

## Use of Approximate Dynamic Programming to solve problems that arises in appointment system in the health care facility.

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**Abstract:** With the goal to develop a model that prescribes the optimal appointment date for a patient at the moment this patient makes his request. We modelled the scheduling process as an MDP. By standard, value iteration is used to solve MDPs but it is computationally infeasible to solve our MDP to optimality due to the curse of dimensionality Value iteration suffers from. We therefore employed an ADP technique, in order to derive an estimate of the optimal value function of our MDP. We simulated our initial policy to determine  $g$  and we keep track of the last  $S$  states that are visited. These states are added to what we call the list of important states with probability  $0.2$ . We apply the k-means clustering algorithm to the list of important states to determine the set of representative states. From the scheduling process over three working days, we see no substantial difference between the number of last states ( $S$ ) and clusters( $K$ ).

[Odunayo Ajala. **Use of Approximate Dynamic Programming to solve problems that arises in appointment system in the health care facility.** *Rep Opinion* 2021;13(11):27-32]. ISSN 1553-9873 (print);ISSN 2375-7205 (online). <http://www.sciencepub.net/report>. 5.doi:[10.7537/marsroj131121.05](https://doi.org/10.7537/marsroj131121.05).

**Keywords:** Use; Approximate; Dynamic; Programming; solve; problem; appointment; system; health; Facility;

### CHAPTER FOUR ANALYSIS AND RESULT

#### 4.1 Four working days

##### 4.1.1 Bellman Error Minimisation

Following the procedure from Section 3.13, for each combination of the parameters in Table 3.1 we apply the BEM method. We can make  $5 \times 3 \times 3 = 45$  combinations, for each combination we apply the BEM method with 8 different approximation functions. Each time we apply the BEM method, we

compare  $g$  obtained from our initial policy with  $g$  obtained after the one-step policy improvement and compute the improvement that is made. We refer to this as the improvement of the BEM method. Figure 4.1 shows for each  $K$  a box plot of the improvement of the BEM method of the results. Each point in this box plot indicates a BEM technique improvement accomplished using one of the combinations of  $S$ ,  $T$  and the remaining approximation functions.

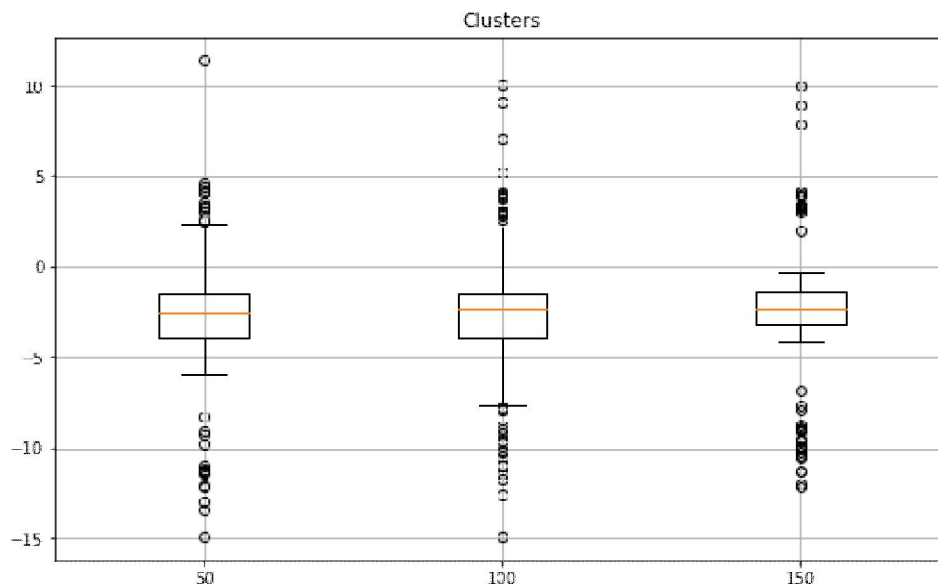


Figure 4.1: Box plots of the improvement for each K.

In Figure 4.2, the box plots show how the improvement of the BEM approach for each number of last states (S). There appears to be no significant

variation in the number of last states (S). There seems to be no substantial difference between the number of last states (S).

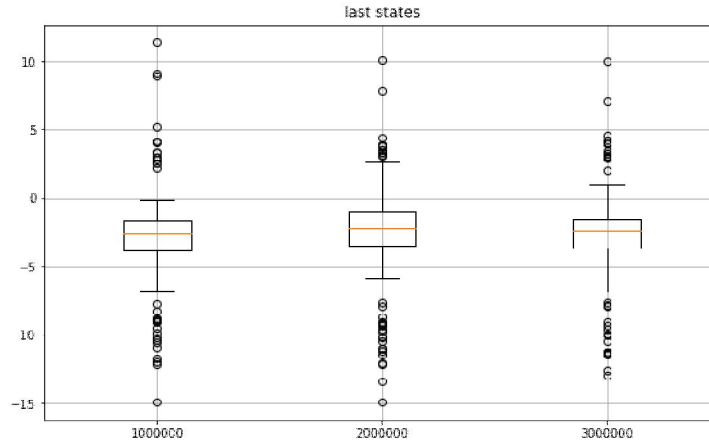


Figure 4.2: Box plots of the improvement for each number of last states (S).

Figure 4.3 shows for each T a box plot of the improvement of the BEM method. It shows that the lower the value of T, the worse the outcome. We therefore remove the value of T = 31, 33 In other to get a better result we apply the bottom up approach in order to find a good approximation function that shows in general the best improvement.

#### 4.1.2 Approximation function

We use the BEM method for a reduced number of T. we have  $3 \times 3 \times 3 = 27$  combinations remaining. Table 4.2 shows for the remaining approximation functions the average and variance of the improvement of the BEM method over 27 combinations. As can be seen functions 07 give the best results with an average improvement of 7.16%. we start with our bottom up approach with all of the functions over the 27 combinations.

**Table 4.2** Median and average of the improvement of the remaining results for each approximation function.

Function	Median (%)	Mean (%)
0	6.06	6.25
01	6.17	4.62
02	6.28	5.85
03	6.45	6.01
04	5.25	4.95
05	5.94	4.67
06	5.47	3.91
<b>07</b>	<b>7.16</b>	<b>5.36</b>

Table 4.3 shows the results from the first step of the bottom up approach. For each approximation function the median, average and variance of the improvement of the BEM method are given. For function 04 it holds that the mean increases slightly from 5.25% to 6.20% when function 1 is added. Adding one of the other functions does not improve the average or median. For function 05, an improvement is made when function 3 is added. We also have an improvement made with function 06 when function 1 or 3 is added. Therefore, in our second step of the bottom up approach we start with 041, 053, 061 and 063.

**Table 4.3** Median and average of the improvement of the remaining results for each approximation function after one step.

(a) Function 01

Function	Median (%)	Mean (%)
012	4.66	3.56
013	4.99	3.26
014	4.24	3.24
015	5.22	3.07
016	4.94	3.20
017	4.30	2.95

(b) Function 02

Function	Median (%)	Mean (%)
021	4.85	4.04
023	5.12	3.68
024	5.08	3.95
025	4.85	4.02
026	4.78	3.88
027	5.44	4.07

(c) Function 03

Function	Median (%)	Mean (%)
031	5.06	4.12
032	5.17	4.00
034	4.66	3.71
035	5.53	3.76
036	5.19	4.12
037	5.19	4.24

(d) Function 04

Function	Median (%)	Mean (%)
041	6.20	6.11
042	4.89	3.75
043	2.37	1.55
045	4.59	3.75
046	5.14	4.16
047	4.72	3.84

(e) Function 05

Function	Median (%)	Mean (%)
051	4.95	4.12
052	5.32	4.13
053	8.29	6.48
054	5.49	3.77
056	-4.89	-3.79
057	4.66	3.68

(f) Function 06

Function	Median (%)	Mean (%)
061	5.67	4.04
062	5.32	4.22
063	9.21	5.74
064	5.21	4.16
065	5.24	3.58
067	4.86	3.91

(g) Function 07

Function	Median (%)	Mean (%)
071	5.00	3.87
072	5.04	3.67
073	1.03	0.95
074	5.09	3.86
075	5.10	4.04
076	5.02	3.54

Table 4.4 shows the results of the second step of the bottom up approach. As can be seen, no further improvement is obtained.

**Table 4.4** Median and average of the improvement of the remaining results for each approximation function after two steps.

(a) Function 041

Function	Median (%)	Mean (%)
0412	5.37	3.84
0413	5.16	3.73
0415	5.77	4.03
0416	5.34	3.87
0417	5.05	3.76

(b) Function 053

Function	Median (%)	Mean (%)
0531	7.80	5.79
0532	4.63	3.42
0534	1.45	1.43
0536	0.78	1.00
0537	5.09	3.77

(c) Function 061

Function	Median (%)	Mean (%)
0612	1.00	1.00
0613	-4.04	-2.13
0614	1.00	1.00
0615	1.00	1.00
0617	1.00	1.00

(d) Function 063

Function	Median (%)	Mean (%)
0631	5.09	3.95
0632	4.77	3.79
0634	5.34	3.26
0635	5.05	3.78
0637	5.55	4.18

Function 041, 053, 061 and 063 are the function that gives the best improvements during the one-step policy improvement, based on the median and average for the scheduling process over four working days. Therefore, we apply these functions to the scheduling process over six, and eight working days. Since function 07 also performs very good for the scheduling process over four working days, we also apply these functions to the scheduling process over six, and eight working days. To simplify the figures in the following sections, we create a translation table, see Table 4.5. From here, if we write about function A, we actually mean function 07.

**Table 4.5** Translation table for the different functions.

New Function Name	Old Function Name
A	07
B	041
C	053
D	061
E	063

Figure 4.4 shows for each  $\lambda$  the average improvement of the BEM method by each of the approximation function. By improvement, we refer to the improvement made when we compare  $g$  obtained from our initial policy with  $g$  obtained after the one-step policy improvement. We see that if  $\lambda \leq 12$ , the average improvement for each function increases as  $\lambda$  increases. When  $\lambda > 12$  the average improvement for each function seem to decrease as  $\lambda$  decreases.

The lower  $\lambda$ , the lower the load of the system which infer the better our initial policy performs and hence, less improvement is possible. Whereas on the other hand, the higher  $\lambda$ , the higher the load of the system which infer the worse our initial policy performs and hence, the more important our one-step policy improvement. But if  $\lambda$  reaches a certain value, the load of the system becomes that high that it does not matter what policy is applied, since it will be imperative to reject many patients.

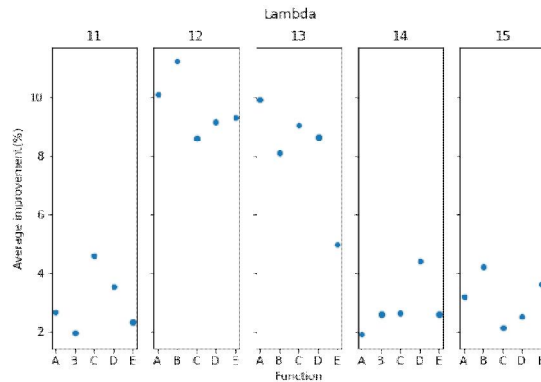


Figure 4.4 Average improvement by  $\lambda$  for different function

#### 4.2 Six, Eight working days

We apply the BEM method over six and eight working days. The results of the approximation functions {A, B, C, D, E} of the scheduling process over six, eight days are given. The parameters S and K needed for the BEM method are fixed to 1000000 and 50 respectively. Figure 4.5 shows for each  $\lambda$  the average improvement of the BEM method by the different functions for the scheduling process over six working days. It shows more or less the same pattern as the results of the scheduling process over four working days. We see that if  $\lambda \leq 12$ , the average improvement for each function increases as  $\lambda$  increases. When  $\lambda > 12$  the average improvement for each function seem to decrease as  $\lambda$  decreases.

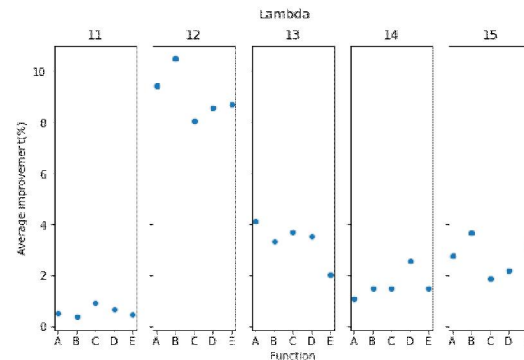


Figure 4.5 Average improvement by T for different functions for the scheduling process over six working days

Figure 4.6 shows for each  $\lambda$  the average improvement of the BEM method by the different functions for the scheduling process over 8 working days. Function {A, B, E} shows the same pattern as the results for four and six working days. For function {A, B, E}, the threshold is at  $\lambda = 12$ . Function C shows a different pattern than have been seen before. It has its highest result at  $\lambda = 11$  and

continues with a decline thereafter. At  $\lambda = 11$  the average improvement gives 4.19% which apparently shows the importance of the one-step policy with this load of the system. Function D also shows a similar pattern to Function C. It has its highest result at  $\lambda = 11$ . The average improvement then decreases to 2.76% After which the average improvement is increased at  $\lambda = 14$

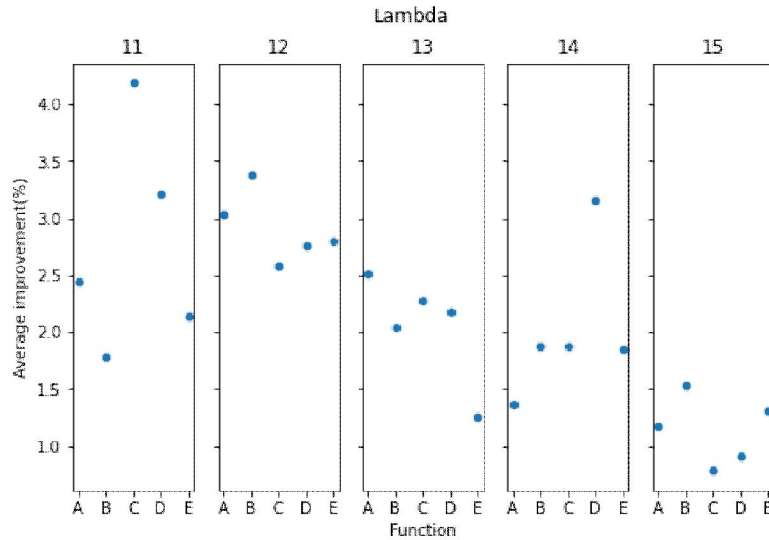


Figure 4.5 Average improvement by  $\lambda$  for different functions for the scheduling process over eight working days

**Table 4.6** Average of the improvement by functions for four, six and eight working days.

Function	$N = 4$	$N = 6$	$N = 8$
	Avg (%)	Avg (%)	Avg (%)
A	5.57	3.60	2.11
B	5.63	3.89	2.12
C	5.41	3.22	2.34
D	5.66	3.53	2.45
E	4.58	3.18	1.87

Table 4.6 shows for the scheduling process over four, six and eight working days for each function the average and variance of the improvement of the BEM method relative to the initial policy. Overall, function B and D give the overall best improvement over four, six and eight working days

## CHAPTER FIVE SUMMARY, CONCLUSION AND RECOMMENDATION

### 5.1 Summary

With the goal to develop a model that prescribes the optimal appointment date for a patient at the moment this patient makes his request. We modelled the scheduling process as an MDP. By standard, value iteration is used to solve MDPs but it is computationally infeasible to solve our MDP to

optimality due to the curse of dimensionality Value iteration suffers from. We therefore employed an ADP technique, in order to derive an estimate of the optimal value function of our MDP. We simulated our initial policy to determine  $g$  and we keep track of the last  $S$  states that are visited. These states are added to what we call the list of important states with probability  $0.2$ . We apply the k-means clustering algorithm to the list of important states to determine the set of representative states. From the scheduling process over three working days, we see no substantial difference between the number of last states ( $S$ ) and clusters( $K$ ).

From all combinations of the set of basis functions, the following two combination outperforms all other combinations:

Approximation function B:

$$d + \sum_{p=1}^3 \sum_{n=1}^N i_n^p + \sum_{n=1}^N i_n * d + \sum_{n=1}^{N-1} i_n * i_{n+1}$$

Approximate function D:

$$d + \sum_{p=1}^3 \sum_{n=1}^N i_n^p + \sum_{n=1}^{N-1} i_n * i_{n+1} * d + d^2$$

The average improvement of the Approximation Function B compared to the initial policy is 5.63%. while the average improvement of the Approximation Function D compared to the initial policy is 5.66%. These functions also outperform all other combination in the overall average of scheduling processes over four, six and eight working days. In general it holds that the lower  $\lambda$ , the lower the low load of the system, the better our initial policy performs and hence, less improvement is obtained. The higher the load of the system, the worse our initial policy performs and the more important is our one-step policy improvement. But if  $\lambda$  reaches a certain value the load of the system becomes that high that it does not matter what policy is applied, since many patients have to be rejected.

## 5.2 Conclusion

In this research, we have been able to make use of Approximate Dynamic Programming to solve problems that arises in appointment system in the health care facility.

## 5.3 Recommendation

The following recommendation arising from this research is as follows;  
Our model has to do with inter-day and not intra-day. Intra-day is recommended as an extension, which

prescribes the optimal time and sequence of appointment on a given day.

Patients may have preferences for a certain date and/or time for their appointment. Our model only gives patient one appointment date. An extension is recommended to allow and return multiple appointment options, giving patients several options to choose from.

Cancellation and no shows are common experiences in appointment system. Including this would be great. Too many no shows and cancellation would make disturb the system significantly.

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10/2/2021