prod_{P|N}\left(1-\frac{1}{P^3-3P+3}\right)\frac{N^2}{\log^3 N}(1+o(1)) .$$

Example 5. Prime equation $P_3=P_1P_2+2$.
From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}\left(P^2-3P+2\right)\neq 0$$

$J_3(\omega)$ denotes the number of pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Since $J_3(\omega)\neq 0$ in (2) exist infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.
From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\le N:P_1P_2+2\text{ prime}\}\right|=\frac{J_3(\omega)\omega}{4\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)).$$

Note. deg $(P_1P_2)=2$.

Example 6 [12]. Prime equation $P_3=P_1^3+2P_2^3$.
From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}\left[(P-1)^2-\chi(P)\right]\neq 0 ,$$

where $\chi(P)=3(P-1)$ if $2^{\frac{P-1}{3}}\equiv 1(\bmod P)$; $\chi(P)=0$ if $2^{\frac{P-1}{3}}\not\equiv 1(\bmod P)$; $\chi(P)=P-1$ otherwise.
Since $J_3(\omega)\neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.
From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\le N:P_1^3+2P_2^3\text{ prime}\}\right|=\frac{J_3(\omega)\omega}{6\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)).$$

Example 7 [13]. Prime equation $P_3=P_1^4+(P_2+1)^2$.
From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}\left[(P-1)^2-\chi(P)\right]\neq 0$$

where $\chi(P)=2(P-1)$ if $P\equiv 1(\bmod 4)$; $\chi(P)=2(P-3)$ if $P\equiv 1(\bmod 8)$; $\chi(P)=0$ otherwise.
Since $J_3(\omega)\neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.
From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\le N:P_3\ \text{prime}\}\right|=\frac{J_3(\omega)\omega}{8\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)).$$

Example 8 [14-20]. Arithmetic progressions consisting only of primes. We define the arithmetic progressions of length k.

$$P_1,P_2=P_1+d,P_3=P_1+2d,\cdots,P_k=P_1+(k-1)d,(P_1,d)=1. \quad (10)$$

From (8) we have the best asymptotic formula

$$\pi_2(N,2)=\left|\{P_1\le N:P_1,P_1+d,\cdots,P_1+(k-1)d \text{ are primes}\}\right|$$
$$=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}(1+o(1))..$$

If $J_2(\omega)=0$ then (10) has finite prime solutions. If $J_2(\omega)\ne 0$ then there are infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes.

To eliminate d from (10) we have

$$P_3=2P_2-P_1,\quad P_j=(j-1)P_2-(j-2)P_1,3\le j\le k.$$

From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P<k}(P-1)\prod_{k\le P}(P-1)(P-k+1)\ne 0$$

Since $J_3(\omega)\ne 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that $P_3,\cdots,P_k$ are prime equations. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that $P_3,\cdots,P_k$ are primes.

From (8) we have the best asymptotic formula

$$\pi_{k-1}(N,3)=\left|\{P_1,P_2\le N:(j-1)P_2-(j-2)P_1 \text{ prime},3\le j\le k\}\right|$$
$$=\frac{J_3(\omega)\omega^{k-2}}{2\phi^k(\omega)}\frac{N^2}{\log^k N}(1+o(1))$$
$$=\frac{1}{2}\prod_{2\le P<k}\frac{P^{k-2}}{(P-1)^{k-1}}\prod_{k\le P}\frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}}\frac{N^2}{\log^k N}(1+o(1)).$$

Example 9. It is a well-known conjecture that one of $P,P+2,P+2^2$ is always divisible by 3. To generalize above to the $k-$primes, we prove the following conjectures. Let n be a square-free even number.

1. $P,P+n,P+n^2$,

where $3\mid(n+1)$.

From (6) and (7) we have $J_2(3)=0$, hence one of $P,P+n,P+n^2$ is always divisible by 3.

2. $P,P+n,P+n^2,\cdots,P+n^4$,

where $5\mid(n+b),b=2,3$.

From (6) and (7) we have $J_2(5)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^4$ is always divisible by 5.

3. $P,P+n,P+n^2,\cdots,P+n^6$,

where $7\mid(n+b),b=2,4$.

From (6) and (7) we have $J_2(7)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^6$ is always divisible by 7.

4. $P,P+n,P+n^2,\cdots,P+n^{10}$,

where $11\mid(n+b),b=3,4,5,9$.

From (6) and (7) we have $J_2(11)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^{10}$ is always divisible by 11.

5. $P,P+n,P+n^2,\cdots,P+n^{12}$,

where $13\,|(n+b), b=2,6,7,11$.

From (6) and (7) we have $J_2(13)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^{12}$ is always divisible by 13.

6. $P,P+n,P+n^2,\cdots,P+n^{16}$,

where $17\,|(n+b), b=3,5,6,7,10,11,12,14,15$.

From (6) and (7) we have $J_2(17)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^{16}$ is always divisible by 17.

7. $P,P+n,P+n^2,\cdots,P+n^{18}$,

where $19\,|(n+b), b=4,5,6,9,16.17$.

From (6) and (7) we have $J_2(19)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^{18}$ is always divisible by 19.

Example 10. Let n be an even number.

1. $P,P+n^i, i=1,3,5,\cdots,2k+1$,

From (6) and (7) we have $J_2(\omega)\neq 0$. Therefore we prove that there exist infinitely many primes P such that $P,P+n^i$ are primes for any k.

2. $P,P+n^i, i=2,4,6,\cdots,2k$.

From (6) and (7) we have $J_2(\omega)\neq 0$. Therefore we prove that there exist infinitely many primes P such that $P,P+n^i$ are primes for any k.

Example 11. Prime equation $2P_2=P_1+P_3$

From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+2)\neq 0.$$

Since $J_3(\omega)\neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is prime equations. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.

From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\le N: P_3 \text{ prime}\}\right|=\frac{J_3(\omega)\omega}{2\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)).$$

In the same way we can prove $2P_2^2=P_3+P_1$ which has the same Jiang's function.

Jiang's function is accurate sieve function. Using it we can prove any irreducible prime equations in prime distribution. There are infinitely many twin primes but we do not have rigorous proof of this old conjecture by any method [20]. As strong as the numerical evidence may be, we still do not even know whether there are infinitely many pairs of twin primes [21]. All the prime theorems are conjectures except the prime number theorem, because they do not prove the simplest twin primes. They conjecture that the prime distribution is randomness [12-26], because they do not understand theory of prime numbers.

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The Hardy-Littlewood prime *k*-tuple conjecture is false

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Abstract

Using Jiang function we prove Jiang prime k-tuple theorem. We prove that the Hardy-Littlewood prime k-tuple conjecture is false. Jiang prime k-tuple theorem can replace the Hardy-Littlewood prime k-tuple conjecture.

(A) Jiang prime k-tuple theorem [1, 2].

We define the prime k-tuple equation

$$p,\ p+n_i, \tag{1}$$

where $2|n_i, i=1,\cdots k-1$.
we have Jiang function [1, 2]

$$J_2(\omega)=\prod_P(P-1-\chi(P)), \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i)\equiv 0 \pmod P, \quad q=1,\cdots,p-1. \quad (3)$$

If $\chi(P)<P-1$ then $J_2(\omega)\neq 0$. There exist infinitely many primes P such that each of $P+n_i$ is prime. If $\chi(P)=P-1$ then $J_2(\omega)=0$. There exist finitely many primes P such that each of $P+n_i$ is prime. $J_2(\omega)$ is a subset of Euler function $\phi(\omega)$ [2].

If $J_2(\omega)\neq 0$, then we hae the best asymptotic formula of the number of prime P [1, 2]

$$\pi_k(N,2)=\left|\{P\le N: P+n_i=prime\}\right|\sim \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}=C(k)\frac{N}{\log^k N} \quad (4)$$

$$\phi(\omega)=\prod_P(P-1),$$

$$C(k)=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)\left(1-\frac{1}{P}\right)^{-k} \quad (5)$$

Example 1. Let $k=2, P, P+2$, twin primes theorem.
From (3) we have

$$\chi(2)=0, \quad \chi(P)=1 \text{ if } P>2, \quad (6)$$

Substituting (6) into (2) we have

$$J_2(\omega)=\prod_{P\ge 3}(P-2)\neq 0 \quad (7)$$

There exist infinitely many primes P such that $P+2$ is prime. Substituting (7) into (4) we have the best asymptotic pormula

$$\pi_k(N,2)=\left|\{P\le N: P+2=prime\}\right|\sim 2\prod_{P\ge 3}(1-\frac{1}{(P-1)^2})\frac{N}{\log^2 N}. \quad (8)$$

Example 2. Let $k=3, P, P+2, P+4$.
From (3) we have

$$\chi(2)=0, \quad \chi(3)=2 \quad (9)$$

From (2) we have

$$J_2(\omega)=0. \quad (10)$$

It has only a solution $P=3$, $P+2=5$, $P+4=7$. One of $P, P+2, P+4$ is always divisible by 3.
Example 3. Let $k=4, P, P+n$, where $n=2,6,8$.
From (3) we have

$$\chi(2)=0, \chi(3)=1, \chi(P)=3 \text{ if } P>3. \quad (11)$$

Substituting (11) into (2) we have

$$J_2(\omega)=\prod_{P\ge 5}(P-4)\neq 0, \quad (12)$$

There exist infinitely many primes P such that each of $P+n$ is prime.
Substituting (12) into (4) we have the best asymptotic formula

$$\pi_4(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim \frac{27}{3}\prod_{P\ge 5}\frac{P^3(P-4)}{(P-1)^4}\frac{N}{\log^4 N} \quad (13)$$

Example 4. Let $k=5$, P, $P+n$, where $n=2,6,8,12$.

From (3) we have

$$\chi(2)=0,\ \chi(3)=1,\ \chi(5)=3,\ \chi(P)=4 \text{ if } P>5 \quad (14)$$

Substituting (14) into (2) we have

$$J_2(\omega)=\prod_{P\ge 7}(P-5)\ne 0 \quad (15)$$

There exist infinitely many primes P such that each of $P+n$ is prime. Substituting (15) into (4) we have the best asymptotic formula

$$\pi_5(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim \frac{15^4}{2^{11}}\prod_{P\ge 7}\frac{(P-5)P^4}{(P-1)^5}\frac{N}{\log^5 N} \quad (16)$$

Example 5. Let $k=6$, P, $P+n$, where $n=2,6,8,12,14$.

From (3) and (2) we have

$$\chi(2)=0,\ \ \chi(3)=1,\ \ \chi(5)=4,\ \ J_2(5)=0 \quad (17)$$

It has only a solution $P=5$, $P+2=7$, $P+6=11$, $P+8=13$, $P+12=17$, $P+14=19$. One of $P+n$ is always divisible by 5.

(B) The Hardy-Littlewood prime k-tuple conjecture[3-14].

This conjecture is generally believed to be true,but has not been proved(Odlyzko et al.exp.math.8(1999)107-118).

We define the prime k-tuple equation

$$P,P+n_i \quad (18)$$

where $2|n_i, i=1,\cdots,k-1$.

In 1923 Hardy and Littlewood conjectured the asymptotic formula

$$\pi_k(N,2)=\left|\{P\le N: P+n_i=prime\}\right|\sim H(k)\frac{N}{\log^k N}, \quad (19)$$

where

$$H(k)=\prod_P\left(1-\frac{\nu(P)}{P}\right)\left(1-\frac{1}{P}\right)^{-k} \quad (20)$$

$\nu(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i)\equiv 0 \pmod P, \quad q=1,\cdots,P. \quad (21)$$

From (21) we have $\nu(P)<P$ and $H(k)\ne 0$. For any prime k-tuple equation there exist infinitely many primes P such that each of $P+n_i$ is prime, which is false.

Conjectore 1. Let $k=2, P, P+2$, twin primes theorem

Frome (21) we have

$$\nu(P)=1 \quad (22)$$

Substituting (22) into (20) we have

$$H(2)=\prod_P\frac{P}{P-1} \quad (23)$$

Substituting (23) into (19) we have the asymptotic formula

$$\pi_2(N,2)=\left|\{P\le N: P+2=prime\}\right|\sim\prod_P\frac{P}{P-1}\frac{N}{\log^2 N} \quad (24)$$

which is false see example 1.

Conjecture 2. Let $k=3, P, P+2, P+4$.

From (21) we have

$$\nu(2)=1,\ \ \nu(P)=2 \text{ if } P>2 \quad (25)$$

Substituting (25) into (20) we have

$$H(3)=4\prod_{P\geq 3}\frac{P^2(P-2)}{(P-1)^3} \quad (26)$$

Substituting (26) into (19) we have asymptotic formula

$$\pi_3(N,2)=\left|\{P\leq N: P+2=prime, P+4=prim\}\right|\sim 4\prod_{P\geq 3}\frac{P^2(P-2)}{(P-1)^3}\frac{N}{\log^3 N} \quad (27)$$

which is false see example 2.

Conjecutre 3. Let $k=4$, $P, P+n$, where $n=2,6,8$.

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(P)=3 \text{ if } P>3 \quad (28)$$

Substituting (28) into (20) we have

$$H(4)=\frac{27}{2}\prod_{P>3}\frac{P^3(P-3)}{(P-1)^4} \quad (29)$$

Substituting (29) into (19) we have asymptotic formula

$$\pi_4(N,2)=\left|\{P\leq N: P+n=prime\}\right|\sim\frac{27}{2}\prod_{P>3}\frac{P^3(P-3)}{(P-1)^4}\frac{N}{\log^4 N} \quad (30)$$

Which is false see example 3.

Conjecture 4. Let $k=5$, $P, P+n$, where $n=2,6,8,12$

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(5)=3,\ \nu(P)=4 \text{ if } P>5 \quad (31)$$

Substituting (31) into (20) we have

$$H(5)=\frac{15^4}{4^5}\prod_{P>5}\frac{P^4(P-4)}{(P-1)^5} \quad (32)$$

Substituting (32) into (19) we have asymptotic formula

$$\pi_5(N,2)=\left|\{P\leq N: P+n=prime\}\right|\sim\frac{15^4}{4^5}\prod_{P>5}\frac{P^4(P-4)}{(P-1)^5}\frac{N}{\log^5 N} \quad (33)$$

Which is false see example 4.

Conjecutre 5. Let $k=6$, P, $P+n$, where $n=2,6,8,12,14$.

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(5)=4,\ \nu(P)=5 \text{ if } P>5 \quad (34)$$

Substituting (34) into (20) we have

$$H(6)=\frac{15^5}{2^{13}}\prod_{P>5}\frac{(P-5)P^5}{(P-1)^6} \quad (35)$$

Substituting (35) into (19) we have asymptotic formula

$$\pi_6(N,2)=\left|\{P\leq N: P+n=prime\}\right|\sim\frac{15^5}{2^{13}}\prod_{P>5}\frac{(P-5)P^5}{(P-1)^6}\frac{N}{\log^6 N} \quad (36)$$

which is false see example 5.

Conclusion.

The Hardy-Littlewood prime k-tuple conjecture is false. The tool of addive prime number theory is basically the Hardy-Littlewood prime tuples conjecture. Jiang prime k-tuple theorem can replace Hardy-Littlewood prime k-tuple Conjecture. There cannot be really modern prime theory without Jiang function.

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