Time-Dependent Analysis Of Waiting And Service Time Behaviour In Discrete Time Queue; A Case Study Of First Bank Plc

Akomolafe Abayomi. A.

Department of Statistics, Federal University of Technology, Akure, Nigeria.
akomolafeayotade@gmail.com

Abstract: In our daily life we generally find a long queue at the Automated Teller Machine [ATM], as a result of this customers have to spend considerable amount of time in queue. In such a situation if instead of using a single ATM machine if we use double ATM machine then it will decrease the waiting time in queue. Against this background, the queuing process is employed with inter arrival time and service time. The data for this study was collected from primary source and is limited to ATM service point of a standard Bank where the data collected incorporate the attributes of queuing on the number of customers for four weeks. Eight ATMs at a steady service rate of 0.18 customers per minute is found to be optimal and the waiting time are found to be relatively higher during the hours of 09:00am to 12:00 noon. The research thus reveals that queue exist although its theory is applicable in finding optimal service levels, waiting time might still be lengthy because of some factors. We then derive the arrival rate, service rate, utilization rate, waiting time in the queue and the average number of customers in the queue based on the data Service unavailability was observed to be a contributory factor to queue formation at the ATM point. The queuing process employed with interarrival time and service time do not follow exponential distribution hence, in reducing queue problem, high routine maintenance regime should be actively implemented, backup-staffs could be engaged during peak periods to handle any additional demand instead of the alternative of installing the rather capital-intensive ATM which might be of less utility for most business hours. Queue management should also be made an active part of the bank's overall strategic queue management processes.
[Akomolafe Abayomi. A. **Time-Dependent Analysis Of Waiting And Service Time Behaviour In Discrete Time Queue; A Case Study Of First Bank Plc.** *Researcher* 2018;10(10):35-43]. ISSN 1553-9865 (print); ISSN 2163-8950 (online). http://www.sciencepub.net/researcher. 5. doi:10.7537/marsrsj101018.05.

Keywords: Queuing Theory, Waiting Time, Service Rate, Arrival Rate, ATM, Optimal Service Level

1. Introduction

Bankley *et al.* (2006) cited in Musara and Fatoki (2010) opine that technology provides enhanced insight into handling old and new tasks. Technology has changed not only the way we do business but has also changed virtually all sphere of human life. Abor (2005) affirms that technology affects even the direction of an economy and its capacity for continued growth. Human beings consciously or unconsciously interact with products of technology in almost all their daily activities. Although there has been significant reforms in recent times all in an effort to maximize profit, reduce cost and satisfy customers optimally in the most generally acceptable international standard. Despite these entire sterling efforts one phenomenon remains inevitable: queue. It is a common practice to see a very long waiting line of customers to be serviced either at the Automated Teller Machine (ATM) or within the banking hall. Though similar waiting lines are seen in places like; busstop, fast food restaurants, clinics and hospitals, traffic light, supermarket, etc. but long waiting line in the banking sector is worrisome be. Queue is a general phenomenon in everyday life. Queues are formed when customers (human or not) demanding service have to wait because their number exceeds the number of servers available; or the facility doesn't work efficiently or takes more than the time prescribed to service a customer. Some customers wait when the total number of customers requiring service exceeds the number of service facilities, some service facilities stand idle when the total number of service facilities exceeds the number of customers requiring service. The population of customer could be finite or infinite Waiting line management has the greatest dilemma for managers seeking to improve the on investment of their operation; as customers don't tolerate waiting intensely. Whenever customer feels that he/she has waited too long at a station for a service, they would either opt out prematurely or may not come back to the station next time when needed a service. This would of course reduce customer demand and in the long run revenue and profit. Moreover, longer waiting time might increase cost because it equals to more space or facilities, which mean additional cost on the management. Despite being in the technology era; line is experienced at within and Banks ATMs in developing nations than elsewhere. ATM are adopted so as to reduce waiting time., offers considerable ease to both the bank and their customers; as it enables customers to make financial transactions at more convenient times and locations, during and after banking hours. Most importantly, ATM, are designed to provide efficient and improved services to customers at the shortest possible time. Yet customers spend a considerable time before they are finally served. Businesses especially banks are striving very hard to provide the best level of service possible, minimizing the service time, giving the customer a much better experience. However, in situations where queue arises in a system, it is appropriate to attempt to minimize the length of the queue rather than to eliminate it completely; complete elimination may be infeasible Therefore, a systematic study of waiting line system would assist the management of the Bank in making certain decisions in an effort minimize the time a customer spends in a service facility.

The input process describes the sequence of requests for service. Often, for example, the input process is specified in terms of the distribution of the lengths of time between consecutive customer arrival instants. The service mechanism includes such characteristics as the number of servers and the length of time that the customers put the server in use. For example, customers might be served by a single-server, each customer putting the server to use at the length of time.

1.1 Queuing Models

Model as an idealized representation of the real life situation; in order to keep the model as simple as possible however, some assumptions need to be made.

1.2 Assumptions Made on the System

(1) Single channel queue.

(2) There is an infinite population from which customers originate.

(3) Poisson arrival (Random arrivals).

(4) Exponential distribution of service time.

(5) Arrival in group at the same time (i.e. bulk arrival) is treated as single arrival.

(6) The waiting area for customers is adequate.

(7) The queue discipline is First In First Out (FIFO).

1.3 M/M/s: (∞ /FIFO) Queuing System

Here the inter arrival time and service time both has an exponential distribution,s server, FIFO is the queue discipline and infinite population size from which the system can draw from.

To be in state E_j (j=0,1,………,s). Let P_j be the proportion of time that the system spends in state E_j; P_j is therefore also the proportion of time that j trunks are busy. Denote by **λ** the caller arrival rate; **λ** is the average number of requests for service per unit time. Consider first the case j<s. Since callers arrive with overall rate **λ**, and since the proportion of time the system spends in state E_j is P_j, the rate at which the transition $E_j \rightarrow E_{j+1}$ occurs (the average number of such transitions per unit time) is therefore λP_j. Now consider the case when j=s. Since the state E_{s+1} represents a physically impossible state (there are only ***s*** trunks), the transition $E_s \rightarrow E_{s+1}$ cannot occur, so the rate of transition $E_s \rightarrow E_{s+1}$ is zero. Thus, the rate at which the upward transition $E_j \rightarrow E_{j+1}$ occurs is λPj when j=0,1,……..s-1 and is zero when j=s.

Let us no consider the downward transition $E_{j+1} \rightarrow E_j$ (j=0,1,……..s-1), also suppose that the mean holding time (the average length of time a caller is served by a trunk) is **τ**. Then, if a single trunk is busy, the average number of calls terminating during an elapsed time **τ** is 1; the termination rate for a single call is therefore 1/**τ**. Similarly, if two calls are in progress simultaneously and the average duration of a call is **τ** is 2; the termination rate for a single call is therefore 2/**τ**. By this reasoning, the termination rate for j+1 simultaneous calls are (j+1)/**τ**. Since the system is in state E_{j+1} with a proportion time of P_{j+1}, we conclude that the downward transition $E_{j+1} \rightarrow E_j$ occurs at the rate of (j+1) $\tau^{-1} P_{j+1}$ per unit time (j=0,1,………,s-1). We now apply the principle of conservation of flow: We equate, for each value of the index ***j***, the rate of occurrence of the occurrence of the upward transition $E_j \rightarrow E_{j+1}$ to the rate of occurrence of the downward transition $E_{j+1} \rightarrow E_j$. Thus, we have the equations of ***statistical equilibrium*** or ***conservation of flow***:

$$\lambda P_j = (j+1)\,\tau^{-1} P_{j+1} \quad \text{(j=0, 1,………,s-1)} \qquad \textbf{(1)}$$

These equations can be solved recurrently; the result, which expresses each P_j in terms of the value P_o is

$$P_j = \frac{(\lambda\tau)^j}{j!} P_o \quad \text{(j=1,………,s)} \qquad \textbf{(2)}$$

Since the numbers $\{P_j\}$ are proportions, they must sum to unity:

$$P_o + P_1 + \ldots\ldots\ldots + P_s = 1 \qquad \textbf{(3)}$$

Using the normalization equation (**3**) together with (**2**), we can determine P_o:

$$P_o = \left(\sum_{k=0}^{s} (\lambda\tau)^k / k!\right)^{-1} \qquad \textbf{(4)}$$

Thus we obtain for the proportion P_j of time that j trunks are buy the formula

$$P_j = \frac{(\lambda\tau)^j / j!}{\sum_{k=0}^{s} (\lambda\tau)^k / k!} \quad \text{(j=0,1,………,s)} \qquad \textbf{(5)}$$

An important observation to be made from the formula (**5**) is that the proportions $\{P_j\}$ depends on the arrival rate λ and mean serving time **τ** only through the product **λτ**. This product is a measure of the demand made on the system; it is often called the offered load and given the symbol ***a*** (***a*=λτ**). The numerical values of ***a*** are expressed in units called erlangs (erl), after the Danish mathematician and teletraffic theorist **A. K. Erlang**, who first published the formula (**5**) in 1917.

When j=s in (**5**), the right- hand side becomes the well-known ***Erlang loss formula***, denoted in the US by B (s,a) and in Europe by $E_{1,s}$ (a):

$$B(s,a) = \frac{a^s/s!}{\sum_{k=0}^{s} a^k/k!} \qquad \textbf{(6)}$$

We shall derive these results more carefully later in future in this study. The point to be made here is that some potentially useful mathematical results have been derived using only heuristic reasoning.

2. Methodology

The method used for data collection based on attributes is to capture a particular customer by labeling such client as shall be explained in the table below with the corresponding details and taken note the time at arrival on each ATM as categorized in the design.

We also went further to measure the traffic intensity (**ρ**) and to establish the indicators with its respective interpretation and this is expressed below:

$$\text{Traffic Intensity} = \begin{cases} \text{When } \boldsymbol{\rho} < \mathbf{1} \text{ is interprets as no queue is built up} \\ \text{When } \boldsymbol{\rho} = \mathbf{1} \text{ is interprets as service rate is equivalent to arrival rate} \\ \text{When } \boldsymbol{\rho} > \mathbf{1} \text{ is interprets as queue is built up} \end{cases}$$

2.1 Arrival Model (Poisson Distribution)

A Poisson distribution is a discrete probability distribution that can be used to predict the number of arrival in a given time. It involves the probability of the occurrences and it is independent of what has happened in the previous observation. The probability distribution function is given as

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

It has E (X) = λ and V (X) = λ. The model for the arrival of customer is assumed to be a Poisson distribution.

2.2 Service Time Model (Exponential Distribution)

Service time is the arrival between the commencement of service and its completion. Service time can be modeled as exponential distribution with mean I/μ and variance I/μ². This model for service is assumed in this study. The probability density function is given as

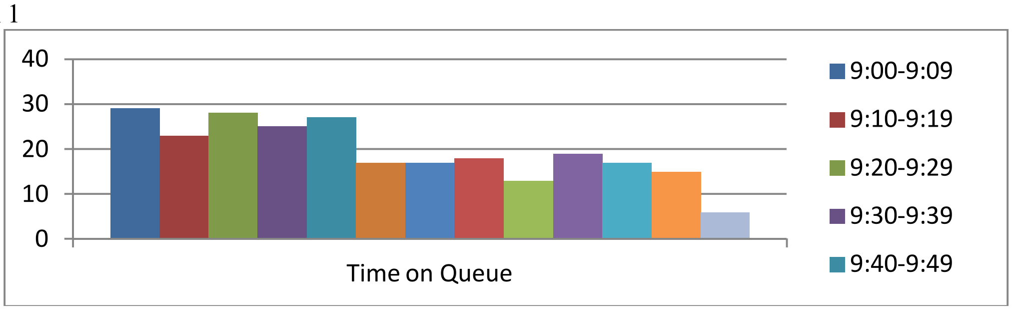
$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

3. Data Analysis

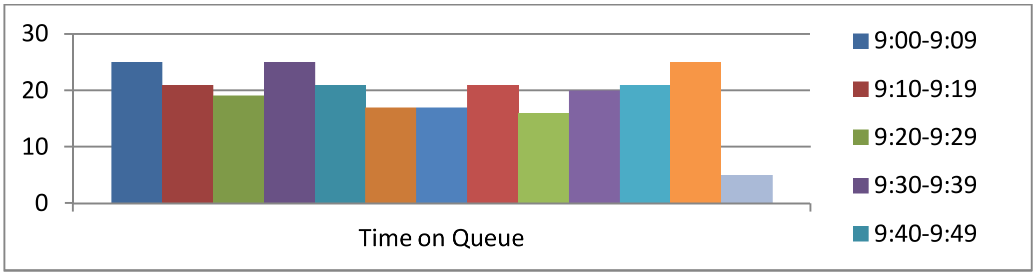
The collection of these data was done by working day visits to the First Bank of Nigeria Plc, University of Ibadan branch, Ibadan for this project. The information collected at the Automated Teller Machine (ATM) for four consecutive weeks in five working days of each week are: The arrival time, Time service commences, Time service ends.

Analysis Of Customer Arrival

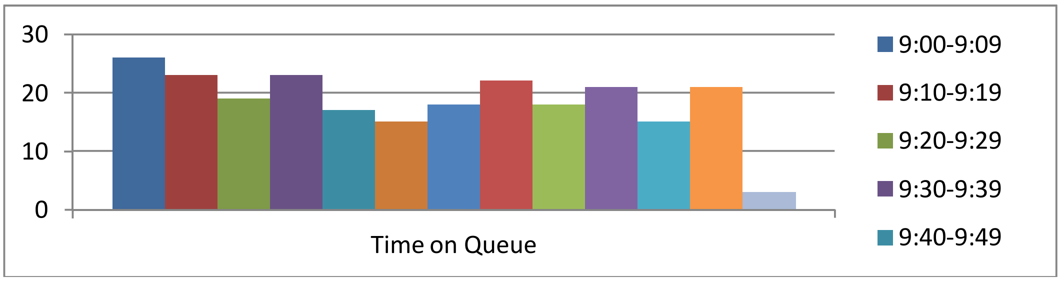
Week 1



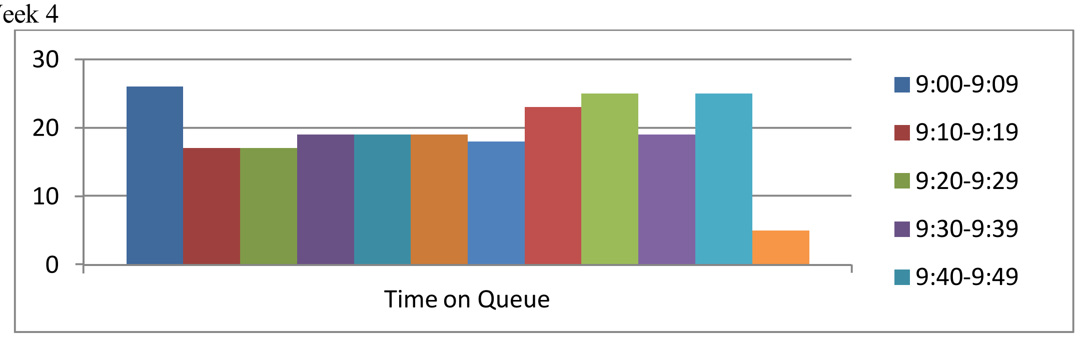
Week 2



Week 3



Week 4



3.1 Analysis on Arrival of Customers

Estimation of mean rate of Arrival of Customers $\lambda = \frac{\Sigma fx}{\Sigma f}$

Test of hypothesis on the Arrival time

Tables1: observed and expected frequencies of no of Arrivals

Week 1 [The mean value (λ_1) = 3.9539]

No of Arrival in 10 min	Observed frequency (O_i)	P (X=x)	Expected frequency (ε_i)	χ^2
≤1	7	0.095015	6.175975	0.10994494
2	2	0.149922	9.74493	6.155399855
3	16	0.197592	12.84348	0.7757725
4	17	0.193145	12. 55425	1. 5743428
5	12	0.154451	10.03932	0.382921
6	7	0.101781	6.615765	0.0223159
≥7	4	0.108094	7.02611	1.3033303
Total	**65**			**10.32403**

Week 2 [The mean value (λ_2) = 3.83871]

No of Arrival in 10 min	Observed frequency (O_i)	P (X=x)	Expected frequency (ε_i)	χ^2
≤1	6	0.1041355	6.456401	0.032262846
2	2	0.1565989	9.7091318	6.121115084
3	15	0.2028986	12.5797132	0.465653556
4	22	0.194715	12.07233	8.164010728
5	8	0.1494909	9.2684358	0.173592331
6	8	0.095642	5.929004	0.723397122
≥7	1	0.0965191	5.9841842	4.1 512914
Total	**62**			**19.831326**

Week 3 [The mean value (λ_3) = 3.86364]

No of Arrival in 10 min	Observed frequency (O_i)	P (X=x)	Expected frequency (E_i)	χ^2
≤1	6	0.010295	0. 67947	41. 66194
2	2	0.156678	10.340748	6.727567
3	15	0.2017814	13.3176	0.212536
4	22	0.1949027	12.86358	6.489187
5	15	0.1506068	9.9400488	2.575753
6	5	0.096982	6.400812	0.3065665
≥7	1	0.188741	12.45691	10.37187
Total	**66**			**68.34542**

Week 4 [The mean value (λ_4) = 4.03175]

No of Arrival in 10 min	Observed frequency (O_i)	P (X=x)	Expected frequency (E_i)	χ^2
≤1	3	0.089279	5.624577	1.224697
2	0	0.144208	9.085104	9.085104
3	13	0.193804	12.209652	0.05116
4	25	0.195342	12.30655	13.092514
5	20	0.157514	9.923382	10.23222
6	2	0.105843	6. 668109	3.2679792
≥7	0	0.11401	7.18263	7.18263
Total	**63**			**44.136304**

From the obtained calculated value as compared to our chi–square value at different level of significant, we have the following:

Hypothesis Testing

H_o: Arrival pattern follows a Poisson distribution

H_1: Arrival pattern does not follow a Poisson distribution

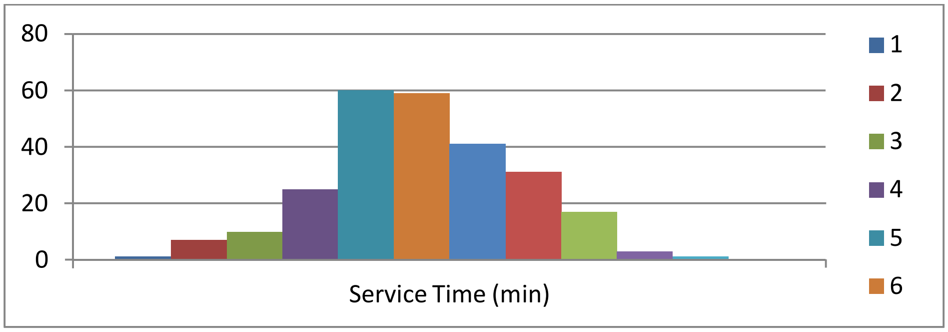
Week	Weekly Calculated Value	$\chi_{5;0.05}$	$\chi_{5;0.025}$	$\chi_{5;0.01}$
1	**10.32403**	**11.07**	**12.833**	**15.086**
2	**19.831326**	**11.07**	**12.833**	**15.086**
3	**68.34542**	**11.07**	**12.833**	**15.086**
4	**44.136304**	**11.07**	**12.833**	**15.086**

Decision Rule: Reject H_o if calculated chi–square is greater than tabulated chi–square at a pre–specified level of significance.

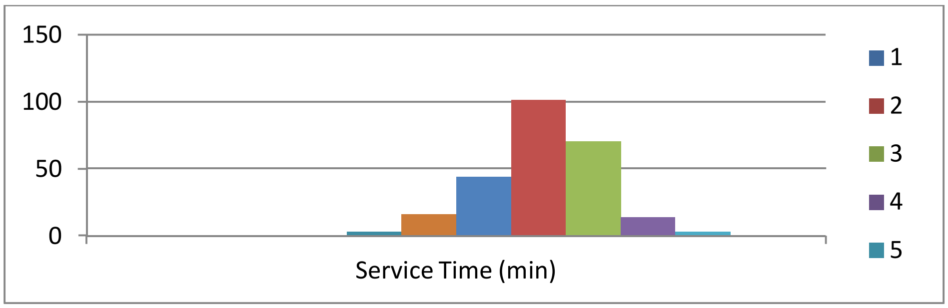
Conclusion: It was observed from the table displayed above that it was only the first week arrival pattern that follows a Poisson distribution.

3.2 Analysis of Service Time

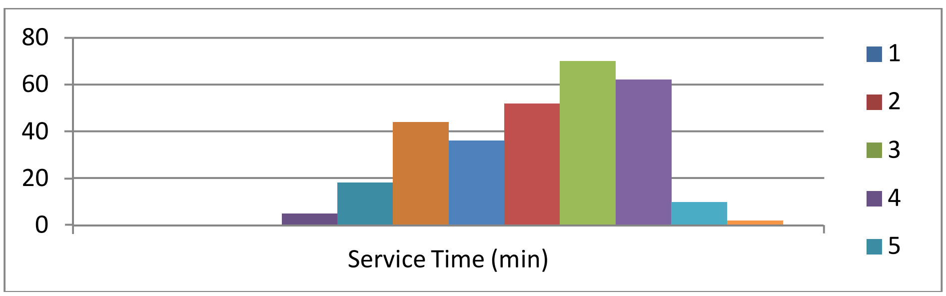
Week 1



Week 2



Week 3



Week 4

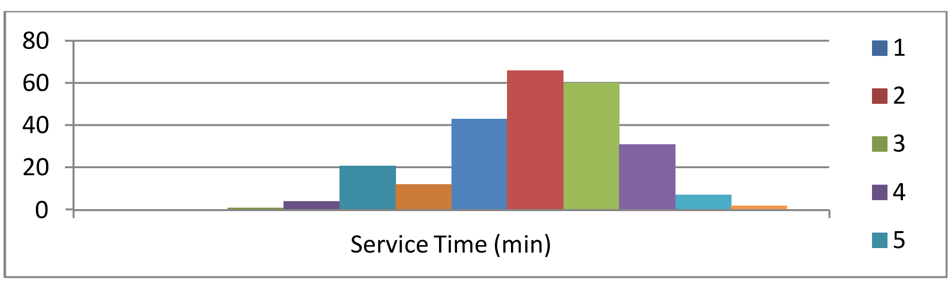


Table 2: observed and expected frequencies of no of Service time

Week 1[$1/\mu_1 = \frac{1400.5}{255} = 5.4922 \rightarrow \mu_1 = 1/\ 5.4922 = 0.18208$]

Int. Of Service	Mid pt	Obs. Freq. (O_i)	fx	P (X=x)	Expected freq. (ε_i)	χ^2
$0 < t < 1$	0.5	1	0. 5	0.16647	42.44985	40.47341
$1 < t < 2$	1.5	7	10. 5	0.13876	35.3838	22.76862
$2 < t < 3$	2.5	10	25	0.11566	29.4933	12.8839
$3 < t < 4$	3.5	25	87. 5	0.0964	24. 582	0.007108
$4 < t < 5$	4.5	60	270	0.08036	20.4918	76.17183
$5 < t < 6$	5.5	59	324. 5	0.06698	17.0799	102.8867
$6 < t < 7$	5.5	41	266.5	0.05583	14.23665	50.31218

Int. Of Service	Mid pt	Obs. Freq. (O_i)	fx	P ($X=x$)	Expected freq. (ε_i)	χ^2
7 < t < 8	7.5	31	232.5	0.04654	11.8677	30.8438
8 < t < 9	8.5	17	144.5	0.03879	9.89145	5.108602
9 < t < 10	9.5	3	28.5	0.03233	8.24415	3.335833
10 < t < 11	10.5	1	10. 5	0.02695	6.87225	5.017763
> 11	11.5	0	0	0.13493	34.40715	34.40715
Total		**255**	**1400. 5**			**333.90472**

Week 2$[1/\mu_2 = \frac{1860.5}{251} = 7.41235 \rightarrow \mu_2 = 1/\ 7.41235 = 0.13491]$

Int. Of Service	Mid pt	Obs. Freq. (O_i)	fx	P ($X=x$)	Expected freq. (ε_i)	χ^2
0 < t < 1	0.5	0	0	0.126206	31.677706	31.677706
1 < t < 2	1.5	0	0	0.110278	27.679778	27.679778
2 < t < 3	2.5	0	0	0.096356	24.185356	24.185356
3 < t < 4	3.5	0	0	0.084199	24. 582	24. 582
4 < t < 5	4.5	3	13. 5	0.073573	18.46683	12.95419
5 < t < 6	5.5	16	88	0.064287	16.136037	0.001147
6 < t < 7	5.5	44	242	0.056174	14.099674	63.407813
7 < t < 8	7.5	101	757.5	0.049084	12.320084	638.3177
8 < t < 9	8.5	70	595	0.04289	10.76539	325.92772
9 < t < 10	9.5	14	133	0.037477	9.406727	2.24288
10 < t < 11	10.5	3	31. 5	0.032747	8.219497	3.314455
> 11	11.5	0	0	0.226729	56.908979	56.908979
Total		**251**	**1860. 5**			**1147.7912**

Week 3$[1/\mu_3 = \frac{2276.5}{299} = 7.\ 613712 \rightarrow \mu_3 = 1/\ 7.41235 = 0.131342]$

Int. Of Service	Mid pt	Obs. Freq. (O_i)	fx	P ($X=x$)	Expected freq. (ε_i)	χ^2
0 < t < 1	0.5	0	0	0.123082	36.801518	36.801518
1 < t < 2	1.5	0	0	0.107933	32.271967	32.271967
2 < t < 3	2.5	0	0	0.094648	28.299752	28.299752
3 < t < 4	3.5	5	17.5	0.082999	24. 816701	15.824087
4 < t < 5	4.5	18	81	0.072783	21.762117	0.650374
5 < t < 6	5.5	44	242	0.063825	19.083675	32. 53164
6 < t < 7	6.5	36	234	0.055969	16.734731	22.178462
7 < t < 8	7.5	52	390	0.04908	14. 67492	94.934868
8 < t < 9	8.5	70	595	0.043039	12.868661	253.638657
9 < t < 10	9.5	62	589	0.037742	11.284858	227.918298
10 < t < 11	10.5	10	105	0.033097	9.896003	0.0010929
> 11	11.5	2	23	0.235803	70. 505097	66.5618305
Total		**299**	**2276.5**			**789.4340685**

Week 4$[1/\mu_4 = \frac{1852.5}{247} = 7.5 \rightarrow \mu_4 = 1/\ 7.5 = 0.13333]$

Int. Of Service	Mid pt	Obs. Freq. (O_i)	Fx	P ($X=x$)	Expected freq. (ε_i)	χ^2
0 < t < 1	0.5	0	0	0.124824	30.831528	36.801518
1 < t < 2	1.5	0	0	0.109243	26.983021	32.271967
2 < t < 3	2.5	1	2.5	0.095607	23.614929	21. 657275
3 < t < 4	3.5	4	14	0.083673	20. 667231	13.441403
4 < t < 5	4.5	21	94. 5	0.073228	18.087316	0.469043
5 < t < 6	5.5	12	66	0.064088	15.829736	0.9265396
6 < t < 7	6.5	43	279. 5	0.056088	13.853736	61.319539
7 < t < 8	7.5	66	495	0.049087	12.124489	239.39736
8 < t < 9	8.5	60	510	0.042959	10.610873	229.8855
9 < t < 10	9.5	31	294. 5	0.037597	9.286459	50.770467
10 < t < 11	10.5	7	73.5	0.032904	8.127288	0.1563594
> 11	11.5	2	23	0.230702	56.983394	53.0535899
Total		**247**	**1852.5**			**740.150561**

From the obtained calculated value as compared to our chi–square value for the hypothesis testing we have the following:

Hypothesis Testing

H_o: Service time follows an exponential distribution

H_1: Service time does not follow an exponential distribution

Week	Weekly Calculated Value	$\chi_{10;0.05}$	$X_{10;0.025}$	$X_{10;0.01}$
1	**333.90472**	**18.307**	**20.483**	**23.209**
2	**1147.7912**	**18.307**	**20.483**	**23.209**
3	**789.4340685**	**18.307**	**20.483**	**23.209**
4	**740.150561**	**18.307**	**20.483**	**23.209**

Decision Rule: Reject H_o if calculated chi–square is greater than tabulated chi–square at a pre–specified level of significance.

Conclusion: It was observed from the table displayed above that none of the service time follows an exponential distribution, although the observed convergence at ATM may follow other pattern which is suggested for future study.

Traffic Intensity measure for each week

Week	Arrival Rate (λ)	Service Rate (1/μ)	Traffic Intensity (ρ)
1	**3.9539**	**5.4922**	**0.7199**
2	**3.83871**	**7.41235**	**0.5179**
3	**3.86364**	**7.613712**	**0.5075**
4	**4.03175**	**7.5**	**0.5376**

3.3 E*valuation*

Generally, arrivals do not occur at fixed regular intervals of times but tend to be clustered for a duration of a week. The Poisson distribution involves the probability of occurrence of an arrival are random and independent of all other operating conditions. The inter arrival rate (i.e., the number of arrivals per unit of time) λ is calculated by considering arrival time of the customers to that of the number of customers.

Service time is the time required for completion of a service i.e., it is the time interval between beginning of a service from ATM machine and its completion. In this research the researcher has calculated mean service time μ of customers by considering different service time for customers to that of the number of customers. Estimated values and the analysis of the data shows that for the first week captured data follows a Poisson distribution with the mean rate of 10.32403 and the service time between **9:00 am** and **12:00 noon** for four weeks does not follow an exponential distribution while the estimated ratio of the mean of arrival rate to the mean of the service rate for the different weeks as displayed above are all less than one indicate that there is no queue.

4. Conclusion

The experience of waiting long hours at ATMs to say the least is annoying to customers and unprofessional on the part of providers who superintend over this. Long waiting time in queue accounts for the loss of customers and attendant losses such as loss of goodwill and reduction in customer satisfaction. This paper has provided a queuing theory-based approach to optimize ATM service level in the midst of fluctuating demands. The case bank operates two ATMs which are considered in queuing terms as the servers. ATM users are considered customers (a term used generically to refer to entities that request for service at a particular service centre). After running a series of goodness of fit test on arrival times and service times it was considered appropriate to represent the interaction between customers and the ATMs in a typical queuing system arrangement as an M/M/s queuing model. The prevailing operation characteristics are thus computed as output of the M/M/s model via an application of a decision support system. Service times and arrival times served as basic inputs to the system. Customer service times and arrival times were measured by conducting an observational study at the designated bank.

From the analysis carried out, it was observed that the arrival rate of customers for first week follows a Poisson distribution while further findings in the weeks that shows a huge drift away from a queue but an immeasurable convergence of customers at the ATM point, however the service time does not follow an exponential distribution which is as a result of more than eight ATM service point and effective network. The traffic intensity which is a good index also shows no queue exists.

References

1 Abor, J. (2005), "Technological innovations and banking in Ghana: An evaluation of customers'

perceptions". Legon: University of Ghana. Available: http://www.financialcertified.com/Technological%20Innovations%20and%20Banking%20in%20Ghana%20- %20AAFM.doc (May 11, 2012).
2 Aldajani, M. A., and Alfares, H. K. (2009), Location of banking automatic teller machines based on convolution. Systems Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia.
3 Baxtex, P. and Jack, S. (2008), Quantitative case study: study design and implementation for novice researchers. McMaster University, West Hamilton, Ontario, Canada.
4 Copper, R. B. (1981), Introduction of queuing theory. 2nd ed. Elsevier North Holland, Inc., New York.
5 Dharmawirya, M. and Adi, E. (2011), "Case Study for Restaurant Queuing Model". International Conference on Management and Artificial Intelligence, IPEDR, vol.6.
6 J. Hiray, Waiting Lines and Queuing System, Article of Business Management, 2008.
7 E. Anderson, A note on managing waiting lines. UT McCombs School of Business, 2007.
8 Dr B.S. Grewal, Higher engineering mathematics, 40Thedition (India, Khanna publishers).
9 Ford, J. L. (1980), Two applications of queuing theory. Department of Human Service Studies, Cornell University, New York.
10 Lariviere, M. and Van Mieghem, J. (2004), "Strategically seeking service: how competition can generate Poisson arrivals". Manufacturing & Service Operations Management, vol. 6, pp. 23-40.
11 Musara, M. and Fatoki, O. (2010), "Has technological innovations resulted in increased efficiency and cost savings for banks' customers?" African Journal of Business Management, vol. 4(9), pp. 1813-1821.

10/25/2018