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Analysis Of Differential Equations By Using Fourier Transformation

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Abstract: The linear non-homogeneous differential equations are generally solved by adopting by Laplace transform method or by method of variation of parameters or by method of undetermined coefficients. The paper inquires the linear non-homogeneous differential equations by applying Fourier Transformation. The purpose of paper is to prove the applicability of Fourier Transformations for the analysis of linear non-homogeneous differential equations.

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Introduction:

It has been noticed that Fourier transformation is helpful for scientists, researches and engineers in number of ways. It is a mathematical tool which is used in the solving the linear non-homogeneous differential equations by converting it from one form into another form. It is used in solving different types of problems in Physics, material sciences etc. [1-15]. It is also used to convert the signal system into frequency domain for solving it on a simple and easy way [16-27]. We can also apply it to analyze non-homogeneous differential equations without solving the corresponding homogeneous differential equations. It has wide applications in different fields of engineering and technology besides basic sciences and mathematics [28-40].

The Fourier transform is linear transform

$$F\,[f] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)e^{i\alpha t}dt$$

then for any constant a and b.

let f be continuous and piecewise smooth in $(-\infty,\infty)$. let $f(t)$ approch zero as $|t| \to \infty$.

If f and f' are absoloutly integrable, then

$$F\,[f'(t)] = -i\alpha\, F\,[f]$$

Proof:

$$F\,[f'(t)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f'(t)e^{i\alpha t}dt$$

$$= \frac{1}{\sqrt{2\pi}}[f(t)|_{-\infty}^{\infty} - i\alpha\int_{-\infty}^{\infty} f(t)e^{i\alpha t}$$

$$= -i\alpha\, F\,[f]$$

this result can be easily extended as

$$F\,[f^n(t)] = (-i\alpha)^n\, F\,[f] \quad n=0, 1, 2, 3\ldots$$

Formulation

1. Solve by Fourier Transformation

$$\frac{d^2y}{dt^2} + 10a\frac{dy}{dt} + 9a^2y = e^{-a|t|}$$

This can be written as

$$(D^2 + 10aD + 9a^2)y = e^{-a|t|}$$

Taking Fourier Transform on both sides

$$F\{y''\} + 10aF\{y'\} + 9a^2F\{y\} = F\{e^{-a|t|}\}$$

$$[\,(ip)^2 + 10aip + 9a^2]F\{p\} = \frac{2a}{p^2 + a^2}$$

$$\text{let } u(ip) = [\,(ip)^2 + 10aip + 9a^2] \text{ and } F\{e^{-a|t|}\} = f\,(p)$$

Then the solution is

$$y\,(t) = \frac{1}{2\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\left\{\frac{f\,(p)e^{ipt}}{u(ip)}\right\}dp$$

$$y\,(t) = \frac{1}{2\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\frac{2a}{p^2 + a^2}\left\{\frac{e^{ipt}}{(ip)^2 + 10aip + 9a^2}\right\}dp$$

$$= -\frac{1}{2\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\frac{2a}{p^2 + a^2}\left\{\frac{e^{ipt}}{(p-9ai)(p-ai)}\right\}dp$$

or

$$-\frac{1}{2\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\left\{\frac{2ae^{ipt}}{(p-9ai)(p+ai)(p-ai)^2}\right\}dp$$

$$-\frac{a}{\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\left\{\frac{e^{ipt}}{(p-9ai)(p+ai)(p-ai)^2}\right\}dp$$

Case-I: The singularities with in contour are

1. simple pole $p = 9ai$,
2. double pole $p = ai$

Therefore, $t > 0$,

we have by cauchy's residue theorem

$$y(t) = -\frac{a}{\pi}\, 2\,\pi i\,[\text{ sum of the residues of the integral at the singularities}] \ldots\ldots (1)$$

Residue(p = 9ai), a simple pole

$$= \lim_{p\to 9ai}\left(\frac{e^{ipt}}{(p+ai)(p-ai)^2}\right)$$

$$\frac{ie^{-9at}}{640a^3}$$

Residue(p = ai), a simple pole

$$= \lim_{p\to ai}\frac{d}{dp}\left(\frac{e^{ipt}}{(p+ai)(p-9ai)}\right)$$

$$= \frac{ite^{-at}}{16a^2} + \frac{3\,ie^{-at}}{128a^3}$$

Hence, from (1),

$$y(t) = -2ai.\left[\frac{ie^{-9at}}{640a^3} + \frac{ite^{-at}}{16a^2} + \frac{3\,ie^{-at}}{128a^3}\right]$$

Case-I: when $t < 0$

If $t < 0$, then we closed the contour in the Lower half plane and hence the simple pole $p = -ai$ is the only singularity, therefore,

$$y(t) = -(-\frac{a}{\pi})\, 2\,\pi i\,[\text{ residue of integrand at the singularity } p = -ai]$$

$$= 2ai\,[\text{residue at } p = -ai] \ldots\ldots (2)$$

But,

Residue(p = ai), a simple pole

$$= \lim_{p\to -ai}\left(\frac{e^{ipt}}{(p-ai)^2(p-9ai)}\right)$$

$$= \lim_{p\to -ai}\left(\frac{e^{ipt}}{(p-ai)^2(p-9ai)}\right)$$

$$= \frac{ie^{-at}}{40a^3}$$

Hence, from (1),

$$= 2ai\,[\text{residue at } p = -ai$$

$$= 2ai\left[\frac{ie^{-at}}{40a^3}\right]$$

$$y\,(t) = \left[-\frac{e^{-at}}{20a^2}\right]$$

2. Solve by Fourier Transformation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = \text{cost}$$

This can be written as

$$(D^2 + 5D + 4)y = \text{cost}$$

Taking Fourier Transform on both sides

$$F\{y''\} + 5F\{y'\} + 4F\{y\} = F\{\text{cost}\}$$

$$\text{or } [\,(ip)^2 + 5ip + 4]F\{p\} = \frac{ip}{1-p^2}$$

$$\text{let } u(ip) = [\,(ip)^2 + 5ip + 4] \text{ and } F\left\{\frac{ip}{1-p^2}\right\} = f\,(p)$$

Then the solution is

$$y\,(t) = \frac{1}{2\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\left\{\frac{f\,(p)e^{ipt}}{u(ip)}\right\}dp$$

Or

$$y\,(t) = \frac{1}{2\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\frac{ip}{1-p^2}\left\{\frac{e^{ipt}}{[\,(ip)^2 + 5ip + 4]}\right\}dp$$

$$= -\frac{1}{2\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\frac{ip}{1-p^2}\left\{\frac{e^{ipt}}{-[p^2 - 5ip - 4]}\right\}dp$$

Or

$$\frac{i}{2\pi}\int_{-\infty+i\gamma}^{\infty+i\gamma}\left\{\frac{pe^{ipt}}{(p-1)(p+1)(p-4i)(p-i)}\right\}dp$$

$$y(t) = \frac{i}{2\pi}\, 2\,\pi i\,[\text{ sum of the residues of the integral at the singularities}]$$

$$y(t) = -[\text{ sum of the residues of the integral at the singularities}] \ldots\ldots (3)$$

$$\text{Residue}(p = 1), \text{a simple pole} = \lim_{p\to 1}\left\{\frac{pe^{ipt}}{(p+1)(p-4i)(p-i)}\right\}$$

$$\frac{e^{it}}{-2(3+5i)}$$

Now,

$$\text{Residue}(p = -1), \text{a simple pole} = \lim_{p\to -1}\left\{\frac{pe^{ipt}}{(p-1)(p-4i)(p-i)}\right\}$$

$$\frac{e^{-it}}{2(-3+5i)}$$

And,

$$\text{Residue}(p = i), \text{a simple pole} = \lim_{p\to i}\left\{\frac{pe^{ipt}}{(p-1)(p-4i)(p+1)}\right\}$$

$$\frac{e^{-t}}{6}$$

And,

$$\text{Residue}(p = 4i), \text{a simple pole} = \lim_{p\to i}\left\{\frac{pe^{ipt}}{(p-1)(p+i)(p+1)}\right\}$$

$$\frac{-4e^{-4t}}{51}$$

From (3),

$$y(t) = -[\text{ sum of the residues of the integral at the singularities}]$$

$$y(t) = \frac{e^{it}}{-2(3+5i)} + \frac{e^{-it}}{2(-3+5i)} + \frac{e^{-t}}{6} - 4\frac{e^{-4t}}{51}$$

Conclusion:

In this paper we have applied the Fourier transformation Fourier Transformations for the analysis of linear non-homogeneous differential equations. It has been noticed that this technique is very much capable in finding solutions of linear non-homogeneous differential equations.

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