



Fuzzy Coloring Or Fuzzy Graphs Of Fuzzy Chromatic Number Of Middle And Total Fuzzy Graphs

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Abstract: In this a portion of the fuzzy graphs on which are $\chi_f(G) = \chi(G^*)$ talked about. At that point fundamental and adequate condition of a fuzzy graph of four vertices on a complete graph to be regular is found and the adequate condition of a fuzzy graph on a complete graph to be regular is likewise talked about. We will likewise read for the given fuzzy graph $G: (\sigma, \mu)$, development fuzzy graph $sd(G): (\sigma_{sd}, \mu_{sd})$, square fuzzy graph $S^2(G): (\sigma, \mu_{sq})$ and absolute fuzzy graph $T(G): (\sigma T, \mu T)$ of G are characterized. Additionally the isomorphism between the square of the development fuzzy graph and $T(G)$ is talked about. Additionally the middle fuzzy graph $M(G)$ is presented. The relationship of $M(G)$ with $sd(G)$, and $T(G)$ are considered.

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Introduction

Coloring of graph is one of the most significant exploration regions of combinatorial streamlining because of its wide applications, all things considered, viz. the board sciences, wiring printed circuits, asset distribution, planning issues, and so forth. These issues are demonstrated by appropriate fresh graphs and settled by coloring of these graphs. In the customary graph coloring issue, least number of colors is given to the vertices of the graphs to such an extent that no two neighboring vertices have similar colors. Fuzzy graph portrayal is more appropriate to reality than fresh graph portrayal. Each occasion in genuine world can be spoken to by fuzzy graphs appropriately. Fuzzy graph hypothesis is progressed with enormous number of branches.

Right off the bat in the writing, one of valuable issues, the traffic light issue was unraveled by utilizing fresh graph coloring technique. In any case, in rush hour gridlock light issue, a few courses are busy contrasted with different courses. Likewise, here and there, two courses can be opened at the same time with some caution. Here "busy", "caution" is fuzzy terms. Specialist structured the traffic light issue by fuzzy graphs and presented the technique for coloring in fuzzy graphs. In that paper, the fuzzy graphs are considered with fresh vertices and fuzzy edges. At that point α -cut of these fuzzy graphs are colored by the strategy for fresh graph coloring. So for various estimations of α , we have diverse fresh graphs and these fresh graphs are colored. In this way, the chromatic number fluctuates for the equivalent fuzzy

graphs for various estimations of α . Additionally, Researchers proposed an alternate technique to color fuzzy graphs. In that paper, a term division level of fuzzy graphs has been characterized and dependent on the estimation of detachment degree, number of least color is found. Here, "fuzzy color" is characterized. A fuzzy graph is colored utilizing fuzzy color dependent on the quality of an edge episode to a vertex. This new coloring idea is utilized to color political guide and to fathom new kind of traffic light coloring issue. In this paper 'quality cut graph' of fuzzy graphs have been thought of. Some fascinating outcomes on this kind of graphs have been contemplated. At that point fuzzy coloring of fuzzy graphs has been presented.

3.3 Fuzzy graphs with $\chi_f(G) = \chi(G^*)$

3.3.1 Theorem

If G is a strong fuzzy graph, then $\chi_f(G) = \chi(G^*)$

Proof:

Leave G be a strong fuzzy graph. At that point all edges of G are strong edge. Presently build a graph $G_1=(V, E')$ such that an edge (x, y) is strong edge in G if and just that it is an edge in G_1 . Clearly G_1 is a crisp graph.

Since all edges of G are strong and there is no edge which isn't strong, each edge of G is likewise an edge of G_1 thus $\chi_f(G) = \chi(G_1)$. We know that G_1 is same as G^* .

Hence $\chi_f(G) = \chi(G^*)$.

3.3.2 Corollary

In the event that G is a complete fuzzy graph, at that point $\chi_f(G) = \chi(G^*)$.

Proof:

Let G be a complete fuzzy graph. Since each complete fuzzy graph is strong, by theorem 3.3.1

$$\chi_f(G) = \chi(G^*)$$

3.3.3 Theorem

If G is a fuzzy cycle then $\chi_f(G) = \chi(G^*)$.

Proof:

Let G be a fuzzy cycle of length n. At that point either n is odd or even.

Case 1:

Expect that n is odd. We realize that fuzzy chromatic number of fuzzy pattern of odd length is 3. The fundamental crisp graph G* is the crisp pattern of odd length whose chromatic number is 3. Thus

$$\chi_f(G) = \chi(G^*)$$

Case 2:

In the event that n is even, at that point fuzzy chromatic number of G is 2. Since G* is the pattern of even length, we have $\chi_f(G) = \chi(G^*)$.

3.3.4 Corollary

Let G be a regular fuzzy graph with the end goal that G* is a cycle of length n. At that point $\chi_f(G) = \chi(G^*)$.

Proof:

On the off chance that G is a regular fuzzy graph where G* is a cycle, at that point G is a fuzzy cycle. Thusly by theorem 3.3.3, $\chi_f(G) = \chi(G^*)$.

3.3.5 Theorem

Let G be a fuzzy graph with the end goal that each vertex is strong adjoining all different vertices.

Then $\chi_f(G) = n$. In this case $\chi_f(G) = \chi(G^*)$.

Proof:

Let G be a fuzzy graph with the end goal that each vertex is strong adjoining all different vertices. Obviously this is a fuzzy graph where all edges are strong. In this kind of fuzzy graph, every vertex will get a special color. Since there are n vertices in G, $\chi_f(G) = n$.

Obviously the basic crisp graph of this fuzzy graph is a complete graph. Thus $\chi_f(G) = \chi(G^*)$.

$$\text{Let } \mu(e_1) = \mu(e_3) = a, \mu(e_2) = \mu(e_4) = b \ \& \ \mu(e_5) = \mu(e_6) = c$$

What's more, assume that $a < b < c$.

At that point, if $i=5, 6$ and, if $i=1, 2, 3$ and 4. Along these lines edges $e_2, 4, e_5$ and e_6 are strong edges and e_1, e_3 are not strong edges. Presently allot color 1 to

3.3.6 Proposition

In the event that G contains no cycle, at that point

$$\chi_f(G) = \chi(G^*)$$

3.4 Fuzzy Coloring Of Regular Fuzzy Graphs On Complete graphs

Each complete graph K_n is the association of cycle of length n and its supplement i.e.

$$K_n = C_n \cup \bar{C}_n$$

3.4.1 Theorem

Let G be a fuzzy graph with the end goal that G* is a complete graph on four vertices. At that point G is regular fuzzy graph if and just if exchange edges of C_4 have a similar participation esteems and edges of C_4 have same enrollment esteem.

Proof:

Expect that G is a fuzzy graph on a complete graph with four vertices v_1, v_2, v_3 and v_4 .

Let $e_1=(v_1, v_2), e_2=(v_2, v_3), e_3=(v_3, v_4)$ and $e_4=(v_4, v_1)$ are edge so fC_4 and $e_5=(v_1, v_3), e_6=(v_2, v_4)$ are edges of C_4 .

Obviously every vertex has degree 3 in G* i.e., 3 edges are episode on every vertex. By our assumption, edges e_1, e_4, e_5 are occurrence on v_1 , edges $e_1, e_2, 6$ are episode on v_2 , edges $e_2, e_3, 5$ are occurrence on v_3 and edges e_3, e_4, e_6 are episode on v_4 . Presently G is regular if and just if the accompanying equations hold

$$\mu(e_1) + \mu(e_4) + \mu(e_5) = a$$

$$\mu(e_1) + \mu(e_2) + \mu(e_6) = a$$

$$\mu(e_2) + \mu(e_3) + \mu(e_5) = a$$

$$\mu(e_3) + \mu(e_4) + \mu(e_6) = a$$

On solving these equations,

We

get

$$\mu(e_1) = \mu(e_3), \mu(e_2) = \mu(e_4) \text{ and } \mu(e_5) = \mu(e_6)$$

Thus G is regular if and only if

$$\mu(e_1) = \mu(e_3), \mu(e_2) = \mu(e_4) \text{ and } \mu(e_5) = \mu(e_6)$$

3.4.2 Corollary

In theorem 3.4.1, on the off chance that the three participation esteems are in carefully expanding (carefully diminishing) arrangement, at that point $\chi_f(G) = 2$.

Proof:

v_1, v_2 and color 2 to v_3, v_4 . Plainly this is fuzzy coloring of least cardinality. Thus, Likewise we can demonstrate this outcome for carefully diminishing arrangement.

3.4.3 Corollary

Let G be a fuzzy graph given in the theorem 3.4.1. In the event that all edges of G have same participation esteems, at that point $\chi_f(G) = 4$. In this case, $\chi_f(G) = \chi_f(G^*)$.

Proof:

On the off chance that all enrollment estimations of edges are same, at that point G ought to be strong fuzzy graph. By theorem 3.3.1, every vertex will get interesting color. Henceforth $\chi_f(G) = 4$.

3.4.4 Example

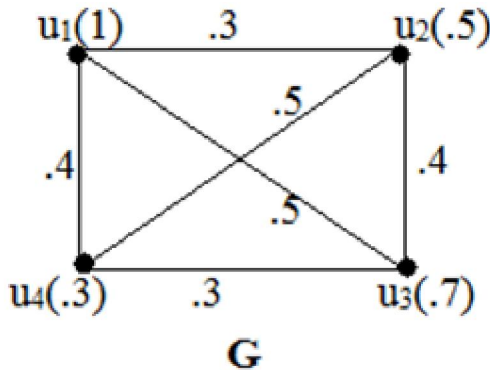


Figure 3.1: REGULAR FUZZY GRAPH

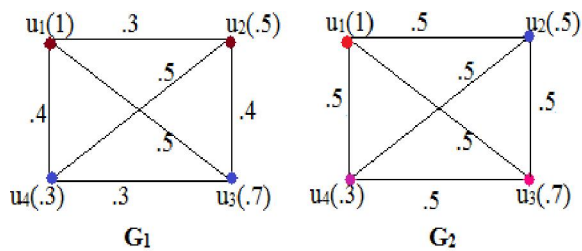


Figure 3.2: Regular Fuzzy Graphs With Different Fuzzy Chromatic Number

3.4.5 Theorem

The complete graph $K_{2n+1}, \geq 1$ has n edge-disjoint Hamiltonian cycles.

3.4.6 Theorem

Let G be a fuzzy graph with the end goal that G^* is K_{2n+1} . At that point G is regular fuzzy graph if the edges of each Hamiltonian cycles in G^* have same enrollment esteems.

Proof:

Let G be a fuzzy graph with the end goal that G^* is K_{2n+1} and name the vertices of G as $v, 0, 1, \dots, 2n-1$. By theorem 3.4.5, for any $n \geq 1$ K_{2n+1} has n Hamiltonian edge-disjoint cycles. They are given underneath

- $C_1: v, 0, 2_{n-1}, 1, 2_{n-2}, 2, 2_{n-3} \dots n-1, n, v.$
- $C_2: v, 1, 0, 2, 2_{n-1}, 3, 2_{n-2} \dots n, n+1, v.$
- $C_3: v, 2, 1, 3, 0, 4, 2_{n-1} \dots n+1, n+2, v.$
- $C_n: v, n-1, n-2, n, n-3, n+1, n-4 \dots 2_{n-2}, 2_{n-1}, v.$

From these cycles, each vertex is neighboring precisely two vertices of each cycle. That is on the off chance that $v \in V$, at that point

$$d_i^*(v) = 2, i = 1, 2, \dots, n. (d_i^*(v) \text{ denotes the degree of } v \text{ in } i^{\text{th}} \text{ cycle of } G^*) \text{ so that}$$

$$d^*(v) = d_1^*(v) + d_2^*(v) + d_3^*(v) + \dots + d_n^*(v) = 2n, \forall v \in V.$$

In fuzzy graph G, assume that the edges of n Hamiltonian edge-disjoint cycles have same participation esteems.

Let a_1, a_2, \dots, a_n be the membership values of C_1, C_2, \dots, C_n respectively.

We realize that each vertex of G is neighboring 2n vertices and precisely two edges of each Hamiltonian cycle are incident with every vertex. Since there are n Hamiltonian edge-disjoint cycles, the degree of v in G is $(v) = 2a_1 + 2a_2 + \dots + 2a_n = 2a$,

Where $a = a_1 + a_2 + \dots + a_n$.

Similarly $(0) = (1) = \dots = (2n-1) = 2a$. In this manner degree of each vertex is 2a, for nearly a.

Hence G is regular.

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