# Beta-Binomial Mixture Models: Its Consistent And Efficient Performance Over Binomial Model.

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**Abstract:** Beta binomial Model is a standard choice for modeling multiple sequences of binary responses. This research was carried out based on the efficiency and consistency of Beta-binomial Model (BBM) in tracking and forecasting purchasing pattern of consumers of Soft drink using secondary data collected from whole sales outlet of a standard bottling Company, The model (BBM) as compared to binomial model (BM) was fitted to the data. Akaike Criterion, Bayesian Criterion and $\chi^2$ goodness of fit were used to establish the efficiency and flexibility of BBM over BM in predicting the customers' purchasing pattern. The analysis shows that Beta-Binomial Model fitted better coupled with it low standard error in predicting future purchasing when compared with Binomial Model. We can therefore say that the predictive efficiency of this model is high. The usefulness of BBM as illustrated using real data depicts that it can be relied on for consistent planning and decision making.
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**Key words:** Beta-binomial model, Predictive Beta-binomial Model, Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Coefficient of variation

## 1 Introduction

Generating accurate, valid and reliable consumer behavioral pattern is very crucial to the producers of economics products. In fact, many renowned economics had studied how consumers react to increase in price of a certain commodity; this has laid a solid foundation for the law of demand and supply which state that the higher the price the lower the quantity demanded and vice versa. To study the consumer buying behavior, it therefore becomes imperative to the producers and the economy planner to track their behavior as regards their product, for future planning, and optimum profitability. This can be modeled using a beta-binomial distribution, if the probability of success parameter, $p$, of a Binomial distribution has a beta distribution with shape parameters $\alpha > 0$ and $\beta > 0$, the resulting distribution is known as a beta binomial distribution. For a binomial distribution, $p$ is assumed to be fixed for successive trials or periods. For the beta-binomial distribution, the value of $p$ changes for each trial or period and it is said to be random variable having a beta-binomial distribution. Many researchers have contributed to the theory of beta binomial distribution and its applications in various fields, among them are Akomolafe et al (2009), Pearson (1925), Skellam (1948), Lord (1965), Greene (1970), Massy et. al. (1970), Griffiths (1973), Williams (1975), Huynh (1979), Wilcox (1979), Smith (1983), Lee and Sabavala (1987), Hughes and Madden (1993), and Shuckers (2003), are notable. Since study consumer behavior is very important to the business men and entrepreneurs around the world, because it determines to a great extents the going concern of their company, capacity of production and their net and capital investment as well as their total profit, then this project discusses the efficiency of the beta-binomial model as compared to the traditional binomial model in forecasting the consumer buying behavioral pattern; a case study of Seven up Bottling Company Plc (Ibadan Depot), and its application in predicting the future consumers of beverages. Beta-Binomial Model is being employed in this study because of its capability of capturing buying behavior pattern of consumers. The computed predictions are also compared for the four products under investigation so as to determine the efficiency of beta-binomial model compared to the conventional binomial model at different proposed production volume. The coefficient of variation at different sales volume showed the efficiency with respect to the lower relative standard deviation. It is hoped that the finding of this paper will be useful for practitioners in various fields, most especially the researchers and various economics planner such as market researchers, and business administrators.

### 1.1 Source Of The Data

The result reported in this research work is based on secondary data available from wholesales standard outlets of beverages outfit for two years. These reports have been generated with the help of sales record (purchase data), precisely Seven up Bottling Company Plc, available in the outlet which showcases the identity of customers as they patronize the outlets and eventually buy the products in large quantity.

**2 Derivation Of Mean, Variance And Posterior Predictive Mean**

The binomial model is $P(x) = {}^{n}C_{x}P^{n}q^{n-x}$

For $O \leq P \leq 1$, where x is a random variable.

Now, suppose p of binomial distribution varies from trial to trial and follow beta distribution. It is in this case, we can say beta distribution is a conjugate distribution of binomial distribution. Then this can be modeled using beta binomial model.

$$= P(x = p) = \frac{P^{\alpha-1}(1-P)^{\beta-1}}{B(\alpha,\beta)} \quad \alpha > 0\beta > 0,\ 0 \leq P \leq 1$$

Let P be represented by θ, then the beta distribution can be re-written as;

$$= P(x = p) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha,\beta)} \quad \alpha > 0\ \beta > 0,\ 0 \leq \theta \leq 1 \qquad (1)$$

The mean and variance of this BBD can be therefore be obtained by the following expectation procedures. The equation (1) above represent the beta-distribution when P of binomial varies from period to period.

$$= E(x) = E(x = \theta) = \int_0^1 \theta f(x) = \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} d\theta$$

Since $\Gamma\alpha+1\,\Gamma(\beta)\,\Gamma\alpha+\ \beta+1. \quad =1$ ,

then= $\Gamma\alpha+1\,\Gamma(\beta)\,\Gamma\alpha+\ \beta+1. \quad 01\,\Gamma\alpha+\beta\,\Gamma\alpha\,\Gamma(\beta)\theta^{\alpha-1}1-\theta^{\beta-1}d\theta$ (2)

$$= \frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+\beta+1)} \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 \frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+1)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} d\theta \qquad (3)$$

$$= \frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+\beta+1)} \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \qquad (4)$$

$$= \frac{\alpha\Gamma(\alpha)\Gamma(\beta)}{(\alpha+\beta)\Gamma(\alpha+\beta)} x \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \qquad (5)$$

$$E(\theta) = \frac{\alpha}{\alpha+\beta} \qquad (6)$$

The mean of beta distribution is given in (2) above

Since $P = {}^{x}/_{n}$, it can be written as

$\theta = {}^{x}/_{n}$; $x = n\theta$

Taking the expectation of both sides

$E(x) = E(n\theta)$

$E(x) = nE(\theta)$

Mean: $E(x) = n\,\alpha/(\alpha+\beta)$ (7)

The mean of BBM is represented by equation (3) above

Now, for the variance, we follow the same approach

Var $(\theta) = E^{\theta 2} - [(E^{\theta})]^2$

$$= \int_0^1 \theta^2 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} d\theta - \left(\int_0^1 \theta \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} d\theta\right)^2$$

$$= \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^2 \theta^{\alpha-1}(1-\theta)^{\beta-1} d\theta - \left(\frac{\alpha}{\alpha+\beta}\right)^2 \qquad (8)$$

$$= \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^2 \theta^{\alpha-1}(1-\theta)^{\beta-1} d\theta - \frac{\alpha^2}{(\alpha+\beta)^2} \qquad (9)$$

$$= \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+\beta+2)} \cdot \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+2)\Gamma(\beta)} \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha+1}(1-\theta)^{\beta-1} d\theta - \frac{\alpha^2}{(\alpha+\beta)^2} \qquad (10)$$

$$= \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+\beta+2)} \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma\beta} \cdot \int_0^1 \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+2)\Gamma(\beta)} \theta^{\alpha+1}(1-\theta)^{\beta-1} d\theta - \frac{\alpha^2}{(\alpha+\beta)^2} \qquad (11)$$

$$= \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+\beta+2)} \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma\beta} - \frac{\alpha^2}{(\alpha+\beta)^2} \qquad (12)$$

$$= \frac{\alpha(\alpha+1)\Gamma(\alpha)\Gamma(\beta)}{(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)} - \qquad (13)$$

$$= - \frac{\alpha^2}{(\alpha+\beta)^2} \quad (14)$$

$$= \frac{\alpha^2+\alpha}{(\alpha+\beta+1)(\alpha+\beta)} - \frac{\alpha^2}{(\alpha+\beta)^2} \quad (15)$$

$$= \alpha^2 + \alpha(\alpha+\beta) - (\alpha+\beta+1)^{(\alpha+\beta+1)(\alpha+\beta)^2} \quad (16)$$

$$= \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2} \quad (17)$$

The variance of beta distribution is represented by (17) above. To prove the variance of conditional distribution, it follows that,

$$var(x/\theta) = \frac{\theta(1-\theta)}{n} \quad (18)$$

Where $x$ is a binomial variable and $\theta$ follows beta distribution

$$E\big(var(x/\theta)\big) = \int_0^1 \frac{\theta(1-\theta)}{n} f(\theta)\, d\theta = \frac{1}{n}\int_0^1 \theta(1-\theta) d\theta \quad (19)$$

$$= \frac{1}{n}\int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta(1-\theta)\theta^{\alpha-1}(1-\theta)^{\beta-1} d\theta \quad (20)$$

$$= \frac{1}{n}\int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta\theta^{\alpha-1}(1-\theta)(1-\theta)^{\beta-1} d\theta \quad (21)$$

$$= \frac{1}{n}\int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha}(1-\theta)^{\beta} d\theta \quad (22)$$

Since $\frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} x \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)}$ =1, then we can say

$$= \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} x \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} \frac{1}{n}\int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha}(1-\theta)^{\beta} d\theta \quad (23)$$

$$= \frac{1}{n}\frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} x \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} \theta^{\alpha}(1-\theta)^{\beta} d\theta \quad (24)$$

$$= \frac{1}{n}\frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} x \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \quad (25)$$

$$= \frac{1}{n}\frac{\alpha\Gamma(\alpha)\beta\Gamma(\beta)}{(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)} x \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \quad (26)$$

$$= E[var(x/\theta)] = \frac{\alpha\beta}{n(\alpha+\beta)(\alpha+\beta+1)} \quad (27)$$

Now,

$$var(x) = var(\theta) + E\big(var(x/\theta)\big)$$

$$= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} + \frac{\alpha\beta}{n(\alpha+\beta)(\alpha+\beta+1)} \quad (28)$$

$$= \frac{n\alpha\beta + \alpha\beta(\alpha+\beta)}{n(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{\alpha\beta(n+\alpha+\beta)}{n(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{\alpha\beta(n+\alpha+\beta)}{n(\alpha+\beta)^2(\alpha+\beta+1)} \quad (30)$$

Therefore, variance of beta binomial is given by (30) above, the posterior predictive mean can be obtained as follows:

$$E(\theta; x, n, \alpha, \beta) = \int_0^1 \theta\, P(\theta; x, n, \alpha, \beta)\, d\theta \quad (31)$$

$$\int_0^1 \theta \,.\, \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+x)\Gamma(\beta+n-x)} \theta^{\alpha+x-1}(1-\theta)^{\beta+n-x-1} d\theta \quad (32)$$

$$= \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+x)\Gamma(\beta+n-x)} \int_0^1 \theta\theta^{\alpha+x-1}(1-\theta)^{\beta+n-x-1} d\theta \quad (33)$$

$$= \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+x)\Gamma(\beta+n-x)} \int_0^1 \theta^{\alpha+x}(1-\theta)^{\beta+n-x-1} d\theta \quad (34)$$

$$= \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+x)\Gamma(\beta+n-x)}, \frac{\Gamma(\alpha+x+1)\Gamma(\beta+n-x)}{\Gamma(\alpha+\beta+n+1)} \quad (35)$$

$$= \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+x)} \cdot \frac{\Gamma(\alpha+x+1)}{\Gamma(\alpha+\beta+n+1)} \quad (36)$$

$$= \frac{\Gamma(\alpha+x+1)}{\Gamma(\alpha+x)} \cdot \frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+\beta+n+1)} \quad (37)$$

$$= \frac{(\alpha+x)\,\Gamma(\alpha+x)}{\Gamma(\alpha+x)} \cdot \frac{\Gamma(\alpha+\beta+n)}{\alpha+\beta+n\,\Gamma(\alpha+\beta+n)} = \frac{(\alpha+x)}{\alpha+\beta+n} \quad (38)$$

$$= \frac{\alpha}{\alpha+\beta+n} + \frac{x}{\alpha+\beta+n} \quad (39)$$

The (39) above is the posterior predictive mean of BBM

To mix binomial distribution with beta distribution: it is assumed that the data collected on purchased data from 7up Bottling Company follows binomial distribution and P denotes the probability that a purchase will be made and (1-P) when no purchase is made. When purchase of any of Pepsi, 7up, Teem and Mirinda is made, there arise a binomial with unvarying P. but if the probability that a purchase is made is changing from period to period, this can be modeled by an Hybrid Model called Beta-binomial Hybrid Distribution. The efficiency of this model in tracking the pattern of purchase of consumers which result to determination of psychology of consumer's behavior can thereafter be detected by getting the main behavioral pattern and attitude towards consuming the 7up, Pepsi, Teem and Mirinda. The implementation of the BBM was done by written simple r codes, and the parameter estimation based on principle of both method of moment and maximum likelihood with the developed codes. The necessary goodness of fit test was also done by the AIC and BIC generated through the method of maximum likelihood in the GAMLSS and VGAM R-library.

The beta distribution is given as $P(x=p) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha,\beta)}$ $\alpha > 0$ $\beta > 0$, $0 \le \theta \le 1$ and B (α, β)= $\int_0^1 y^{\alpha-1}(1-y)^{\beta-1}\,dy$ is a beta function

Since θ = P is probability that purchase will be made and it varies from period to period, therefore the set of P form random sample which follows beta distribution.

$$= P(x) = \int_0^1 \binom{n}{x} \theta^{\alpha-1}(1-\theta)^{\beta-1} f(\theta)\,d\theta \quad (40)$$

$$= \int_0^1 \binom{n}{x} \theta^{\alpha-1}(1-\theta)^{\beta-1} \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha,\beta)}\,d\theta \quad (41)$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 \binom{n}{x} \theta^{\alpha+x-1}(1-\theta)^{n+\beta-x+1}\,d\theta \quad (42)$$

From beta function: B (α, β)= $\int_0^1 y^{\alpha-1}(1-y)^{\beta-1}\,dy$, we can conclude that the BBD is expressed as shown below:

$$= P(x) = \binom{n}{x} \frac{B(\alpha+x, n+\beta-x)}{B(\alpha,\beta)} \quad (43)$$

The equation (43) above is the beta-binomial model

## 3 Numerical Results And Discussion

### 3.1 Fitting BBM to Pepsi Flavor

**Table 1: Comparision Criteria for Binomial and Predictive Betabinomial Model**

| | Binomial (BM) | Betabinomial (BBM) |
|---|---|---|
| AIC | 40.82 | -61.13645 |
| BIC | 0.001 | -70.1628 |

From the table1 above, it can be deduced that the Predictive Beta-Binomial Model fits the Pepsi purchase data reasonably well. AIC and BIC showed that Beta-binomial is better than Binomial Model in modeling the consumers' preference for soft drinks. Therefore, BBM provides good fit regarding the prediction of future sales of the Pepsi and the model is efficient to describe the behavior of consumers. Since $^2$ calculated (25986.63)> $^2$ tabulated (12.592), we thereby have utmost statistical reason not to accept Ho, and conclude that the goodness of fit is significant for BBM, and it therefore modeled the Pepsi data accurately and reasonably well.

**Table 2: Estimation of Parameters**

| Parameter | Binomial Model (BM) | Betabinomial Model (BBM) |
|---|---|---|
| $\hat{p}$ | 0.5092 | |
| $\hat{\alpha}$ | | 0.8144925 |
| $\hat{\beta}$ | | 0.7850609 |

The parameter $\hat{\alpha}$ and $\hat{\beta}$ determine the shape of the distribution. With these two parameters greater than zero, we have beta binomial distribution. Since probability of success, p =*0.5092*, of a Binomial distribution has a beta distribution with shape parameters $\hat{\alpha} > 0$ and $\hat{\beta} > 0$, with this, the resulting distribution is known as a beta binomial distribution.

**Table 3: Efficiency of BBM at Different Proposed Sales Volume of Pepsi Flavor**

| Future sales ('0000') | Mean of BM | Mean of BBM | Standard Error of BM | Standard Error of BBM | CV (%) of BM | CV (%) of BBM |
|---|---|---|---|---|---|---|
| 30 | 4.305429 | 15.276 | 1.453652 | 0.4255340 | 33.76324 | 2.7856377 |
| 40 | 4.823770 | 20.368 | 1.538670 | 0.4228329 | 31.89767 | 2.0759667 |
| 50 | 5.673684 | 25.460 | 1.668725 | 0.4212039 | 29.41167 | 1.6543752 |
| 60 | 6.674332 | 30.552 | 1.809907 | 0.4201144 | 27.11742 | 1.3750799 |
| 70 | 7.847269 | 35.644 | 1.962509 | 0.419334 | 25.00881 | 1.1764518 |
| 80 | 9.302026 | 40.736 | 2.136688 | 0.4187485 | 22.97013 | 0.9879570 |
| 90 | 11.362336 | 45.828 | 2.361490 | 0.4182922 | 20.78349 | 0.912739 |
| 100 | 15.655153 | 50.920 | 2.771922 | 0.4179269 | 17.70613 | 0.8207520 |

From the table 3 above, we can conclude that coefficient of variation of BBM decreases with increasing in the sales volume (purchase). Although, the CV of BM is significantly larger than that of BBM, the BBM produces best estimate for predicting the probability of getting the future targeted sales for Pepsi. At volume 900,000crates, the CV is 0.9280664% and shows a decrease of 0.0935987% at volume 1,000,000 crates. The decrease in the CV of BBM is as a result that the model fitted the future sales the most as compared to its binomial counterpart, so BBM is efficient at different future sale and it is very efficient at significantly higher sales. The average sale of Pepsi increase with increasing production volume, and at sales 1million crates the coefficient of variation for BM and BBM are 17.70613% and 0.8207520% respectively which reflect that the CV of BBM is significantly reduced at sales one million. The BBM stands to be the best model for Pepsi due to its lower coefficient of variation.

**3.2 Fitting BBM for 7up flavor**

**Table 4: Estimation of model's parameter**

| Parameter | Binomial | Betabinomial | |
|---|---|---|---|
| $\hat{p}$ | 0.4975977 | | |
| $\hat{\alpha}$ | | 0.8659094 | |
| $\hat{\beta}$ | | 0.8742703 | $^2$=41308.5 |

The parameter $\hat{\alpha}$ and $\hat{\beta}$ determine the shape of the distribution. With these two parameters greater than zero, the resulting distribution is BBM. Since probability of success, *p=0.5092*, of a Binomial distribution has a beta distribution with shape parameters $\hat{\alpha} > 0$ and $\hat{\beta} > 0$, with this, the resulting distribution is known as a beta binomial distribution.

**3.2.1 Comparison criteria for binomial and Predictive BBM**

Since $^2$ calculated (41308.5) > $^2$ tabulated (12.592), we thereby have utmost statistical reason not to accept $H_o$, and conclude that the goodness of fit is significant for BBM. BBM therefore modeled the 7up data reasonably well. The result obtained in BM cannot be totally relied upon in decision making as regarding consumers' altitude to 7up Flavor. Also the AIC of -81.075 and 41.269 for BBM and BM respectively revealed that BBM provides consistent numerical evidence for predicting consumers' habit regarding 7up.

**Table 5: Efficiency of BBM at Different Proposed Sales Volume of 7up Flavor**

| Number of proposed sales for 7up ('0000') | Mean of BM | Mean of BBM | Standard error of BM | Standard error of BBM | CV% of BM | CV (%) of BBM |
|---|---|---|---|---|---|---|
| 30 | 4.255 | 14.927 | 1.462261 | 0.4234337 | 34.355790 | 2.8365200 |
| 40 | 5.004 | 19.9039 | 1.58500 | 0.4205219 | 31.68730 | 2.1127602 |
| 50 | 5.9600 | 24.879 | 1.730415 | 0.4187650 | 29.0336 | 1.6831468 |
| 60 | 7.008 | 29.856 | 1.876470 | 0.4175896 | 26.77377 | 1.3986856 |
| 70 | 8.159 | 34.832 | 2.024631 | 0.4167481 | 24.81451 | 1.1964573 |
| 80 | 9.472 | 39.808 | 2.181402 | 0.4161158 | 23.03116 | 1.40453118 |
| 90 | 11.116 | 44.784 | 2.363214 | 0.4856234 | 21.25928 | 0.9280664 |
| 100 | 13.856 | 49.759 | 2.63851 | 0.4152290 | 19.04113 | 0.8344677 |

From the table 5 above, we can conclude that CV decreases with increasing in the sales volume (purchase). Although, the CV of BM is significantly larger than that of BBM, the BBM produces best estimate for predicting future sales of 7up. At volume 900,000crates, the CV is 0.9280664% and shows a decrease of 0.0935987% at volume 1,000,000 crates. Although the average sales increase for both models, but those of BBM is significantly higher as compared to BM. It can be buttressed that at sale 1million crates, the CV of BBM is 0.8344677% while that of BM is 19.04113% weakening BM estimates in making future decision.

**3.3 Fitting BBM for Teem Flavor**

**Table 6: Estimation of model's parameter**

| Parameter | Binomial Model | Beta-binomial Model |
|---|---|---|
| $\hat{P}$ | 0.4967201 | |
| $\hat{\alpha}$ | | 0.897378 |
| $\hat{\beta}$ | | 0.9092291 |

The parameter $\hat{\alpha}$ and $\hat{\beta}$ determine the shape of the distribution. Since probability of success parameter, $p$=0.4967201, of a Binomial distribution has a beta distribution with shape parameters $\hat{\alpha}$> 0 and $\hat{\beta}$> 0, the resulting distribution is known as a beta binomial distribution.

**Table 7: Comparison Criteria for binomial and predictive BBM**

| | Binomial | Predictive BBM | |
|---|---|---|---|
| AIC | 41.19 | -127.2757 | |
| BIC | 0.01223 | -121.0373 | $^2$= 41305.83 |

From the table 7 above, it can be deduced that the Predictive Beta-Binomial Model fits the Teem purchase data accurately well. AIC and BIC supported BBM being better than BM in modeling Teem's data. The lower value of BIC and AIC for BBM as compared to BM shows that BBM is consistent as compared to BM in predicting buying behavior of consumers. Also, $^2$ calculated (41305.83)> $^2$ tabulated (12.592), we thereby have statistical reason not to accept $H_o$ and conclude that the fit is not good for binomial, therefore BBM modeled Teem's data more reasonably well. The result obtained in BM cannot be totally relied upon in decision making as regarding behavior of consumers towards to Teem flavor.

**Table 8: Efficiency of BBM at Different Proposed Sales Volume of Teem Flavor**

| Proposed sales teem ('0000') | mean of BM | Mean of BBM | Standard Error of BM | Standard Error of BBM | CV (%) of BM | CV (%) of BBM |
|---|---|---|---|---|---|---|
| 30 | 4.147249 | 14.90119 | 1.444744 | 0.4228144 | 34.83619 | 2.8374 |
| 40 | 5.027573 | 19.86826 | 1.590706 | 0.4198014 | 31.63964 | 2.1129 |
| 50 | 6.053631 | 24.83532 | 1.745495 | 0.4179833 | 28.83386 | 1.6830 |
| 60 | 7.14557 | 29.80238 | 1.896395 | 0.4167667 | 26.53950 | 1.3984 |
| 70 | 8.311104 | 34.76945 | 2.045220 | 0.4158956 | 24.60829 | 1.1962 |
| 80 | 9.593507 | 39.73651 | 2.197351 | 0.4152411 | 22.90456 | 1.0449 |
| 90 | 11.111975 | 44.70357 | 2.364865 | 0.4147313 | 21.28213 | 0.9277 |
| 100 | 13.37882 | 49.67064 | 2.594897 | 0.414320 | 19.39551 | 0.8342 |

From the table 8 above, we can conclude that coefficient of variation decreases with increasing in the sales volume. BBM produces best estimate for predicting future sales of Teem. At volume 900,000crates, the CV is 0.9277% and shows a decrease of 0.0935% at volume 1,000,000 crates. This means BBM would be the better fit for future purchasing pattern. The higher the production volume, the higher the sales, and it is hard for consumers to lose taste for this product because the probability of future sales keep increasing and the coefficient of variation of the Predictive BBM is very low at future sales. At sale 1milion, CV is 0.8342% and shows decrease of 0.0935% in the preceding sales. Probabilities of sales keep increasing and its coefficient of variation at each sales point decreases; this gives the BBM its unique predictive efficiency.

**3.4 Fitting BBM to Mirinda Flavor**

**Table 9: Estimation of Model's Parameter**

| PARAMETER | BM | BBM |
|---|---|---|
| $\hat{P}$ | 0.5578697 | |
| $\hat{\alpha}$ | | 0.8646325 |
| $\hat{\beta}$ | | 0.6852501 |

Since probability of success, $p$=0.4967201, of a Binomial distribution has a beta distribution with shape parameters $\hat{\alpha} > 0$ and $\hat{\beta} > 0$, with this, the resulting distribution is known as a beta binomial distribution.

**Table 10: Comparison Criteria for the Models**

| | BBM | BM | |
|---|---|---|---|
| AIC | -80.678 | 39.04 | |
| BIC | -80.4413 | 0.1453 | $^2$=473.1387 |

From the table10 above, it can be deduced that the Predictive BBM fits the Mirinda purchase data reasonably well. The Predictive BBM is efficient for forecasting and it is also consistent. Since $^2$ calculated (473.1387)> $^2$ tabulated (12.592), we thereby have utmost statistical reason not to accept $H_0$, and conclude that the fit is not good for binomial. Then BBM therefore modeled Mirinda data accurately well. The result obtained from BM cannot be totally relied upon in decision making regarding the behavior of consumers.

**Table 11: Efficiency of BBM at Different Proposed Sales Volume of Teem Flavor**

| Proposed Sales for mirinda ('0000') | Mean of BM | Mean of BBM | Standard Error of BM | Standard Error of BBM | CV (%) of BM | CV (%) of BBM |
|---|---|---|---|---|---|---|
| 30 | 3.484943 | 16.7361 | 1.241289 | 0.4279647 | 35.6186 | 2.5571 |
| 40 | 4.206747 | 22.3148 | 1.363793 | 0.4253287 | 32.4192 | 1.9060 |
| 50 | 5.173940 | 27.8935 | 1.512467 | 0.4237391 | 29.2324 | 1.5191 |
| 60 | 6.326849 | 33.4722 | 1.672511 | 0.4226761 | 26.4351 | 1.2628 |
| 70 | 7.740616 | 39.0509 | 1.849962 | 0.4219152 | 23.8994 | 1.0804 |
| 80 | 9.615769 | 44.6296 | 2.061896 | 0.42134436 | 21.4429 | 0.9441 |
| 90 | 12.548498 | 50.2083 | 2.355434 | 0.4208985 | 18.7707 | 0.8383 |
| 100 | 19.953516 | 55.7869 | 2.970194 | 0.4205451 | 14.8856 | 0.7538 |

**Table 12: Diagnosis of the Predictive Efficiency BBM (Posterior mean)**

| Future sales ('0000') | Pepsi | 7up | Teem | Mirinda |
|---|---|---|---|---|
| 20 | 0.9592902 | 0.9597855 | 0.9583008 | 0.9682217 |
| 30 | 0.9724107 | 0.9724554 | 0.9714110 | 0.9782804 |
| 40 | 0.9790286 | 0.9790545 | 0.97824493 | 0.9835078 |
| 50 | 0.9830859 | 0.9831027 | 0.9824478 | 0.9867070 |
| 60 | 0.9858277 | 0.9858395 | 0.9852876 | 0.9888668 |
| 70 | 0.9878046 | 0.9878134 | 0.9873365 | 0.9904228 |
| 80 | 0.9892975 | 0.9893043 | 0.988845 | 0.9915972 |
| 90 | 0.9904648 | 0.9904701 | 0.990095 | 0.9925150 |
| 100 | 0.9914025 | 0.9914068 | 0.991061 | 0.9932521 |

From table 11 above, we can conclude that CV decreases with increasing in sales volume. Although, CV of BMl is significantly larger than of BBM, BBM produces best result for predicting future sales of Mirinda. At volume 900,000crates, the CV is 0.8383% and shows a decrease of 0.0845% at volume 1million-crates. It means BBM produces better estimate as compared to BM.

In general, BBM provides good fit for all the four flavors under investigation and its predictive efficiency can be relied upon in prospective planning. For this reason, it provides excellent evidence regarding the prediction made about the four flavors. It can be concluded BBM performed exceedingly better for Pepsi and Mirinda because the CV% of both Pepsi and Mirinda are lower than that of other two flavors.

The table 12 above shows that the probability of purchases increases as the production volume increases. For instances, if the production volume is 1million crate, then the probability that all the produced 1million-crates will be purchased can be modeled by the above predictive probabilities. It can then be deduced from the table above that out of one million-crates produced, the probability that all the 1million crates will be purchased are 0.9914025, 0.9914068, 0.991061 and 0.9932521 for pepsi,7up, Teem and Mirinda respectively. We now have statistical reason to say, as the production volume increases, the probability of purchase is tending to unity. The purchasing power of the consumer for the four flavors at production volume one million is 96.752417%. The populace is having good taste for the flavors and they are ready to buy all the produced flavors at any given period. That is, when production volume is sufficiently large say (N), so also the probability of purchasing all the products will be extremely large. From the predictive probabilities above, one can conclude that the probabilities of purchasing Mirinda product are higher as compared to other three products. Since there are no significant differences between all the predicted probabilities for all the flavors, then we can conclude that the four flavors, rolling out of 7up Bottling Company Plc, will continue to gain public acceptance of the populace of Ibadan because the products had met their taste and have been satisfying their refreshing taste for long period of time.

**Conclusion**

The BBM provides a good basis for relying on the predicted values for adequate and consistent decision making and planning based on the study; BBM predicted that the probability of purchase increases with increase in the number of future sales. In general, BBM has stood the test of Akaike and Bayesian Information Criteria, for this reason, it provides efficient, consistence and reliable evidence based on the prediction obtained from the analytical expressions about the four flavors.

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