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Properties of Solutions to the Cauchy Problem for Degenerate Nonlinear Cross-Systems with Absorption

Sadullaeva Shahlo Azimbayevna[1], Saida Safibullayevna Beknazarova[2], Abdurakhmanov Kaxar Patahovich[3]

[1.] doctor of physical –mathematics science, vice rector of Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi, Tashkent, Uzbekistan
sh.sadullaeva@tuit.uz
[2.] doctor of technical science, professor Audiovisual technologies of Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi, Tashkent, Uzbekistan
saida.beknazarova@gmail.com
[3.] doctor of physical –mathematics science, professor of Physic of Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi, Tashkent, Uzbekistan
k.abdurakhmanov@mail.ru

Abstract: This article describe the ways of development of methods and algorithms for visualization of processes in multicomponent media with two nonlinearities, for the divergent case taking into account convective transport. Visualization of nonlinear processes describing systems of reaction-diffusion processes with double nonlinearity in one-dimensional and multidimensional cases. To establish the properties of global solvability and time-unsolvability of solutions of a nonlinear mathematical filtration model in two-component media associated with non-local boundary conditions. Establish critical exponents of the Fujita type for a mathematical model for nonlinear filtration systems in two component media with source or absorption. The object of research is the qualitative properties of solutions of polytropical filtration systems in a two-component medium with non-local boundary conditions. In the article uses the nonlinear splitting algorithm, the method of reference equations, the method of comparing solutions, iterative numerical methods, methods of variable directions and runs.
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1. Introduction

Recently, there has been a great interest in the study of nonlinear modeling of a wide variety of phenomena and processes occurring in mechanics, physics, technology, Biophysics, biology, ecology and many other fields of knowledge described by nonlinear differential equations. Especially for ultra-fast processes occurring in blow-up modes, in which the studied function (solution) turns to infinity in a finite time at individual points or in an entire region of space [13]. In many physical processes, these solutions occur (for example, Gorenje processes). In this regard, A. A. Samarsky, S. P. Kurdyumov, V. A. Galaktionov and others began to develop the theory and practice of studying such problems. Unlimited solutions were called the mode with aggravation (in foreign literature called blow-up). In this case, a special place is occupied by quasi-linear equations of parabolic type modeling various processes occurring in the natural Sciences.

The widespread mathematical model of processes described by quasi-linear parabolic equations is explained by the fact that they are derived from fundamental conservation laws (energy, mass, number of particles, etc.). Therefore, it is possible that two physical processes that at first glance have nothing in common (for example, thermal conductivity in semiconductors and the process of magnetic field propagation in a medium with finite conductivity) are described by the same nonlinear diffusion equation, only with different numerical parameters.

One of the most relevant areas of mathematical modeling is the study of nonlinear mathematical models of various physical, biological, chemical and other phenomena and processes. Examples include such physical theories as nonlinear quantum mechanics, nonlinear electrodynamics and optics, nonlinear plasma theory, nonlinear acoustics, nonlinear thermal conductivity, nonlinear diffusion, and other theories based on nonlinear partial differential equations.

In the General case the difference of quasilinear parabolic equations that underlie the mathematical models of various processes is the dependence of equation coefficients (thermal conductivity, transfer, power, volumetric energy sources and sinks) of variables that determine the state of the environment, i.e. temperature, density, magnetic field, etc.

It is hardly possible to characterize even a simple enumeration of the main results obtained in the theory of nonlinear parabolic equations. Note only that for wide classes of equations, the principal problems of solvability and uniqueness of solutions to various boundary value problems have been solved, and the differential properties of solutions have been studied in detail. The General results of the theory allow us to analyze entire classes of equations of the same type from these positions.

The main problems that arise in the study of a mathematical model of complex real physical processes are primarily related to the non-linearity of the equations underlying the mathematical model. The first consequence of non-linearity is the absence of the superposition principle inherent in linear homogeneous problems. This ensures the inexhaustibility of the set of possible directions of the dissipative process evolution, and also determines the appearance of discrete space-time scales in a continuous environment. They characterize properties of a nonlinear medium that do not depend on external influence. Nonlinear dissipative environments may exhibit some internal ordering, which is characterized by the spontaneous appearance of complex dissipative structures in the environment.

A special place in the theory of nonlinear equations is occupied by the circle of studies of unlimited solutions, or, as they are otherwise called, modes with aggravation. Nonlinear evolutionary problems that allow unlimited solutions are globally (in time) unsolvable: solutions increase indefinitely over a finite period of time. For a long time, they were considered in theory as some exotic examples, suitable only for establishing the degree of optimality of global solvability conditions as a natural "physical" requirement. However, we should note that the first successful attempts to derive unbounded solutions to nonlinear parabolic problems were made more than 50 years ago. Today, the theory of modes with aggravation is a vast field in which thousands of works have been performed, in which there are outstanding achievements.

Today, in the world practice of the field of natural Sciences, the study of nonlinear mathematical models is considered one of the most urgent tasks in the development of methods for improving the efficiency of the reaction-diffusion control system. According to the Elsevier database, the number of scientific papers by scientists around the world devoted to the study of the nonlinear reaction-diffusion equation, as well as the Cauchy problem and boundary value problems for this equation, and the practical application of the research results is constantly growing.

In the Republic of Uzbekistan, large-scale work has been carried out on the effective organization of events dedicated to the creation of automated systems for computer visualization of diffusion processes, mathematical modeling of diffusion processes described by nonlinear equations with double nonlinearity in an inhomogeneous environment. From this point of view, a number of research works are being carried out to improve the methods of research and visualization of nonlinear processes, to create automated production systems that play an important role in the study of mathematical models of nonlinear processes.

Currently, a number of fundamental problems in the world require mathematical modeling of nonlinear processes, improvement of visualization methods and tools, as well as application to practice of the obtained important results of reaction-diffusion problems with double nonlinearity. Currently, for the study of equations with double non-linearity and practical application, conducting targeted research in the following areas is considered one of the important tasks: development of visualization methods in the study of nonlinear models; creating software systems that help study nonlinear processes; creating a technology for conducting computational experiments, monitoring the evolution of the process over time, creating a computerized system for determining properties that depend on the dynamics of changes in parameters.

2. Material and Methods

On properties of solutions to the Cauchy problem for degenerate nonlinear cross-systems with absorption

In the domain $Q=\{(t,x): t>0, x\in R^N\}$ properties of solutions of a nonlinear diffusion-reaction system with variable density are studied:

$$L_1(u,v) = -|x|^l \frac{\partial u}{\partial t} + div\left(|x|^n v^{m_1-1}|\nabla u^k|^{p-2}\nabla u\right) - |x|^l \gamma_1(t) u = 0$$
$$L_2(u,v) = -|x|^l \frac{\partial v}{\partial t} + div\left(|x|^n u^{m_2-1}|\nabla v^k|^{p-2}\nabla v\right) - |x|^l \gamma_2(t) v = 0 \quad (1)$$

$$u(0,x)=u_0(x)\ge 0, v(0,x)=v_0(x)\ge 0,\ x\in R^N, \quad (2)$$

where $k\ge 1, n, l, p, m_i, i=1,2$ - specified numerical parameters of a nonlinear medium, $\nabla(.) - grad_x(.)$, functions $u_0(x),\ v_0(x)\ge 0$, $x\in R^N$, $0<\gamma_i(t)\in C(0,\infty),\ i=1,2$.

The system (1) describes a set of physical processes, such as the process of mutual reaction-diffusion, thermal conductivity, polytrophic filtration of liquid and gas in a nonlinear medium whose capacity is the same $\gamma_1(t)u$, $\gamma_2(t)v$. Various special cases of the problem (1) - (2) are considered in many. In particular, we obtained conditions for the existence of solutions to the Cauchy problem in time. The system (1) in the domain of degeneracy and in the domain of degeneracy may not have a classical solution. Therefore, weak solutions of the system (1) with physical meaning are studied: $0\le u,v\in C(Q)$ and also $|x|^n v^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u,\ |x|^n u^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v\in C(Q)$ satisfying some integral identity [8]. Various qualitative properties of a nonlinear degenerate system and a dual nonlinear equation, in particular the equations of porous media, have been intensively studied (see [8] and the literature in them). For example, the asymptotic behavior of the destroyed interface solutions of a one-dimensional nonlinear equation of a porous medium in the inhomogeneous case $|x|^{-l}\frac{\partial u}{\partial t}=\Delta u^m$, where $m>1,\ \Delta=\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}$ (3) when l> 2 studied in [1,2,5,6,7]. Relative to the initial data, it is assumed that they are smooth, bounded, compactly supported, symmetrical, and monotonous. Behavior over a long time for the equation of an inhomogeneous porous medium (3) in a medium with a slowly decreasing density at $0<l<2$ have been considered in other works of the authors. Case $l=2$ it is called a critical case [8(119 p.)]. In this paper, on the basis of reducing the original system (3.71) with variable density to a "radially symmetric" form, we can consider the following cases $l<2,\ l>1$ and also $l=2$ from one point of view, this makes it relatively easy to build solutions of the Zeldovich-Companeitz type, thanks to which new nonlinear effects are established. Based on the method of the standard equation and the principle of comparing the conditions of emerging phenomena, the final velocity of perturbation propagation and the localization of space are established. An estimation of the weak solution and free boundary for solutions of the Cauchy problem for a cross-system with variable density is obtained. The naturally established approach is valid as for the porous medium equation (2).

Below we will show the phenomenon of propagation perturbation at the finite velocity and localization of space.

3. Results

The phenomenon of propagation violation at a finite speed and localization of space.

Consider the following self-similar system

$$\xi^{1-S}\frac{d}{d\xi}\left(\xi^{S-1}\psi^{m_1-1}\left|\frac{df^k}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right)+\frac{\xi}{p}\frac{df}{d\xi}=0$$
$$\xi^{1-S}\frac{d}{d\xi}\left(\xi^{S-1}f^{m_2-1}\left|\frac{d\psi^k}{d\xi}\right|^{p-2}\frac{d\psi}{d\xi}\right)+\frac{\xi}{p}\frac{d\psi}{d\xi}=0 \quad (4)$$

It is obtained by transformations

$$u(t,x)=\bar{u}(t)w(\tau(t),x),$$
$$v(t,x)=\bar{v}(t)\,z(\tau(t),x), \quad (5)$$

Where

$$\bar{u}(t)=e^{-\int_0^t\gamma_1(y)dy},\quad \bar{v}(t)=e^{-\int_0^t\gamma_2(y)dy)} \quad (6)$$

and function $\tau(t)$ - was selected as $\tau(t)=\int_0^t\bar{v}^{m_1-1}(\eta)\bar{u}^{p-2}(\eta)d\eta=\int_0^t\bar{u}^{m_2-1}(\eta)\bar{v}^{p-2}(\eta)d\eta$

Then the functions $w(\tau(t),x),\ z(\tau(t),x),$ satisfy the system

$$\frac{\partial w}{\partial\tau}=div\left(z^{m_1-1}\left|\nabla w^k\right|^{p-2}\nabla w\right)$$
$$\frac{\partial v}{\partial\tau}=div\left(u^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v\right) \quad (7)$$

It is obvious that the system (6) the so-called re-symmetric form $w(\tau,x)=w(\tau,\varphi(|x|)),\ z(\tau,x)=z(\tau,\varphi(|x|)),\ |x|=\sum_{i=1}^{N}x_i^2$ - which satisfy the so-called "radially symmetric" system

$$\frac{\partial w}{\partial\tau}=\varphi^{1-S}\frac{\partial}{\partial\varphi}\left(\varphi^{S-1}(z^{m_1-1}\left|\frac{\partial}{\partial\varphi}w^k\right|^{p-2}\frac{\partial\omega}{\partial\varphi}\right)$$
$$\frac{\partial v}{\partial\tau}=\varphi^{1-S}\frac{\partial}{\partial\varphi}\left(\varphi^{S-1}(u^{m_2-1}\left|\frac{\partial}{\partial\varphi}v^k\right|^{p-2}\frac{\partial v}{\partial\varphi}\right) \quad (8)$$

If

$$\varphi(|x|)=\frac{p-(n+l)}{p}|x|^{\frac{p}{p-(n+l)}},\ n+l<p \quad (9)$$

Set in (8) $w(\tau,x)=f(\xi),\ z(\tau,x)=\psi(\xi),$ $\xi=\frac{\varphi(|x|)}{[\tau(t)]^{1/p}}$

we have the following self-similar system

$$\xi^{1-S}\frac{d}{d\xi}\left(\xi^{S-1}\psi^{m_1-1}\left|\frac{df^k}{d\xi}\right|^{p-2}\frac{df}{d\xi}\right)+\frac{\xi}{p}\frac{df}{d\xi}=0$$

$$\xi^{1-S}\frac{d}{d\xi}\left(\xi^{S-1}f^{m_2-1}\left|\frac{d\psi^k}{d\xi}\right|^{p-2}\frac{d\psi}{d\xi}\right)+\frac{\xi}{p}\frac{d\psi}{d\xi}=0 \quad (10)$$

where $S=\frac{(N-l)p}{p-(n+l)},\ p>n+l,\ N>l$ Note that here the role of space measurement is played by a number S.

If $S=1,\ i.e.\ p(1+l)+l-n=pN$ then the system (10) it comes down to the shape of the plate. In this case $\varphi(|x|)=|x|^N,\ N\geq 2$ i.e. $\varphi(|x|)$ depends only on N space dimension. Obviously in a critical case $n+l=p$ function $\varphi(|x|)=\ln(|x|)$ and the "radially symmetric" system has the form

$$\frac{d}{d\varphi}\left((z^{m_1-1}\left|\frac{d}{d\varphi}w^k\right|^{p-2}\frac{d\omega}{d\varphi}\right)+z^{m_1-1}\left|\frac{d}{d\varphi}w^k\right|^{p-2}\frac{d\omega}{d\varphi}=0$$

$$\frac{d}{d\varphi}\left((w^{m_2-1}\left|\frac{d}{d\varphi}v^k\right|^{p-2}\frac{dv}{d\varphi}\right)+w^{m_2-1}\left|\frac{d}{d\varphi}v^k\right|^{p-2}\frac{dv}{d\varphi}=0 \quad (11)$$

Consider the functions

$$\bar{f}(\xi)=A\left(a-\xi^{\gamma}\right)_+^{\gamma_1},\ \bar{\psi}(\xi)=B\left(a-\xi^{\gamma}\right)_+^{\gamma_2},\ \xi=\varphi(|x|)[\tau(t)]^{-1/p} \quad (12)$$

$(b)_+\ means\ (b)_+=\max(0,b)$

$$\gamma=\frac{p}{p-1},\gamma_i=\frac{(p-1)[k(p-2]-(m_i-1)}{[k(p-2)]^2-(m_1-1)(m_2-1)},\ p\geq 2,\ i=1..2$$

We notice the function $\bar{f}(\xi)$, $\bar{\psi}(\xi)$ - are compact carrier (finite) weak solutions of a degenerate type system

$$\xi^{1-S}\frac{d}{d\xi}\left(\xi^{S-1}\bar{\psi}^{m_1-1}\left|\frac{d\bar{f}^k}{d\xi}\right|^{p-2}\frac{d\bar{f}}{d\xi}\right)+\frac{\xi}{p}\frac{d\bar{f}}{d\xi}+\frac{S}{p}\bar{f}=0$$

$$\xi^{1-S}\frac{d}{d\xi}\left(\xi^{S-1}\bar{f}^{m_2-1}\left|\frac{d\bar{\psi}^k}{d\xi}\right|^{p-2}\frac{d\bar{\psi}}{d\xi}\right)+\frac{\xi}{p}\frac{d\bar{\psi}}{d\xi}+\frac{S}{p}\bar{\psi}=0 \quad (13)$$

if the number of $A,\ B$ - are the solution of the following equation of an algebraic system

$$A^{k(p-2)}B^{m_1-1}=1/\,p\left(\gamma k\gamma_1\right)^{p-2}\gamma\gamma_1$$

$$A^{m_2-1}B^{k(p-2)}=1/\,p\left(\gamma k\gamma_2\right)^{p-2}\gamma\gamma_2 \quad (14)$$

By design, function $\bar{f}(\xi),\bar{\psi}(\xi)$ has the property $\bar{f}(\xi)\equiv 0$, $\bar{\psi}(\xi)\equiv 0$ in the domain $|\xi|\geq a^{\frac{p-1}{p}}$. So we can continue with the functions $\bar{f}(\xi),\bar{\psi}(\xi)$ by 0 for the value $\xi\in(a^{(p-1)/p},\infty)$.

It is clear that the functions $\bar{f},\bar{\psi}$ - they are classic solutions of a self-similar system (13) in the range $|\xi|<a^{(p-1)/p}$.

Since the functions $u_+(t,x)=\bar{f}(\xi),\ v_+(t,x)=\bar{\psi}(\xi)$ - they are a self-similar solution of the system and have the property of the finite velocity of perturbation propagation $i.e.$ $u_+(t,x)\equiv 0,\ v_+(t,x)\equiv 0$ when $|x|\geq a^{(p-(n+l))}\frac{p}{p-(n+l)}[\tau(t)]^{1/(p-(n+l))},\ (n+l)<p$.

4. Discussions

Theorem 1. Allow $(n+l)<p$ and also

$$\tau(t)=\int_0^t\bar{v}^{m_1-1}(\eta)\bar{u}^{p-2}(\eta)d\eta=\int_0^t\bar{u}^{m_2-1}(\eta)\bar{v}^{p-2}(\eta)d\eta$$

(15) this is a monotonous growing function, and accept the conditions $u(0,x)\leq u_+(0,x),\ v(0,x)\leq v_+(0,x)$,

$$A^{k(p-2)}B^{m_1-1}\leq 1/\,p\left(\gamma k\gamma_1\right)^{p-1},\ A^{m_2-1}B^{k(p-2)}\leq 1/\,p\left(\gamma k\gamma_2\right)^{p-1}$$

they were satisfied. T when solving the problem (1), (2), the phenomena of the finite velocity of distribution propagation occur under the conditions $k(p-2)+m_i-1>0,\ p\geq 2,\ i=1..2,\ n+l<p$ be the truth.

Conclusion. Suppose $\gamma_i>0,\ p\geq 2,\ i=1..2,\ n+l<p$,

$$\tau(\infty)=\int_0^\infty\bar{v}^{m_1-1}(\eta)\bar{u}^{p-2}(\eta)d\eta=\int_0^t\bar{u}^{m_2-1}(\eta)\bar{v}^{p-2}(\eta)d\eta<\infty$$

then the problem is (1), (2) it has the property of localization of the solution space [13].

Evidence. Consider the functions

$$u(t,x)=\bar{u}(t)w_+(\tau(t),\varphi(x)),$$

$$v(t,x)=\bar{v}(t)z_+(\tau(t),\varphi(x))$$

Substituting these functions in system (1), we have

$$L_1(u_+,v_+)=u_1(t)\{(\alpha_1/[1-(m_1-1)\alpha_2-(p-2)\alpha_1]-N/p)\}\bar{f}(\xi),$$

$$L_2(u_+,v_+)=u_2(t)\{\alpha_2/[1-(m_2-1)\alpha_1-(p-2)\alpha_2]-N/p\}\bar{\psi}(\xi),$$

According to the condition of theorem 1 we have

$$L_1(u_+,v_+)\leq 0, L_2(u_+,v_+)\leq 0\ \ in\ D=\{(t,x):t>0,|x|<l(t),$$

$$l(t)=a^{(p-1)/p}[\tau(t)]^{1/[p-(n+l)]}\}$$

where $\tau(t)$ - certain function (15).

Therefore, according to the principles of comparison, we have

$$u(t,x) \le \bar{u}(t) w_+(\tau(t), \varphi(x)),$$
$$v(t,x) \le \bar{v}(t) z_+(\tau(t), \varphi(x))$$ in Q.

Because $w_+(\tau(t),\varphi(x)) \equiv 0,\ z_+(\tau(t),\varphi(x)) \equiv 0$

Because $\frac{p-(n+l)}{p}|x|^{\frac{p}{p-(n+l)}} \ge a^{(p-1)/p}[\tau(t)]^{1/p}$ and

$|x| \ge \left(\frac{a^{(p-1)/p}[\tau(t)]^{1/p}}{\frac{p-(n+l)}{p}}\right)^{\frac{p-(n+l)}{p}}$ this means the final velocity of propagation of the perturbation of solutions. Please note that when $\tau(t) \to \infty$ then set the final velocity of propagation of the perturbation of solutions.

Consequence. If the conditions are $k(p-2)+m_i-1>0,\ p\ge 2,\ i=1..2,\ n+l<p,$

$$\tau(\infty)=\int_0^\infty \bar{v}^{m_1-1}(\eta)\bar{u}^{p-2}(\eta)d\eta=\int_0^t \bar{u}^{m_2-1}(\eta)\bar{v}^{p-2}(\eta)d\eta<\infty$$

completed, then the task (1), (2) it has the property of spatial localization of the solution.

2. Case $\mathbf{n+l=p}$

In this case, the following theorem holds.

Theorem 2. Allow $n+l=p$ be and the conditions of theorem 1 were met. If $u(0,x)\le u_1(0,x),\ v(0,x)\le v_1(0,x), x\in R^N\setminus 0$ then to solve the problem (1), (2) obtained estimate $u(t,x)\le u_1(t,x),\ v(t,x)\le v_1(t,x), t>0, x\in R^N\setminus 0$,

$$u_1(t,x)=A\bar{u}(t)\bar{f}(\xi),\ v_1(t,x)=B\bar{v}(t)\bar{\psi}(\xi),\ \ \xi=\ln(|x|)[\tau(t)]^{-1/p}$$

constants and functions $\bar{u}(t), \bar{v}(t), \bar{f}(\xi),\ \bar{\psi}(\xi)$ were defined above.

3. Asymptotes of self-similar solutions

Now we proceed to study the asymptotic behavior of solutions of self-similar systems (3) under the following boundary conditions.

$$f'(0)=0,\ f(\infty)=0,\ \psi'(0)=0,\ \ \psi(\infty)=0 \quad (16)$$

$$f(0)=a_1>0,\ f(d)=0,\ \psi(0)=a_2>0,\ \ \psi(d)=0,\ d<\infty \quad (17)$$

$$f(0)=a_1>0,\ f(\infty)=0,\ \psi(0)=a_2>0,\ \ \psi(\infty)=0 \quad (18)$$

Theorem 3. Allow $\gamma_1>0,\ \gamma_2>0$ then the solution to the problem (3) - (14) has an asymptotic representation as $\eta\to\infty\ \ (\eta=-\ln(a-\xi^{p/(p-1)})$

$$f(\xi)=A\tilde{f}(\xi)(1+o(1)),$$
$$\psi(\xi)=B\tilde{\psi}(\xi)(1+o(1)) \quad (19)$$

where are the coefficients $A,B-$ they are the solution of a system of algebraic equations

$$A^{p-2}B^{m_1-1}=1/p(\gamma\gamma_1)^{p-1}$$
$$A^{m_2-1}B^{p-2}=1/p(\gamma\gamma_2)^{p-1} \quad (20)$$

Evidence. To prove theorem (3.73) in (3.84), we will make a replacement $f(\xi)=\tilde{f}(\xi)w(\eta),\ \psi(\xi)=\tilde{\psi}(\xi)z(\eta),\ \eta=-\ln(a-\xi^{p/(p-1)})$ (21)

Then for unknown functions $w(\eta), z(\eta)$ in (14) we have

$$\frac{d}{d\eta}L_3(w,z)+d_1L_3(w,z)+d(\eta)\left(\frac{dw}{d\eta}-\gamma_1 w\right)=0$$
$$\frac{d}{d\eta}L_4(w,z)+d_2L_4(w,z)+d(\eta)\left(\frac{dz}{d\eta}-\gamma_2 z\right)=0 \quad (22)$$

Where

$$L_3(w,z)=z^{m_1-1}\left|\frac{dw}{d\eta}-\gamma_1 kw^k\right|^{p-2}\left(\frac{dw}{d\eta}-\gamma_1 w\right),\ L_4(w,z)=w^{m_2-1}\left|\frac{dz}{d\eta}-\gamma_2 kz^k\right|^{p-2}\left(\frac{dz}{d\eta}-\gamma_2 z\right)$$

$$d_i(\eta)=\gamma^{p-1}\left(\frac{N}{\gamma}\frac{e^{-\eta}}{a-e^{-\eta}}-\gamma\gamma_i\right),\ i=1,2, d(\eta)=\frac{\gamma}{p}\frac{e^{-\eta}}{a-e^{-\eta}},$$

Analysis of system solutions (6) shows that the functions, W B Z when $\eta\to\infty$ there must be a solution to the system (22). Theorem 2 is proved.

Note that the solution of this problem is a single parametric Eigen function of a nonlinear medium.

Theorem 4. Allow $\gamma_i<0,\ \ N+\gamma\gamma_i<0,\ i=1,2$. Then the solution to the problem is (7) - (14) when $\eta\to\infty\ \ (\eta=\ln(a+\xi^{p/(p-1)})$ has an asymptotic representation

$$f(\xi)=A_3\left(a+\xi^\gamma\right)^{\gamma_1}(1+o(1)),$$
$$\psi(\xi)=A_4\left(a+\xi^\gamma\right)^{\gamma_2}(1+o(1)) \quad (23)$$

here are the coefficients $A_i>0,\ \ i=1,2$ they are the solution of a system of algebraic equations

$$A_3^{p-2}A_4^{m_1-1}=[-1/\gamma^{p-1}p(N+\gamma\gamma_1)+b_1](|\gamma\gamma_1|)^{2-p}$$
$$A_3^{m_2-1}A_4^{p-2}=[-1/\gamma^{p-1}p(N+\gamma\gamma_2)+b_2](|\gamma\gamma_2|)^{2-p} \quad (24)$$

To prove theorem 3 in (20), we will make a replacement

$$f(\xi)=\tilde{f}(\xi)w(\eta),\ \psi(\xi)=\tilde{\psi}(\xi)z(\eta),$$
$$(\eta=\ln(a+\xi^{p/(p-1)})\qquad(25)$$

Then for an unknown function $w(\eta), z(\eta)$ of (25) has

$$\frac{d}{d\eta}L_5(w,z)+d_1L_5(w,z)+d_3(\eta)(\frac{dw}{d\eta}+\gamma_1 w)+b_1w=0$$
$$\frac{d}{d\eta}L_6(w,z)+d_2L_6(w,z)+d_3(\eta)(\frac{dz}{d\eta}+\gamma_2 z)+b_2z=0\qquad(26)$$

where

$$L_5(w,z)=z^{m_1-1}\left|\frac{dw}{d\eta}+\gamma_1 w\right|^{p-2}\left(\frac{dw}{d\eta}+\gamma_1 w\right),$$
$$L_6(w,z)=w^{m_2-1}\left|\frac{dz}{d\eta}+\gamma_2 z\right|^{p-2}\left(\frac{dz}{d\eta}+\gamma_2 z\right),$$
$$d_i(\eta)=\gamma^p(N\frac{e^\eta}{e^\eta-a}+\gamma\gamma_i),$$
$$i=3,4,d_3(\eta)=\frac{\gamma}{p}\frac{e^\eta}{e^\eta-a}$$

Analysis of solutions of the system (26) shows that A_3, A_4- coefficients must be solutions of an algebraic system (26). Theorem 4 is proved.

5. Conclusion

As we know, there are a number of difficulties in finding solutions for degenerate nonlinear parabolic systems, such as correctly determining the initial approximation. Here, the initial approximation is strongly related to the parameters of the equation. As an initial approximation, we used the self-similar solutions built above.

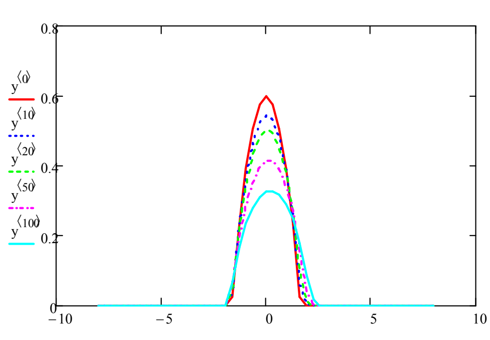
Below are some examples from numerical calculations and comparison of the self-similar approach and approximate solutions for the system's equations (the behavior of the solution in time).

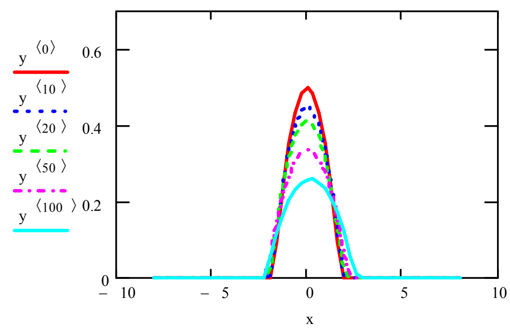
The difference between exact solutions and approximate solutions

Example 1. One-dimensional case:

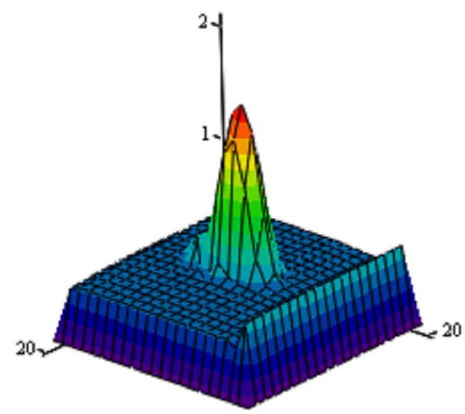
a) Exact solution

b) Approximate solution.



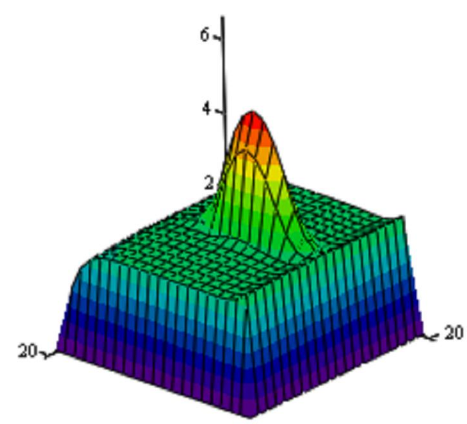


Example 2. Two-dimensional case:



$YY_{100,0}$

Behavior of the solution when t = 100 s.



$YY_{200,0}$

Behavior of the solution when t = 200 s.

Methods and algorithms for visualization of processes in multicomponent media with two nonlinearities are developed for the divergent case taking into account convective transport. Visualization of nonlinear processes describing systems of reaction-diffusion processes with double nonlinearity in one-dimensional and multidimensional cases was performed.

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Corresponding Author:
Dr. Beknazarova Saida Safibullayevna
Doctor of technical science, professor Audiovisual technologies of Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi, Tashkent, Uzbekistan, 100096
Telephone: 998-90-3276666
E-mail: saida.beknazarova@gmail.com

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