



## Quantum mechanics

Mark Herbert, PhD

World Development Institute, 39 Main Street, Flushing, Queens, New York 11354, USA, [ma708090@gmail.com](mailto:ma708090@gmail.com)

**Abstract:** Quantum mechanics is a fundamental theory in [physics](#) that describes the physical properties of [nature](#) at small scales, of the order of [atoms](#) and [subatomic particles](#). It is the foundation of all [quantum physics](#) including [quantum chemistry](#), [quantum field theory](#), [quantum technology](#), and [quantum information science](#). Most contents are from Wikipedia, the free encyclopedia ([https://en.wikipedia.org/wiki/Quantum\\_mechanics](https://en.wikipedia.org/wiki/Quantum_mechanics)). [Mark Herbert, PhD. **Quantum mechanics**. *Researcher* 2020;12(10):24-37]. ISSN 1553-9865 (print); ISSN 2163-8950 (online). <http://www.sciencepub.net/researcher>. 5. doi:[10.7537/marsrsj121020.05](https://doi.org/10.7537/marsrsj121020.05).

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### Introduction

Most contents are from Wikipedia, the free encyclopedia ([https://en.wikipedia.org/wiki/Quantum\\_mechanics](https://en.wikipedia.org/wiki/Quantum_mechanics)).

Quantum mechanics cannot predict the exact location of a particle in space, only the probability of finding it at different locations.<sup>[1]</sup> The brighter areas represent a higher probability of finding the electron.

Quantum mechanics is a fundamental theory in [physics](#) that describes the physical properties of [nature](#) at small scales, of the order of [atoms](#) and [subatomic particles](#).<sup>[2]</sup> It is the foundation of all [quantum physics](#) including [quantum chemistry](#), [quantum field theory](#), [quantum technology](#), and [quantum information science](#).

[Classical physics](#), the description of physics that existed before the [theory of relativity](#) and quantum mechanics, describes many aspects of nature at an ordinary (macroscopic) scale, while quantum mechanics explains the aspects of nature at small (atomic and [subatomic](#)) scales, for which classical mechanics is insufficient.

Most theories in classical physics can be derived from quantum mechanics as an approximation valid at large (macroscopic) scale.<sup>[3]</sup> Quantum mechanics differs from classical physics in that [energy](#), [momentum](#), [angular momentum](#), and other quantities of a bound system are restricted to [discrete values](#) ([quantization](#)), objects have characteristics of both [particles](#) and [waves](#) ([wave-particle duality](#)), and there are limits to how accurately the value of a physical quantity can be predicted prior to its measurement, given a complete set of initial conditions (the [uncertainty principle](#)).<sup>[4]</sup>

Quantum mechanics [arose gradually](#), from theories to explain observations which could not be reconciled with classical physics, such as [Max Planck's](#) solution in 1900 to the [black-body radiation](#)

problem, and the correspondence between energy and frequency in [Albert Einstein's 1905 paper](#) which explained the [photoelectric effect](#). [Early quantum theory](#) was profoundly re-conceived in the mid-1920s by [Niels Bohr](#), [Erwin Schrödinger](#), [Werner Heisenberg](#), [Max Born](#) and others. The original interpretation of quantum mechanics is the [Copenhagen interpretation](#), developed by Niels Bohr and Werner Heisenberg in Copenhagen during the 1920s. The modern theory is formulated in various [specially developed mathematical formalisms](#). In one of them, a mathematical function, the [wave function](#), provides information about the [probability amplitude](#) of energy, momentum, and other physical properties of a particle.

### History

Scientific inquiry into the wave nature of light began in the 17th and 18th centuries, when scientists such as [Robert Hooke](#), [Christiaan Huygens](#) and [Leonhard Euler](#) proposed a wave theory of light based on experimental observations.<sup>[5]</sup> In 1803 English [polymath Thomas Young](#) described the famous [double-slit experiment](#).<sup>[6]</sup> This experiment played a major role in the general acceptance of the [wave theory of light](#).

In 1838 [Michael Faraday](#) discovered [cathode rays](#). These studies were followed by the 1859 statement of the [black-body radiation](#) problem by [Gustav Kirchhoff](#), the 1877 suggestion by [Ludwig Boltzmann](#) that the energy states of a physical system can be discrete, and the 1900 quantum hypothesis of [Max Planck](#).<sup>[7]</sup> Planck's hypothesis that energy is radiated and absorbed in discrete "quanta" precisely matched the observed patterns of black-body radiation.

In 1896 [Wilhelm Wien](#) empirically determined a distribution law of black-body radiation,<sup>[8]</sup> called [Wien's law](#). Ludwig Boltzmann independently arrived

at this result by considerations of [Maxwell's equations](#). However, it was valid only at high frequencies and underestimated the radiance at low frequencies.

The foundations of quantum mechanics were established during the first half of the 20th century by [Max Planck](#), [Niels Bohr](#), [Werner Heisenberg](#), [Louis de Broglie](#), [Arthur Compton](#), [Albert Einstein](#), [Erwin Schrödinger](#), [Max Born](#), [John von Neumann](#), [Paul Dirac](#), [Enrico Fermi](#), [Wolfgang Pauli](#), [Max von Laue](#), [Freeman Dyson](#), [David Hilbert](#), [Wilhelm Wien](#), [Satyendra Nath Bose](#), [Arnold Sommerfeld](#), and [others](#). The [Copenhagen interpretation](#) of [Niels Bohr](#) became widely accepted.

[Max Planck](#) corrected this model using Boltzmann's statistical interpretation of thermodynamics and proposed what is now called [Planck's law](#), which led to the development of quantum mechanics. After Planck's solution in 1900 to the black-body radiation problem (reported 1859), [Albert Einstein](#) offered a quantum-based explanation of the [photoelectric effect](#) (1905, reported 1887). Around 1900–1910, the [atomic theory](#) but not the [corpuscular theory of light](#)<sup>[9]</sup> first came to be widely accepted as scientific fact; these latter theories can be considered quantum theories of [matter](#) and [electromagnetic radiation](#), respectively. However, the photon theory was not widely accepted until about 1915. Even until Einstein's Nobel Prize, Niels Bohr did not believe in the photon.<sup>[10]</sup>

Among the first to study quantum phenomena were [Arthur Compton](#), [C. V. Raman](#), and [Pieter Zeeman](#), each of whom has a quantum effect named after him. [Robert Andrews Millikan](#) studied the [photoelectric effect](#) experimentally, and Albert Einstein developed a theory for it. At the same time, [Ernest Rutherford](#) experimentally discovered the nuclear model of the atom, and [Niels Bohr](#) developed a theory of atomic structure, confirmed by the experiments of [Henry Moseley](#). In 1913 [Peter Debye](#) extended Bohr's theory by introducing [elliptical orbits](#), a concept also introduced by [Arnold Sommerfeld](#).<sup>[11]</sup> This phase is known as [old quantum theory](#).

Planck cautiously insisted that this was only an aspect of the processes of absorption and emission of radiation and was not the *physical reality* of the radiation.<sup>[12]</sup> In fact, he considered his quantum hypothesis a mathematical trick to get the right answer rather than a sizable discovery.<sup>[13]</sup> However, in 1905 [Albert Einstein](#) interpreted Planck's quantum hypothesis [realistically](#) and used it to explain the [photoelectric effect](#), in which shining light on certain materials can eject electrons from the material. Einstein won the 1921 Nobel Prize in Physics for this work.

Einstein further developed this idea to show that an [electromagnetic wave](#) such as light could also be

described as a particle (later called the [photon](#)), with a discrete amount of energy that depends on its frequency.<sup>[14]</sup> In his paper "On the Quantum Theory of Radiation," Einstein expanded on the interaction between energy and matter to explain the absorption and emission of energy by atoms. Although overshadowed at the time by his general theory of relativity, this paper articulated the mechanism underlying the stimulated emission of radiation,<sup>[15]</sup> which became the basis of the [laser](#).

In the mid-1920s quantum mechanics was developed to become the standard formulation for atomic physics. In the summer of 1925, Bohr and Heisenberg published results that closed the old quantum theory. Due to their particle-like behavior in certain processes and measurements, light quanta came to be called [photons](#) (1926). In 1926 [Erwin Schrödinger](#) suggested a partial differential equation for the wave functions of particles like electrons. And when effectively restricted to a finite region, this equation allowed only certain modes, corresponding to discrete quantum states – whose properties turned out to be exactly the same as implied by matrix mechanics.<sup>[16]</sup> Einstein's simple postulation spurred a flurry of debate, theorizing, and testing. Thus, the entire field of [quantum physics](#) emerged, leading to its wider acceptance at the Fifth [Solvay Conference](#) in 1927.<sup>[17]</sup>

It was found that [subatomic particles](#) and electromagnetic waves are neither simply particle nor wave but have certain properties of each. This originated the concept of [wave-particle duality](#).<sup>[18]</sup>

By 1930 quantum mechanics had been further unified and formalized by [David Hilbert](#), [Paul Dirac](#) and [John von Neumann](#)<sup>[19]</sup> with greater emphasis on [measurement](#), the statistical nature of our knowledge of reality, and [philosophical speculation about the 'observer'](#).<sup>[20]</sup> It has since permeated many disciplines, including quantum chemistry, [quantum electronics](#), [quantum optics](#), and [quantum information science](#). It also provides a useful framework for many features of the modern [periodic table of elements](#), and describes the behaviors of [atoms](#) during [chemical bonding](#) and the flow of [electrons](#) in computer [semiconductors](#), and therefore plays a crucial role in many modern technologies.<sup>[18]</sup> Its speculative modern developments include [string theory](#) and [quantum gravity](#) theory.

While quantum mechanics was constructed to describe the world of the very small, it is also needed to explain some [macroscopic](#) phenomena such as [superconductors](#)<sup>[21]</sup> and [superfluids](#).<sup>[22]</sup>

The word *quantum* derives from the [Latin](#), meaning "how great" or "how much".<sup>[23]</sup> In quantum mechanics, it refers to a discrete unit assigned to certain [physical quantities](#) such as the [energy](#) of an [atom](#) at rest. The discovery that particles are discrete

packets of energy with wave-like properties led to the branch of physics dealing with atomic and subatomic systems which is today called quantum mechanics. It underlies the [mathematical](#) framework of many fields of [physics](#) and [chemistry](#), including [condensed matter physics](#), [solid-state physics](#), [atomic physics](#), [molecular physics](#), [computational physics](#), [computational chemistry](#), quantum chemistry, [particle physics](#), [nuclear chemistry](#), and [nuclear physics](#).<sup>[24]</sup> Some fundamental aspects of the theory are still actively studied.<sup>[25]</sup>

Quantum mechanics is essential for understanding the behavior of systems at [atomic](#) length scales and smaller. If the physical nature of an atom were solely described by [classical mechanics](#), electrons would not *orbit* the nucleus, since orbiting electrons emit radiation (due to [circular motion](#)) and so would quickly lose energy and collide with the nucleus. This framework was unable to explain the stability of atoms. Instead, electrons remain in an uncertain, non-deterministic, *smear*d, [probabilistic](#) wave-particle [orbital](#) about the nucleus, defying the traditional assumptions of classical mechanics and [electromagnetism](#).<sup>[26]</sup>

Quantum mechanics was initially developed to provide a better explanation and description of the atom, especially the differences in the [spectra](#) of light emitted by different [isotopes](#) of the same [chemical element](#), as well as subatomic particles. In short, the quantum-mechanical atomic model has succeeded spectacularly in the realm where classical mechanics and electromagnetism falter.

Broadly speaking, quantum mechanics incorporates four classes of phenomena for which classical physics cannot account<sup>[20]</sup>:

- [quantization](#) of [certain physical properties](#)
- [quantum entanglement](#)
- [principle of uncertainty](#)
- [wave-particle duality](#)

#### Mathematical formulations

In the mathematically rigorous formulation of quantum mechanics developed by [Paul Dirac](#),<sup>[27]</sup> [David Hilbert](#),<sup>[28]</sup> [John von Neumann](#),<sup>[29]</sup> and [Hermann Weyl](#),<sup>[30]</sup> the possible states of a quantum mechanical system are symbolized<sup>[31]</sup> as [unit vectors](#). Formally, these vectors are elements of a [complex separable Hilbert space](#) – variously called the *state space* or the *associated Hilbert space* of the system – that is well defined up to a complex number of norm 1. In other words, the possible states are points in the [projective space](#) of a Hilbert space, usually called the [complex projective space](#). The exact nature of this Hilbert space is dependent on the system – for example, the state space for position and momentum states is the space of [square-integrable](#) functions, while the state space for

the spin of a single proton is just the product of two complex planes. Each observable is represented by a maximally [Hermitian](#) ([self-adjoint](#)) linear [operator](#) acting on the state space. Each [eigenstate](#) of an observable corresponds to an [eigenvector](#) of the operator, and the associated [eigenvalue](#) corresponds to the value of the observable in that eigenstate. If the operator's spectrum is discrete, the observable can attain only those discrete eigenvalues.

In the formalism of quantum mechanics, the state of a system at a given time is described by a [complex wave function](#), also referred to as state vector in a complex [vector space](#).<sup>[32]</sup> This abstract mathematical object allows for the calculation of [probabilities](#) of outcomes of concrete experiments. For example, it allows one to compute the probability of finding an electron in a particular region around the nucleus at a particular time. Contrary to classical mechanics, one can never make simultaneous predictions of [conjugate variables](#), such as position and momentum, to arbitrary precision. For instance, electrons may be considered to be located somewhere within a given region of space, but with their exact positions unknown. Contours of constant probability density, often referred to as clouds, may be drawn around the nucleus of an atom to conceptualize where the electron might be located with the most probability. Heisenberg's [uncertainty principle](#) quantifies the inability to precisely locate the particle given its conjugate momentum.<sup>[33]</sup>

According to one interpretation, as the result of a measurement, the wave function containing the probability information for a system [collapses](#) from a given initial state to a particular eigenstate. The possible results of a measurement are the eigenvalues of the operator representing the observable – which explains the choice of *Hermitian* operators, for which all the eigenvalues are real. The probability distribution of an observable in a given state can be found by computing the [spectral decomposition](#) of the corresponding operator. Heisenberg's [uncertainty principle](#) is represented by the statement that the operators corresponding to certain observables do not [commute](#).

The [probabilistic](#) nature of quantum mechanics thus stems from the act of measurement. This is one of the most difficult aspects of quantum systems to understand. It was the central topic in the famous [Bohr–Einstein debates](#), in which the two scientists attempted to clarify these fundamental principles by way of [thought experiments](#). In the decades after the formulation of quantum mechanics, the question of what constitutes a measurement has been extensively studied. Newer [interpretations of quantum mechanics](#) have been formulated that do away with the concept of [wave function collapse](#). The basic idea is that when a quantum system interacts with a measuring apparatus,

their respective wave functions become [entangled](#), so that the original quantum system ceases to exist as an independent entity. For details, see the article on [measurement in quantum mechanics](#).<sup>[34]</sup>

Generally, quantum mechanics does not assign definite values. Instead, it makes a prediction using a [probability distribution](#); that is, it describes the probability of obtaining the possible outcomes from measuring an observable. Often these results are skewed by many causes, such as dense probability clouds. Probability clouds are approximate whereby electron location is given by a [probability function](#), the [wave function eigenvalue](#), such that the probability is the squared modulus of the [complex amplitude](#), or [quantum state](#) nuclear attraction.<sup>[35][36]</sup> Naturally, these probabilities will depend on the quantum state at the instant of the measurement. Hence, uncertainty is involved in the value. There are, however, certain states that are associated with a definite value of a particular observable. These are known as [eigenstates](#) of the observable.<sup>[37]</sup>

In the everyday world, it is natural and intuitive to think of everything as being in an eigenstate. Everything appears to have a definite position, a definite momentum, a definite energy, and a definite time of occurrence. However, quantum mechanics does not pinpoint the exact values of a particle's position and momentum or its energy and time. Rather, it provides only a range of probabilities in which that particle might be given its momentum and momentum probability. Therefore, it is helpful to use different words to describe states having [uncertain](#) values and states having [definite](#) values.

Usually, a system will not be in an [eigenstate](#) of the observable we are interested in. However, if one measures the observable, the wave function will instantaneously be an eigenstate of that observable. This process is known as [wave function collapse](#), a controversial and much-debated process<sup>[38]</sup> that involves expanding the system under study to include the measurement device. If one knows the corresponding wave function at the instant before the measurement, one will be able to compute the probability of the wave function collapsing into each of the possible eigenstates.

For example, the free particle in the previous example will usually have a wave function that is a [wave packet](#) centered around some mean position  $x_0$ . When one measures the position of the particle, it is impossible to predict with certainty the result.<sup>[34]</sup> It is probable, but not certain, that it will be near  $x_0$ , where the amplitude of the wave function is large. After the measurement is performed, having obtained some result  $x$ , the wave function collapses into a position eigenstate centered at  $x$ .<sup>[39]</sup>

The time evolution of a quantum state is

described by the [Schrödinger equation](#), in which the [Hamiltonian](#) (the [operator](#) corresponding to the [total energy](#) of the system) generates the time evolution. The [time evolution](#) of wave functions is [deterministic](#) in the sense that – given a wave function at an *initial* time – it makes a definite prediction of what the wave function will be at any *later* time.<sup>[40]</sup>

During a [measurement](#), on the other hand, the change of the initial wave function into another, later wave function is not deterministic, it is unpredictable (i.e., [random](#)).<sup>[41][42]</sup>

Wave functions change as time progresses. The [Schrödinger equation](#) describes how wave functions change in time, playing a role similar to [Newton's second law in classical mechanics](#). The Schrödinger equation, applied to the aforementioned example of the free particle, predicts that the center of a wave packet will move through space at a constant velocity. However, the wave packet will also spread out as time progresses, which means that the position becomes more uncertain with time. This also has the effect of turning a position eigenstate into a broadened wave packet that no longer represents a position eigenstate.<sup>[43]</sup>

Some wave functions produce probability distributions that are constant, or independent of time – such as when in a [stationary state](#) of definite energy, time vanishes in the absolute square of the wave function. Many systems that are treated dynamically in classical mechanics are described by such "static" wave functions. For example, a single [electron](#) in an unexcited [atom](#) is pictured classically as a particle moving in a circular trajectory around the [atomic nucleus](#), whereas in quantum mechanics, it is described by a static, [spherically symmetric](#) wave function surrounding the nucleus.<sup>[44]</sup>

The Schrödinger equation acts on the *entire* probability amplitude, not merely its absolute value. Whereas the absolute value of the probability amplitude encodes information about probabilities, its [phase](#) encodes information about the [interference](#) between quantum states. This gives rise to the wave-like behavior of quantum states.

Analytic solutions of the Schrödinger equation are known for [very few relatively simple model Hamiltonians](#) including the [quantum harmonic oscillator](#), the [particle in a box](#), the [dihydrogen cation](#), and the [hydrogen atom](#). Even the [helium atom](#) – which contains just two electrons – has defied all attempts at a fully analytic treatment.

However, there are techniques for finding approximate solutions. One method, called [perturbation theory](#), uses the analytic result for a simple quantum mechanical model to create a result for a related but more complicated model by the addition of a weak [potential energy](#). Another method is



called semi-classical equation of motion, which applies to systems for which quantum mechanics produces only small deviations from classical behavior. These deviations can then be computed based on the classical motion. This approach is particularly important in the field of [quantum chaos](#).

### Mathematically equivalent formulations

There are many mathematically equivalent formulations of quantum mechanics. One of the oldest and most common is the [transformation theory](#) proposed by [Paul Dirac](#), which unifies and generalizes the two earliest formulations of quantum mechanics – [matrix mechanics](#) and [wave mechanics](#).<sup>[45]</sup>

Especially since Heisenberg was awarded the [Nobel Prize in Physics](#) in 1932 for the creation of quantum mechanics, the role of [Max Born](#) in the development of QM was overlooked until the 1954 Nobel award. The role is noted in a 2005 biography of Born, which recounts his role in the matrix formulation and the use of probability amplitudes. Heisenberg acknowledges having learned matrices from Born, as published in a 1940 *festschrift* honoring [Max Planck](#).<sup>[46]</sup> In the matrix formulation, the instantaneous state of a quantum system encodes the probabilities of its measurable properties, or [observables](#). Examples of observables include [energy](#), [position](#), [momentum](#), and [angular momentum](#). Observables can be either [continuous](#) or [discrete](#).<sup>[47]</sup> An alternative formulation of quantum mechanics is [Feynman's path integral formulation](#), in which a quantum-mechanical amplitude is considered as a sum over all possible classical and non-classical paths between the initial and final states. This is the quantum-mechanical counterpart of the [action principle](#) in classical mechanics.

### Relation to other scientific theories

The rules of quantum mechanics are fundamental. They assert that the state space of a system is a [Hilbert space](#) and that observables of the system are [Hermitian operators](#) acting on vectors in that space – although they do not tell us which Hilbert space or which operators. These can be chosen appropriately in order to obtain a quantitative description of a quantum system. An important guide for making these choices is the [correspondence principle](#), which states that the predictions of quantum mechanics reduce to those of classical mechanics when a system moves to higher energies or, equivalently, larger quantum numbers, i.e. whereas a single particle exhibits a degree of randomness, in systems incorporating millions of particles averaging takes over and, at the high energy limit, the statistical probability of random behaviour approaches zero. In other words, classical mechanics is simply a quantum mechanics of large systems. This

high energy limit is known as the *classical* or *correspondence limit*. One can even start from an established classical model of a particular system, then try to guess the underlying quantum model that would give rise to the classical model in the correspondence limit.

When quantum mechanics was originally formulated, it was applied to models whose correspondence limit was [non-relativistic classical mechanics](#). For instance, the well-known model of the [quantum harmonic oscillator](#) uses an explicitly non-relativistic expression for the [kinetic energy](#) of the oscillator, and is thus a quantum version of the [classical harmonic oscillator](#).

Early attempts to merge quantum mechanics with [special relativity](#) involved the replacement of the Schrödinger equation with a covariant equation such as the [Klein–Gordon equation](#) or the [Dirac equation](#). While these theories were successful in explaining many experimental results, they had certain unsatisfactory qualities stemming from their neglect of the relativistic creation and annihilation of particles. A fully relativistic quantum theory required the development of [quantum field theory](#), which applies quantization to a field. The first complete quantum field theory, [quantum electrodynamics](#), provides a fully quantum description of the [electromagnetic interaction](#). The full apparatus of quantum field theory is often unnecessary for describing electrodynamic systems. A simpler approach, one that has been used since the inception of quantum mechanics, is to treat [charged](#) particles as quantum mechanical objects being acted on by a classical [electromagnetic field](#).

[Quantum field](#) theories for the [strong nuclear force](#) and the [weak nuclear force](#) have also been developed. The quantum field theory of the strong nuclear force is called [quantum chromodynamics](#), and describes the interactions of subnuclear particles such as [quarks](#) and [gluons](#). The weak nuclear force and the [electromagnetic force](#) were unified, in their quantized forms, into a single quantum field theory, by the physicists [Abdus Salam](#), [Sheldon Glashow](#) and [Steven Weinberg](#). These three men shared the Nobel Prize in Physics in 1979 for this work.<sup>[48]</sup>

It has proven difficult to construct quantum models of [gravity](#), the remaining [fundamental force](#). Semi-classical approximations are workable, and have led to predictions such as [Hawking radiation](#). However, the formulation of a complete theory of [quantum gravity](#) is hindered by apparent incompatibilities between [general relativity](#) and some of the fundamental assumptions of quantum theory. The resolution of these incompatibilities is an area of active research. Candidates for a future theory of quantum gravity include [string theory](#).

Classical mechanics has also been extended into

the complex domain, with complex classical mechanics exhibiting behaviors similar to quantum mechanics.<sup>[49]</sup>

### Relation to classical physics

Predictions of quantum mechanics have been verified experimentally to an extremely high degree of accuracy.<sup>[50]</sup> According to the correspondence principle between classical and quantum mechanics, all objects obey the laws of quantum mechanics, and classical mechanics is just an approximation for large systems of objects.<sup>[51]</sup> The laws of classical mechanics thus follow from the laws of quantum mechanics as a statistical average at the limit of large systems or large quantum numbers.<sup>[52][53]</sup> However, chaotic systems do not have good quantum numbers, and quantum chaos studies the relationship between classical and quantum descriptions in these systems.

Quantum coherence is an essential difference between classical and quantum theories as illustrated by the Einstein–Podolsky–Rosen (EPR) paradox – an attack on a certain philosophical interpretation of quantum mechanics by an appeal to local realism.<sup>[54]</sup> Quantum interference involves adding together probability amplitudes, whereas classical waves infer that there is an adding together of intensities. For microscopic bodies, the extension of the system is much smaller than the coherence length, which gives rise to long-range entanglement and other nonlocal phenomena characteristic of quantum systems.<sup>[55]</sup> Quantum coherence is not typically evident at macroscopic scales, except maybe at temperatures approaching absolute zero at which quantum behavior may manifest macroscopically.<sup>[56]</sup> This is in accordance with the following observations:

- Many macroscopic properties of a classical system are a direct consequence of the quantum behavior of its parts. For example, the stability of bulk matter, the rigidity of solids, and the mechanical, thermal, chemical, optical and magnetic properties of matter are all results of the interaction of electric charges under the rules of quantum mechanics.<sup>[57]</sup>
- While the seemingly exotic behavior of matter posited by quantum mechanics and relativity theory become more apparent for extremely small particles or for velocities approaching the speed of light, the laws of classical, often considered Newtonian, physics remain accurate in predicting the behavior of the vast majority of large objects at velocities much smaller than the velocity of light.<sup>[58]</sup>

### Copenhagen interpretation of quantum versus classical kinematics

A big difference between classical and quantum mechanics is that they use very different kinematic descriptions.<sup>[59]</sup>

In Niels Bohr's mature view, quantum mechanical phenomena are required to be experiments, with complete descriptions of all the devices for the system, preparative, intermediary, and finally measuring. The descriptions are in macroscopic terms, expressed in ordinary language, supplemented with the concepts of classical mechanics.<sup>[60][61][62][63]</sup> The initial condition and the final condition of the system are respectively described by values in a configuration space, for example a position space, or some equivalent space such as a momentum space. Quantum mechanics does not admit a completely precise description, in terms of both position and momentum, of an initial condition or state that would support a precisely deterministic and causal prediction of a final condition.<sup>[64][65]</sup> In this sense, a quantum phenomenon is a process, a passage from initial to final condition, not an instantaneous state in the classical sense of that word.<sup>[66][67]</sup> Thus there are two kinds of processes in quantum mechanics: stationary and transitional. For a stationary process, the initial and final condition are the same. For a transition, they are different. Obviously by definition, if only the initial condition is given, the process is not determined.<sup>[64]</sup> Given its initial condition, prediction of its final condition is possible, causally but only probabilistically, because the Schrödinger equation is deterministic for wave function evolution, but the wave function describes the system only probabilistically.<sup>[68][69]</sup>

For many experiments, it is possible to think of the initial and final conditions of the system as being a particle. In some cases it appears that there are potentially several spatially distinct pathways or trajectories by which a particle might pass from initial to final condition. It is an important feature of the quantum kinematic description that it does not permit a unique definite statement of which of those pathways is actually followed. Only the initial and final conditions are definite, and, as stated in the foregoing paragraph, they are defined only as precisely as allowed by the configuration space description or its equivalent. In every case for which a quantum kinematic description is needed, there is always a compelling reason for this restriction of kinematic precision. An example of such a reason is that for a particle to be experimentally found in a definite position, it must be held motionless; for it to be experimentally found to have a definite momentum, it must have free motion; these two are logically incompatible.<sup>[70][71]</sup>

Classical kinematics does not primarily demand experimental description of its phenomena. It allows completely precise description of an instantaneous state by a value in phase space, the Cartesian product of configuration and momentum spaces. This description simply assumes or imagines a state as a

physically existing entity without concern about its experimental measurability. Such a description of an initial condition, together with Newton's laws of motion, allows a precise deterministic and causal prediction of a final condition, with a definite trajectory of passage. [Hamiltonian](#) dynamics can be used for this. Classical kinematics also allows the description of a process analogous to the initial and final condition description used by quantum mechanics. [Lagrangian mechanics](#) applies to this.<sup>[72]</sup> For processes that need account to be taken of actions of a small number of [Planck constants](#), classical kinematics is not adequate; quantum mechanics is needed.

### Relation to general relativity

Even with the defining postulates of both Einstein's theory of general relativity and quantum theory being indisputably supported by rigorous and repeated [empirical evidence](#), and while they do not directly contradict each other theoretically, they have proven extremely difficult to incorporate into one consistent, cohesive model.<sup>[73]</sup>

Gravity is negligible in many areas of particle physics, so that unification between general relativity and quantum mechanics is not an urgent issue in those particular applications. However, the lack of a correct theory of [quantum gravity](#) is an important issue in [physical cosmology](#) and the search by physicists for an elegant [Theory of Everything](#). Consequently, resolving the inconsistencies between both theories has been a major goal of 20th- and 21st-century physics. Many prominent physicists, including [Stephen Hawking](#), worked for many years to create a theory underlying *everything*. The [Theory of Everything](#) would combine not only the models of subatomic physics, but also derive the four [fundamental forces of nature](#) – the [strong force](#), [electromagnetism](#), the [weak force](#), and [gravity](#) – from a single force or phenomenon. However, after considering [Gödel's Incompleteness Theorem](#), Hawking concluded that a theory of everything is not possible, and stated so publicly in his lecture "Gödel and the End of Physics" (2002).<sup>[74]</sup>

### Attempts at a unified field theory

The quest to unify the [fundamental forces](#) through quantum mechanics is ongoing. [Quantum electrodynamics](#), which is the most accurately tested physical theory in competition with general relativity,<sup>[75][76]</sup> has been merged with the weak nuclear force into the [electroweak force](#); work continues, to merge it with the strong force into the [electrostrong force](#). Current predictions state that at around  $10^{14}$  GeV these 3 forces fuse into a single field.<sup>[77]</sup> Beyond this grand unification, it is speculated that it may be possible to merge gravity with the other three gauge

symmetries, expected to occur at roughly  $10^{19}$  GeV. However – and while special relativity is parsimoniously incorporated into quantum electrodynamics – the expanded [general relativity](#), currently the best theory describing the gravitation force, has not been fully incorporated into quantum theory. One of those searching for a coherent [Theory of Everything](#) is [Edward Witten](#), a theoretical physicist who formulated the [M-theory](#), which is an attempt at describing the supersymmetrical based [string theory](#). M-theory posits that our apparent [4-dimensional spacetime](#) is, in reality, actually an 11-dimensional spacetime containing 10 spatial dimensions and 1 time dimension, although 7 of the spatial dimensions are – at lower energies – completely compactified and not readily amenable to measurement or probing.

Another popular theory is [loop quantum gravity](#) proposed by [Carlo Rovelli](#), that describes quantum properties of gravity. It is also a theory of [quantum spacetime](#) and [quantum time](#), because in general relativity the geometry of spacetime is a manifestation of [gravity](#). [Loop quantum gravity](#) is an attempt to merge and adapt standard quantum mechanics and standard [general relativity](#). This theory describes space as granular analogous to the granularity of photons in the quantum theory of electromagnetism and the discrete energy levels of atoms. More precisely, space is an extremely fine fabric or networks woven of finite loops called [spin networks](#). The evolution of a spin network over time is called a spin foam. The predicted size of this structure is the [Planck length](#), which is approximately  $1.616 \times 10^{-35}$  m. According to this theory, there is no meaning to length shorter than this.

### Philosophical implications

Since its inception, the many [counter-intuitive](#) aspects and results of quantum mechanics have provoked strong [philosophical](#) debates and many [interpretations](#). Even fundamental issues, such as [Max Born's](#) basic [rules](#) about [probability amplitudes](#) and [probability distributions](#), took decades to be appreciated by society and many leading scientists.<sup>[78]</sup><sup>[79]</sup>

The [Copenhagen interpretation](#) – due largely to Niels Bohr and Werner Heisenberg – remains most widely accepted some 75 years after its enunciation. According to this interpretation, the probabilistic nature of quantum mechanics is not a *temporary* feature which will eventually be replaced by a deterministic theory, but is instead a *final* renunciation of the classical idea of causality. It also states that any well-defined application of the quantum mechanical formalism must always make reference to the experimental arrangement, due to the [conjugate](#) nature of evidence obtained under different experimental situations.

Albert Einstein, himself one of the founders of quantum theory, did not accept some of the more philosophical or metaphysical interpretations of quantum mechanics, such as rejection of [determinism](#) and of [causality](#). He famously said about this, "God does not play with dice".<sup>[80]</sup> He rejected the concept that the state of a physical system depends on the experimental arrangement for its measurement. He held that a state of nature occurs in its own right, regardless of whether or how it might be observed. That view is supported by the currently accepted definition of a quantum state, which does not depend on the configuration space for its representation, that is to say, manner of observation. Einstein also believed that underlying quantum mechanics must be a theory that thoroughly and directly expresses the rule against [action at a distance](#); in other words, he insisted on the [principle of locality](#). He considered, but rejected on theoretical grounds, a particular proposal for hidden variables to obviate the indeterminism or acausality of quantum mechanical measurement. He believed that quantum mechanics was a currently valid but not a permanently definitive theory for quantum phenomena. He thought its future replacement would require profound conceptual advances, and would not come quickly or easily. The [Bohr-Einstein debates](#) provide a vibrant critique of the Copenhagen interpretation from an [epistemological](#) point of view. In arguing for his views, he produced a series of objections, of which the most famous has become known as the [Einstein–Podolsky–Rosen paradox](#).

[John Bell](#) showed that this [EPR paradox](#) led to [experimentally testable differences](#) between quantum mechanics and theories that rely on local hidden variables. [Experiments](#) confirmed the accuracy of quantum mechanics, thereby showing that quantum mechanics cannot be improved upon by addition of local hidden variables.<sup>[81]</sup> Alain Aspect's experiments in 1982 and many later experiments definitively verified quantum entanglement. Entanglement, as demonstrated in Bell-type experiments, does not violate [causality](#), since it does not involve transfer of information. By the early 1980s, experiments had shown that such inequalities were indeed violated in practice – so that there were in fact correlations of the kind suggested by quantum mechanics. At first these just seemed like isolated esoteric effects, but by the mid-1990s, they were being codified in the field of quantum information theory, and led to constructions with names like [quantum cryptography](#) and [quantum teleportation](#).<sup>[82]</sup> [Quantum cryptography](#) is proposed for use in high-security applications in banking and government.

The [Everett many-worlds interpretation](#), formulated in 1956, holds that *all* the possibilities described by quantum theory *simultaneously* occur in a

[multiverse](#) composed of mostly independent parallel universes.<sup>[83]</sup> This is not accomplished by introducing a new axiom to quantum mechanics, but by *removing* the axiom of the collapse of the wave packet. *All* possible consistent states of the measured system and the measuring apparatus are present in a *real* physical – not just formally mathematical, as in other interpretations – [quantum superposition](#). Such a superposition of consistent state combinations of different systems is called an [entangled state](#). While the multiverse is deterministic, we perceive non-deterministic behavior governed by probabilities, because we can only observe the universe that we, as observers, inhabit. Everett's interpretation is perfectly consistent with [John Bell's](#) experiments and makes them intuitively understandable. However, according to the theory of [quantum decoherence](#), these parallel universes will never be accessible to us. The inaccessibility can be understood as follows: once a measurement is done, the measured system becomes [entangled](#) with *both* the physicist who measured it *and* a huge number of other particles, some of which are [photons](#) flying away at the [speed of light](#) towards the other end of the universe. In order to prove that the wave function did not collapse, one would have to bring *all* these particles back and measure them again, together with the system that was originally measured. Not only is this completely impractical, but even if one *could* theoretically do this, it would have to destroy any evidence that the original measurement took place.

In light of the [Bell tests](#), Cramer in 1986 formulated his [transactional interpretation](#)<sup>[84]</sup> which is unique in providing a physical explanation for the [Born rule](#).<sup>[85]</sup> [Relational quantum mechanics](#) appeared in the late 1990s as the modern derivative of the [Copenhagen interpretation](#).

## Applications

Quantum mechanics has had enormous<sup>[18]</sup> success in explaining many of the features of our universe. Quantum mechanics is often the only theory that can reveal the individual behaviors of the [subatomic particles](#) that make up all forms of matter ([electrons](#), [protons](#), [neutrons](#), [photons](#), etc). Quantum mechanics has strongly influenced [string theories](#), candidates for a [Theory of Everything](#).

Quantum mechanics is also critically important for understanding how individual atoms are joined by covalent bonds to form [molecules](#). The application of quantum mechanics to [chemistry](#) is known as quantum chemistry. Quantum mechanics can also provide quantitative insight into [ionic](#) and [covalent bonding](#) processes by explicitly showing which molecules are energetically favorable to which others and the magnitudes of the energies involved.<sup>[86]</sup> Furthermore, most of the calculations performed in modern



[computational chemistry](#) rely on quantum mechanics.

In many aspects modern technology operates at a scale where quantum effects are significant. Important applications of quantum theory include [quantum chemistry](#), [quantum optics](#), [quantum computing](#), [superconducting magnets](#), [light-emitting diodes](#), the [optical amplifier](#) and the [laser](#), the [transistor](#) and [semiconductors](#) such as the [microprocessor](#), [medical and research imaging](#) such as [magnetic resonance imaging](#) and [electron microscopy](#).<sup>[87]</sup> Explanations for many biological and physical phenomena are rooted in the nature of the chemical bond, most notably the macro-molecule [DNA](#).<sup>[88]</sup>

### Electronics

Many modern electronic devices are designed using quantum mechanics. Examples include the [laser](#), the [transistor](#), the [electron microscope](#), and [magnetic resonance imaging](#). The study of [semiconductors](#) led to the invention of the [diode](#) and the [transistor](#), which are indispensable parts of modern [electronics](#) systems, [computer](#) and [telecommunication](#) devices. Another application is for making laser diodes and [light emitting diodes](#), which are a high-efficiency source of light.

### Cryptography

Researchers are currently seeking robust methods of directly manipulating quantum states. Efforts are being made to more fully develop [quantum cryptography](#), which will theoretically allow guaranteed secure transmission of information.

An inherent advantage yielded by quantum cryptography when compared to classical [cryptography](#) is the detection of passive [eavesdropping](#). This is a natural result of the behavior of quantum bits; due to the [observer effect](#), if a bit in a superposition state were to be observed, the superposition state would collapse into an [eigenstate](#). Because the intended recipient was expecting to receive the bit in a superposition state, the intended recipient would know there was an attack, because the bit's state would no longer be in a superposition.<sup>[89]</sup>

### Quantum computing

Another goal is the development of [quantum computers](#), which are expected to perform certain computational tasks exponentially faster than classical [computers](#). Instead of using classical bits, quantum computers use [qubits](#), which can be in [superpositions](#) of states. Quantum programmers are able to manipulate the superposition of qubits in order to solve problems that classical computing cannot do effectively, such as searching unsorted databases or [integer factorization](#). [IBM](#) claims that the advent of quantum computing may progress the fields of

medicine, logistics, financial services, [artificial intelligence](#) and cloud security.<sup>[90]</sup>

Another active research topic is [quantum teleportation](#), which deals with techniques to transmit quantum information over arbitrary distances.

### Macroscale quantum effects

While quantum mechanics primarily applies to the smaller atomic regimes of matter and energy, some systems exhibit [quantum mechanical effects](#) on a large scale. [Superfluidity](#), the frictionless flow of a liquid at temperatures near [absolute zero](#), is one well-known example. So is the closely related phenomenon of [superconductivity](#), the frictionless flow of an electron gas in a conducting material at sufficiently low temperatures. The [fractional quantum Hall effect](#) is a [topological ordered](#) state which corresponds to patterns of long-range [quantum entanglement](#).<sup>[91]</sup> States with different topological orders cannot change into each other without a phase transition.

### Other phenomena

Quantum theory also provides accurate descriptions for many previously unexplained phenomena, such as [black-body radiation](#) and the stability of the [orbitals](#) of electrons in atoms. It has also given insight into the workings of many different [biological systems](#), including [smell receptors](#) and [protein structures](#).<sup>[92]</sup> Recent work on [photosynthesis](#) has provided evidence that quantum correlations play an essential role in this fundamental process of plants and many other organisms.<sup>[93]</sup> Even so, [classical physics](#) can often provide good approximations to results otherwise obtained by quantum physics, typically in circumstances with large numbers of particles or large [quantum numbers](#). Since classical formulas are much simpler and easier to compute than quantum formulas, classical approximations are used and preferred when the system is large enough to render the effects of quantum mechanics insignificant.

### Examples

#### Free particle

For example, consider a [free particle](#). In quantum mechanics, a free matter is described by a wave function. The particle properties of the matter become apparent when we measure its position and velocity. The wave properties of the matter become apparent when we measure its wave properties like interference. The [wave-particle duality](#) feature is incorporated in the relations of coordinates and operators in the formulation of quantum mechanics. Since the matter is free, its quantum state can be represented as a [wave](#) of arbitrary shape and extending over space as a [wave function](#). The position and momentum of the particle are [observables](#). The [Uncertainty Principle](#) states that

both the position and the momentum cannot simultaneously be measured with complete precision. However, one *can* measure the position of a moving free particle, creating an eigenstate of position with a wave function that is very large at a particular position  $x$ , and zero everywhere else. If one performs a position measurement on such a wave function, the resultant  $x$  will be obtained with 100% probability. This is called an eigenstate of position – or, stated in mathematical terms, a *generalized position eigenstate*. If the particle is in an eigenstate of position, then its momentum is completely unknown. On the other hand, if the particle is in an eigenstate of momentum, then its position is completely unknown.<sup>[94]</sup> In an eigenstate of momentum having a [plane wave](#) form, it can be shown that the [wavelength](#) is equal to  $h/p$ , where  $h$  is [Planck's constant](#) and  $p$  is the momentum of the [eigenstate](#).<sup>[95]</sup>

### Particle in a box

The particle in a one-dimensional potential energy box is the most mathematically simple example where restraints lead to the quantization of energy levels. The box is defined as having zero potential energy everywhere *inside* a certain region, and therefore infinite potential energy everywhere *outside* that region.

### Finite potential well

A finite potential well is the generalization of the infinite potential well problem to potential wells having finite depth. The finite potential well problem is mathematically more complicated than the infinite particle-in-a-box problem as the wave function is not pinned to zero at the walls of the well. Instead, the wave function must satisfy more complicated mathematical boundary conditions as it is nonzero in regions outside the well.

### Rectangular potential barrier

This is a model for the [quantum tunneling](#) effect which plays an important role in the performance of modern technologies such as [flash memory](#) and [scanning tunneling microscopy](#). Quantum tunneling is central to physical phenomena involved in [superlattices](#).

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