The New Prime theorem（15）
$$P_j = (j)^3 P + (k-j)^3, \, j=1,\cdots,k-1$$

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Abstract: Using Jiang function we prove that there exist infinitely many primes P such that each of $(j)^3 P + (k-j)^3$ is a prime.

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Theorem. Let k be a given prime.
$$P_j = (j)^3 P + (k-j)^3 (j=1,\cdots,k-1) \tag{1}$$

There exist infinitely many prime P such that each of $(j)^3 P + (k-j)^3$ is a prime.

Proof. We have Jiang function[1]
$$J_2(\omega) = \prod_P [P-1-\chi(P)], \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}[(j)^3 q + (k-j)^3] \equiv 0 \pmod{P}, q=1,\cdots,P-1. \tag{3}$$

From (3) we have $\chi(2)=0$, if $P<k$ then $\chi(P) \le P-2, \chi(k)=1$, if $k<P$ then $\chi(P) \le k-1$. From (3) we have

$$J_2(\omega) \neq 0. \tag{4}$$

We prove that there exist infinitely many primes P such that each of $(j)^3 P + (k-j)^3$ is a prime. Jiang function is a subset of Euler function: $J_2(\omega) \subset \phi(\omega)$.

We have asymptotic formula [1]

$$\pi_k(N,2) = \left|\left\{P \le N : (j)^3 P + (k-j)^3 = prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}. \tag{5}$$

where $\phi(\omega) = \prod_P (P-1)$.

Example 1. Let $k=3$. From (1) we have
$$P_1 = P+8, \quad P_2 = 8P+1 \tag{6}$$

We have Jiang function
$$J_2(\omega) = \prod_{5 \le P}(P-3) \neq 0 \tag{7}$$

There exist infinitely many primes P such that P_1 and P_2 are all prime.

We have asymptotic formula

$$\pi_3(N,2) = \left|\{P \le N : P_1 = prime, P_2 = prime\}\right| \sim \frac{J_2(\omega)\omega^2}{\phi^3(\omega)} \frac{N}{\log^3 N}$$

（8）

Example 2. Let $k = 5$, from (1) we have

$$P_j = (j)^3 P + (k-j)^3 (j = 1,2,3,4)$$

（9）

We have jiang function

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] ,$$

（10）

where $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{4} [(j)^3 q + (k-j)^3] \equiv 0 \pmod P$$

（11）

From (11) we have $\chi(2) = 0$, $\chi(3) = 1$, $\chi(5) = 1$, $\chi(7) = 2$, $\chi(11) = 4$, $\chi(13) = 3$, $\chi(P) = 4$ otherwise.

Substituting it into (10) we have.

$$J_2(\omega) = 648 \prod_{17 \le P} (P - 5) \ne 0$$

（12）

We prove that there exist infinitely many primes P such that each of $(j)^3 P + (k-j)^3$ is prime.

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series

$$\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \frac{1 + \chi(P)}{P}\right)(1 - \frac{1}{P})^{-k}$$

[1-2], which can count the number of prime number. The

$$\sigma(H) = \prod_P \left(1 - \frac{\nu(P)}{P}\right)(1 - \frac{1}{P})^{-k}$$

is false. [2-5], which

prime number is not random. But Hardy singular series can not count the number of prime numbers.

Note:

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