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Study On New Techniques For Solving Non-Linear Equations

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Abstract: Macroevolution is a new kind of high-level species evolution that can avoid premature convergence that may arise during the selection process of conventional genetic algorithms. The MMGA enriches the capabilities of genetic algorithms to handle multiobjective problems by diversifying the solution set. Monitoring complex environmental systems is extremely challenging because it requires environmental professionals to capture impacted systems' governing processes, elucidate human and ecologic risks, limit monitoring costs, and satisfy the interests of multiple stakeholders (e.g., site owners, regulators, and public advocates).
[Kumar, R. and Yadav, A. **Study On New Techniques For Solving Non-Linear Equations.** *Researcher* 2020;12(7):28-32]. ISSN 1553-9865 (print); ISSN 2163-8950 (online). http://www.sciencepub.net/researcher. 6. doi:10.7537/marsrsj120720.06.

Keywords: Tecniques, Linear, Quadratic Algebraic Equation

Introduction:

Numerical methods are used to approximate solutions of equations when exact solutions can not be determined via algebraic methods. They construct successive approximations that converge to the exact solution of an equation or system of equations. In Math, we focused on solving nonlinear equations involving only a single variable. We used methods such as Newton's method, the Secant method, and the Bisection method. We also examined numerical methods such as the Runge-Kutta methods, that are used to solve initial-value problems for ordinary differential equations. However these problems only focused on solving nonlinear equations with only one variable, rather than nonlinear equations with several variables.

System of nonlinear equations arise in many domains of practical importance such as engineering, mechanics, medicine, chemistry, and robotics. Solving such a system involves finding all the solutions (there are situations when more than one solution exists) of the polynomial equations contained in the mentioned system. The problem is nondeterministic polynomial-time hard, and it is having very high computational complexity due to several numerical issues[27]. There are several approaches for solving these types of problems. Van Hentenryck et al. [27] divided these approaches into two main categories: 1) interval methods that are generally robust but tend to be slow; 2) continuation methods that are effective for problems for which the total degree is not too high[27]. The limitations of Newton's method are pointed out in the aforementioned works.

Bader[5] mentioned that standard direct methods, such as Newton's method, are impractical for large-scale problems because of their high linear algebra Manuscript received September 9, 2006; revised March 22, 2007. Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org. Digital Object Identifier 10.1109/TSMCA.2008.918599 costs and large memory requirements. Bader proposed a tensor method using Krylov subspace methods for solving largescale systems of linear equations. There is a condition to be fulfilled—the equations must be continuously differentiable at least once. Bader's paper also provides a good review of similar research for solving systems of equations. Krylov subspace methods based on moment matching are also used by Salimbahrami and Lohmann[41].

Effati and Nazemi[18] proposed a very efficient approach for solving nonlinear systems of equations. Although there are several existing approaches for solving systems of nonlinear equations, there are still limitations of the existing techniques, and, still, more research is to be done. There is a class of methods for the numerical solutions of the above system, which arise from iterative procedures used for systems of linear equations[2,1].

These methods use reduction to simpler 1-D nonlinear equations for the components f1, f2,...,fn[26]. In a strategy based on trust regions[30], at each iteration, a convex quadratic function is minimized to determine the next feasible point to step to. The convex quadratic function is the squared norm of the original system plus a linear function multiplied by the Jacobian matrix. There is also the approach of homotopy methods, which are sometimes referred to as continuation methods[5,6,7].

This approach begins with a "starting" system of equations (not the true system) whose solution is known. This starting system is gradually transformed to the original system. At each stage, the current system is solved to find a starting solution for the next stage system. The idea is that as the system changes, the solutions trace out a path from a solution of the starting system to a solution of the original system. At each stage, the current system is normally solved by a Newton-type method[8].

Review of Literature:

Sfard and Linchevski (1994) have analysed the nature and growth of algebraic thinking from an epistemological perspective supported by historical observations. They indicated that the development of algebraic thinking was a sequence of ever more advanced transitions from operational (procedural) to structural outlooks.

Mason (2000) has argued that "... the style and the nature of questions encountered by students strongly influences the sense that they make of the subject matter". The questions that come to the mind of an educator are influenced by the perspective and disposition that he/she has towards mathematics and pedagogy. These questions in turn influence the sense learners make of the subject matter. In this article I focus on the outcomes and implications of research on (a) use of symbols in mathematics, (b) algebraic/trigonometric expressions, (c) solving equations, and (d) functions and calculus.

When introducing algebra the use of letters should be withheld until it is evident that learners are ready for their use, and teaching should recognise and prepare learners for the various uses of letters in algebra as the need arises (Harper, 1987; Stols, 1996).

The Historical Roots of Elementary Mathematics (Bunt, Jones, & Bedient, 1976) is very similar in style and information to Math through the Ages. Both books present information in short chapters specific to a main idea (e.g. Greek numeration systems). In addition, both books cover a wide range of topics that are broken down by date. However, The Historical Roots of Elementary Mathematics does not delve into the stories describing the people behind the discoveries. The four volume collection.

The World of Mathematics (Newman, 1956) consists of individual articles compiled together in an effort to convey the "...diversity, the utility and the beauty of mathematics" (Newman, iii). Newman attempted to show the richness and range of mathematics. This collection spans ideas from the Rhind Papyrus to the "Statistics of Deadly Quarrels" (Newman, 1956).

Approximation Techniques by Circles

1.2.1 Case I: Consider the equation $f(x) = 0$ whose one or more roots are to be found. Let $y = f(x)$ represents the graph of the function $f(x)$. Let $x = x_0$ is an initial estimate of the desired root. A circle C_1 with centre $[x_0+h, f(x_0+h)]$ and radius $f(x_0+h)$ is drawn on the curve $y = f(x)$, where h is a small positive or negative quantity. Another circle C_2 with centre $[x_0-h, f(x_0-h)]$ and radius $f(x_0-h)$ is drawn on the curve $y = f(x)$. The authors have considered following two cases:

(a) Circles C_1 and C_2 touch externally

(b) Circles C_1 and C_2 intersect orthogonally

1.2.1. (a) External touch technique: The circle C_2 will touch circle C_1 externally *iff* sum of their radii = distance between their centers, *i.e.*, if

$$f(x_0+h)+f(x_0-h)=\sqrt{(x_0+h-x_0+h)^2+\left[f(x_0+h)-f(x_0-h)\right]^2},$$

$$\Rightarrow 4h^2-4f(x_0+h)f(x_0-h)=0,$$

Expanding by Taylor's theorem, and retaining the terms up to O (h^2),

$$h^2-\left\{\left[f(x_0)+hf'(x_0)+\frac{h^2}{2!}f''(x_0)+\ldots\right]\left[f(x_0)-hf'(x_0)+\frac{h^2}{2!}f''(x_0)-\ldots\right]\right\}=0$$

$$\Rightarrow h^2-\left\{[f(x_0)]^2+hf(x_0)f'(x_0)+\frac{h^2}{2!}f''(x_0)f(x_0)-hf'(x_0)f(x_0)-h^2[f'(x_0)]^2+\frac{h^2}{2!}f(x_0)f''(x_0)\right\}=0$$

$$\Rightarrow h^2[1+f'^2(x_0)-f(x_0)f''(x_0)]=f^2(x_0),$$

which on simplification gives

$$h=\pm\frac{f(x_0)}{\sqrt{1+f'^2(x_0)-f(x_0)f''(x_0)}},$$

where h can be taken positive or negative according as x_0 lies in the left or right of true root. If x_0 lies in the left of true root, then h is taken as positive otherwise, negative. Therefore, first approximation can be written as:

$$x_1=x_0\pm\frac{f(x_0)}{\sqrt{1+f'^2(x_0)-f(x_0)f''(x_0)}}$$

The general formula for successive approximation is, therefore, given by

$$x_{n+1}=x_n\pm\frac{f(x_n)}{\sqrt{1+f'^2(x_n)-f(x_n)f''(x_n)}},\ (n\geq 0). \quad \ldots(1.1)$$

The sufficient condition for convergence is given by

$$f(x_n)f''(x_n)<1+f'^2(x_n).$$

1.2.1(b) Orthogonal intersection technique: The circle C_2 will intersect the circle C_1 orthogonally iff sum of the square of their radii = square of the distance between their centers, *i.e.,* if

$$f^2(x_0+h)+f^2(x_0-h)=(x_0+h-x_0+h)^2+[f(x_0+h)-f(x_0-h)]^2,$$

Solving as in 1.2.1(a) above, one gets,

$$h=\pm\frac{f(x_0)}{\sqrt{2+f'^2(x_0)-f(x_0)f''(x_0)}},$$

And the general approximation to the root is given by

$$x_{n+1}=x_n\pm\frac{f(x_n)}{\sqrt{2+f'^2(x_n)-f(x_n)f''(x_n)}},\ (n\geq 0). \quad \ldots(1.2)$$

The sufficient condition for convergence is given by

$$f(x_n)f''(x_n)<2+f^2(x_n).$$

1.2.1(c) Special circle's formulae: If the Taylor's series are expanded upto $O(h)$, then the formulae (1.1) and (1.2) reduce to the following general formulae:

$$x_{n+1}=x_n\pm\frac{f(x_n)}{\sqrt{1+f'^2(x_n)}}, \quad \ldots(1.3)$$

and

$$x_{n+1}=x_n\pm\frac{f(x_n)}{\sqrt{2+f'^2(x_n)}}. \quad \ldots(1.4)$$

1.2.2 Case II: As in the previous case; again it is assumed that an initial estimate $x=x_0$ is an initial estimate of the desired root. A circle C_1 with centre $[x_0, f(x_0)]$ and radius $f(x_0)$ is drawn on the curve $y=f(x)$, where h is a small positive or negative quantity. Another circle C_2 is drawn with centre $[x_0+h, f(x_0+h)]$ and radius $f(x_0+h)$ on the curve $y=f(x)$. The authors have again considered the cases of external touch and orthogonal intersection of circles.

1.2.2(a) External touch technique: Circle C_2 will touch circle C_1 externally *iff* sum of the radii of two circles =distance between their centers, *i.e.,* if

$$f(x_0+h)+f(x_0)=\sqrt{(x_0+h-x_0)^2+[f(x_0+h)-f(x_0)]^2},$$

Solving as in 1.2.1(a) above, one gets,

$h = 2f(x_0)\,[\,f'(x_0) \pm \sqrt{1+f'^2(x_0)}\,],$

the sign is taken in such a manner that the numerator becomes smallest. To minimize the errors implicit in this formula, the numerator is rationalized as:

$$h = 2f(x_0)\,[\,f'(x_0) \pm \sqrt{1+f'^2(x_0)}\,] \times \frac{f'(x_0) \mp \sqrt{1+f'^2(x_0)}}{f'(x_0) \mp \sqrt{1+f'^2(x_0)}}$$

$$\Rightarrow \quad h = \frac{-2f(x_0)}{f'(x_0) \pm \sqrt{1+f'^2(x_0)}}, \qquad \ldots(1.5)$$

in this the sign is so chosen as to make the denominator largest in magnitude.

The general formula for successive approximation to the root is given by

$$x_{n+1} = x_n - \frac{2f(x_n)}{f'(x_n) \pm \sqrt{1+f'^2(x_n)}}, \; (n \geq 0) \qquad \ldots(1.6)$$

1.2.3(b) Orthogonal touch technique: The circle C_2 will intersect the circle C_1 orthogonally iff sum of the square of radii = square of the distance between centers, *i.e.,* if

$f^2(x_0 + h) + f^2(x_0) = (x_0 + h - x_0)^2 + [f(x_0 + h) - f(x_0)]^2 = 0,$

Solving as in 1.2.1(a) above, one gets, $h = f(x_0)[f'^2(x_0) \pm \sqrt{f'(x_0) + 2}\;]$

The general formula for successive approximation to the root is given by

$$x_{n+1} = x_n - \frac{2f(x_n)}{f'(x_n) \pm \sqrt{2+f'^2(x_n)}} \qquad \ldots(1.7)$$

Geometrically, the methods given above consist of replacing the part of the curve between the point and x-axis by means of circles. These methods repeated until there is a circle with radius sufficiently close to zero, which may be a point circle of the intersection of the curve with x-axis and hence giving the required approximate root.

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7/25/2020