**A Study on parameters related to** σ**-Statistical Convergence**

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**Abstract:** The mappings σ are one-to-one and such that σm(ok) ≠ ok for all advantageous integers ok and m, wherein σm(ok) denotes the mth iterate of the mapping σ at ok. Thus Ф extends the restrict useful on c, the distance of convergent sequences, withinside the experience that Ф(x) = lim ξk for all x ∈ c. In case σ is the interpretation mapping ok→ok+1, an invariant suggest is frequently known as a Banach restrict and Vσ, the set of bounded sequences all of whose invariant approach are equal, is the set of just about convergent sequences.

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**Introduction**

Convergence of random variables (sometimes called stochastic convergence) is where a set of numbers settle on a particular number. It works the same way as convergence anywhere else; For example, cars on a 5-line highway might converge to one specific lane if there’s an accident closing down four of the other lanes. In the same way, a sequence of numbers (which could represent cars or anything else) can [converge](https://calculushowto.com/converge/)(mathematically, this time) on a single, specific number. Certain processes, distributions and events can result in convergence— which basically mean the values will get closer and closer together.

The main object of this paper is to study two more extensions of the concept of statistical convergence namely σ-statistical convergence and lacunary σ-statistical convergence. We also study the concept of Lθ-convergence. In section 1.2 we study some inclusion relations between Lθ-convergence and lacunary σ-statistical convergence and show that these are equivalent for bounded sequences. Further in section 1.3 we study relation between σ-statistical convergence and lacunary σ-statistical convergence.

**Definition 1.1.1.** Let σ be a mapping of the set of positive integers into itself. A continuous linear functional Ф on *l*∞, the space of real bounded sequences x = {ξk}, is said to be an invariant mean or a σ-mean if and only if

1. Ф(x) ≥ 0 if ξk ≥ 0 for all k,
2. Ф({ξσ(k)}) = Ф(x) for all x ∈ *l*∞,
3. Ф(e) = 1 where e = {1,1,1,…}.

The mappings σ are one-to-one and such that σm(k) ≠ k for all positive integers k and m, where σm(k) denotes the mth iterate of the mapping σat k. Thus Ф extends the limit functional on c, the space of convergent sequences, in the sense that Ф(x) = lim ξk for all x ∈ c. In case σ is the translation mapping k→k+1, an invariant mean is often called a Banach limit and Vσ, the set of bounded sequences all of whose invariant means are equal, is the set of almost convergent sequences [19].

If x = {ξk}, set Tx = {Tξk} = {ξσ(k)}. It can be shown [28] that

Vσ = {x = {ξk}: = ξe uniformly in k, ξ = σ-lim ξk}

where  = .

Several authors including Mursaleen [22], Savas [27], Schaefer [31] and others have studied invariant convergent sequences.

**Definition 1.1.2.** A sequence x = {ξk} is said to be strongly σ-convergent [23] to ξ if

– ξ| = 0 uniformly in m.

In this case we write ξk → ξ[Vσ] and [Vσ] denotes the set of all strongly σ-convergent sequences.

**Remark 1.1.3.**

(i) For σ(m) = m+1, the space [Vσ] is the space of strongly almost convergent sequences.

(ii) It is known [23] that c ⊂ [Vσ] ⊂ Vσ ⊂ *l*∞.

**Definition 1.1.4.** A lacunary sequence is an increasing integer sequence θ = {kr} such that k0 = 0 and hr = kr – kr-1 → ∞ as r → ∞.

Throughout this paper the intervals determined by θ will be denoted by Ir = (kr-1, kr].

**Definition 1.1.5.** Let θ be a lacunary sequence. The space denoted by Nθ is defined [9] as

Nθ = {x = {ξk}: for some ξ, – ξ| = 0}.

**Definition 1.1.1.** A sequence x = {ξk} is said to be lacunary strong σ-convergent [28] to ξ if

– ξ| = 0 uniformly in m.

We shall denote by Lθ the set of all lacunary strong σ-convergent sequences.

**Remark 1.1.1.** Lθ ⇔ [Vσ] for every lacunary sequence θ.

**Definition 1.1.8.** A complex number sequence x = {ξk} is said to be σ-statistically convergent or Sσ -convergent to the number ξ if for each ε > 0

|{0 ≤ k ≤ n: |– ξ| ≥ ε}| = 0 uniformly in m.

In this case we write Sσ-lim ξk = ξ or ξk → ξ(Sσ) and Sσ denotes the set of all σ-statistically convergent sequences.

**Definition 1.1.9.** Letθ = {kr} be a lacunary sequence. The complex number sequence x = {ξk} is said to be lacunary σ-statistically convergent or

Sσθ-convergent to the number ξ if for each ε > 0

|{k ∈ Ir : |– ξ| ≥ ε}| = 0 uniformly in m.

In this case we write Sσθ-lim ξk = ξ or ξk → ξ(Sσθ) and Sσθ denotes the set of all lacunary σ-statistically convergent sequences.

**1.2 SOME INCLUSION RELATIONS BETWEEN Lθ-CONVERGENCE AND LACUNARY** σ**-STATISTICAL CONVERGENCE**

In his section we study some inclusion relations between Lθ-convergence and lacunary σ-statistical convergence and show that these are equivalent for bounded sequences.

**Theorem 1.4.1.** Let θ = {kr} be a lacunary sequence. Then

(i) ξk → ξ(Lθ) ⇒ ξk → ξ(Sσθ),

(ii) if x ∈ *l*∞ and ξk → ξ(Sσθ), then ξk → ξ(Lθ),

(iii) Sσθ ∩ *l*∞ = Lθ.

**Proof. (i).** Since ξk → ξ(Lθ), for each ε > 0, we have

– ξ| = 0 uniformly in m. …(1)

If ε > 0, we can write

– ξ| ≥– ξ|

≥ ε|{k ∈ Ir: |– ξ| ≥ ε}|

Consequently,

– ξ| ≥ ε |{k ∈ Ir: |– ξ| ≥ ε}|

Hence by (1) and the fact that ε is fixed number, we have

|{k ∈ Ir: |– ξ| ≥ ε}| = 0 uniformly in m,

i.e. ξk → ξ(Sσθ).

**(ii).** Suppose that ξk → ξ(Sσθ) and x ∈ *l*∞. Then for each ε > 0

|{k ∈ Ir: |– ξ| ≥ ε}| = 0 uniformly in m. ...(2)

Since x ∈ *l*∞, there exists a positive real number M such that  – ξ| ≤ M for all k and m.

For given ε > 0, we have

– ξ| = – ξ| + – ξ|

≤  +  = |{k ∈ Ir: |– ξ| ≥ ε}| + ε[n–(n–hr+1) + 1]

= |{k ∈ Ir: |– ξ| ≥ ε}| + εhr

= |{k ∈ Ir: |– ξ| ≥ ε}| + ε

⇒ – ξ| ≤ M |{k ∈ Ir: |– ξ| ≥ ε}| + ε

Hence by using (2), we get

– ξ| = 0 uniformly in m. …(3)

⇒ ξk → ξ(Lθ).

**Example 1.2.2.** Let θ be given and define ξk to be 1,2,3,…,[] for k = σn(m), n = kr-1 + 1, kr-1 + 2,…,kr-1 + []; m ≥ 1 and ξk = 0 otherwise (where [ ] denotes the greatest integer function).

Note that x is not bounded. Now

|{k ∈ Ir : |– 0| ≥ ε}| =  → 0 as r → ∞,

i.e. ξk → 0(Sσθ). But

– 0| = () →  ≠ 0 as r → ∞,

i.e. ξk ↛0(Lθ).

Thus inclusion in (i) is proper and this example shows that the boundedness condition can not be omitted from (ii).

**(iii).** It follows from (i), (ii), Remark 1.1.7 and the fact that [Vσ]⊂ *l*∞.

This completes the proof of the theorem.

**1.3** In this section we study relation between Sσ-convergence and Sσθ-convergence. First we discuss a lemma which will be used in studying that relation.

**Lemma 1.4.1.** A sequence x = {ξk} is σ-statistically convergent to the number ξ if for given ε1 > 0 and each ε > 0, there exist n0 and m0 such that

|{0 ≤ k ≤ n–1: |– ξ| ≥ ε}| < ε1

for all n ≥ n0 and m ≥ m0.

**Proof.** Let ε1 > 0 be given. For each ε > 0, choose n0**'** and m0 such that

|{0 ≤ k ≤ n–1: |– ξ| ≥ ε}| <  …(4)

for all n ≥ n0**'** and m ≥ m0.

It is enough to prove that there exists n0**''** such that for n ≥ n0**''**, 0 ≤ m ≤ m0,

|{0 ≤ k ≤ n–1: |– ξ| ≥ ε}| < ε1 …(5)

since taking n0 = max {n0**'**,n0**''** }, (5) will hold for n ≥ n0and for all m, which gives the result.

Once m0 has been chosen, 0 ≤ m ≤ m0, m0 is fixed.

So let |{0 ≤ k ≤ m0–1: |– ξ| ≥ ε}| = K.

Now taking 0 ≤ m ≤ m0 and n ≥ m0, we have

|{0 ≤ k ≤ n–1: |– ξ| ≥ ε}| = |{0 ≤ k ≤ m0–1: |– ξ| ≥ ε}|

+ |{m0 ≤ k ≤ n–1: |– ξ| ≥ ε}|

≤ K +  [Using (4)]

< ε1 [Taking n sufficiently large]

which gives (5), and hence the result follows.

**Theorem 1.3.2.** Sσθ = Sσ for every lacunary sequence θ.

**Proof.** Let x ∈ Sσθ. Then from Definition 1.1.9, given ε1 > 0, there exist r0 and ξ such that

|{0 ≤ k ≤ hr –1: |– ξ| ≥ ε}| < ε1

for r ≥ r0 and m = kr-1 + 1 + u, u ≥ 0.

Let n ≥ hr and write n = ihr + t where 0 ≤ t ≤ hr, i is an integer. Since n ≥ hr, it follows that i ≥ 1.

Now

|{0 ≤ k ≤ n–1: |– ξ| ≥ ε}| ≤ |{0 ≤ k ≤ (i+1)hr –1: |– ξ| ≥ ε}|

= {jhr ≤ k ≤ (j+1)hr –1: |– ξ| ≥ ε}|

≤ (i+1)hr ε1

≤ 2i hr [i ≥ 1]

for ≤ 1, since ≤ 1. So

|{0 ≤ k ≤ n–1: |– ξ| ≥ ε}| ≤ 2ε1.

Then, by Lemma 1.4.1, x ∈ Sσ.

Thus Sσθ ⊂ Sσ.

It is easy to see that Sσ ⊂ Sσθ.

Hence Sσθ = Sσ for every lacunary sequence θ.

This completes the proof of the theorem.

**Remark 1.3.3.** When σ(m) = m + 1, from Definition 1.1.8 and Definition 1.1.9, we have the definitions of almost statistical convergence and lacunary almost statistical convergence of a sequence.

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