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State and Disturbance Estimation of a Linear Systems using Proportional Integral Observer

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Abstract: This paper offers a short survey of linear systems Proportional-Integral-Observer design. This observer has the capacity to estimate simultaneously the states and unknown inputs which include disturbances or model uncertainties appearing on the system. The design of state and output estimation using PO and state, output and disturbance estimation using PIO is done using Matlab/Simulink successfully. The simulation is done for estimating using PO and PIO and the results proved that estimates the state variables and output correctly when there is no disturbance in the plant and there is a constant steady-state error in estimation after leading a constant disturbance into the plant for both state variables and plant output for the Proportional Observer and there is ability to estimate state variables, disturbance and system output correctly with or without the disturbance in plant for the Proportional Integral Observer.

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1. Introduction

Observers play a vital rules on control system because some control techniques require the accurate estimation of system to realize the close loop control obligations. Measuring all states is not usually feasible due to the fact a few states aren't measurable or the usage of sensors may additionally too high priced. Meanwhile excessive quality performance may be accomplished thru the estimation of unknown inputs affecting the system inclusive of disturbances or model uncertainties. Beside the estimation of states and unknown inputs, observers are also able to growth the control performance. In this contribution a quick survey and evaluation of PI-Observer design and its formulation for a linear system is presented.

2. State and Disturbance Estimation

The state-space model of an n^{th} order, p input *and* q output plant with l independent disturbance of constant value is described as:

$$\left.\begin{aligned} \dot{x} &= Ax + Bu + Ed \\ \dot{d} &= 0 \end{aligned}\right\} \quad (1)$$

The plant state vector x is $n \times 1$, the plant input vector u is $p \times 1$, and the independent disturbance d is an $l \times 1$ vector. In case of the disturbance model is unknown, the matrix E can be assumed to be identity matrix with the same order of plant. The output y, a $q \times 1$ vector, of the plant is:

$$y = Cx \quad (2)$$

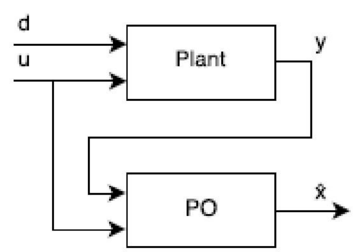

Figure 1. Block diagram of proportional observer

Proportional observers are built to estimate state variables using the plant input and output, as is shown in Figure 1. The state-space model of proportional observer in shown as follows:

$$\dot{\hat{x}} = A\hat{x} + Bu + L\left(y - C\hat{x}\right) \quad (3)$$

In which $\hat{x}$ is the estimated of state variables. Subtract equation (3) from (1), letting $e = x - \hat{x}$, which is the error between actual and estimated variables and disturbances, so that:

$$\dot{e} = \dot{x} - \dot{\hat{x}} = Ax + Ed - \left(A - LC\right)\hat{x} - Ly = \left(A - LC\right)e + Ed \quad (4)$$

Therefore, if there is no disturbance in plant (d = 0), a proportional observer has the ability to estimate the state variables, if ($A-LC$) is Routh-Hurwitz stable which means all eigenvalues of ($A-LC$) have negative real parts. However, if d is a non-zero constant, there will be a constant steady-state error between the estimated and actual state variables.

In order to eliminate this error in estimation, disturbance observer (DO) is described as follows:

$$\left.\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L_P\left(y - C\hat{x}\right) + E\hat{d} \\ \dot{\hat{d}} &= L_I\left(y - C\hat{x}\right) \end{aligned}\right\} \qquad (5)$$

According to the definition [5], the state space mode of proportional integral observer is:

$$\left.\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + Ev + G\left(y - C\hat{x}\right) \\ \dot{v} &= F\left(y - C\hat{x}\right) \end{aligned}\right\} \qquad (6)$$

Comparing equation (5) and equation (6), it is obvious that disturbance observer can be regarded as proportional integral observer in a special case. The block diagram of proportional integral observer is shown in Figure 2.

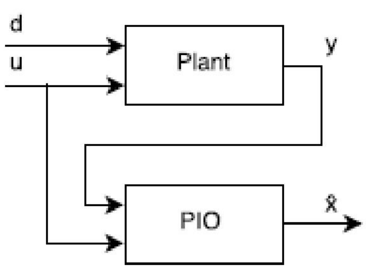

Figure 2. Block diagram of proportional integral observer

Recall the state space model of the plant with disturbance described by equation (1). These two equations can be combined together by defining $z = \begin{bmatrix} x \\ d \end{bmatrix}$, so that

$$\dot{z} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix}u = A_Z z + B_Z u \qquad (7)$$

The output of the plant is given by the following equation:

$$y = Cx = \begin{bmatrix} C & 0 \end{bmatrix}\begin{bmatrix} x \\ d \end{bmatrix} = C_Z z \qquad (8)$$

Similarly, the state space model of disturbance observer described by equation (5) can also be rewritten into the following equation by defining $\hat{z} = \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}$:

$$\dot{\hat{z}} = \begin{bmatrix} A - L_p C & E \\ -L_I C & 0 \end{bmatrix} z + \begin{bmatrix} L_p \\ L_I \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u \qquad (9)$$

By forming

,

$$A_{Z(n+l\times n+l)} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}$$

$$B_{Z(n+l\times p)} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$C_{Z(q\times n+l)} = \begin{bmatrix} C & 0 \end{bmatrix}$$

$$L_{Z(n+l\times q)} = \begin{bmatrix} L_p \\ L_I \end{bmatrix}$$

Equation (9) is equivalent to:

$$\left.\begin{aligned} \dot{\hat{z}} &= \left(\begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} L_p \\ L_I \end{bmatrix}\begin{bmatrix} C & 0 \end{bmatrix}\right) z + \begin{bmatrix} L_p \\ L_I \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u \\ &= \left(A_Z - L_Z C_Z\right)\hat{z} + L_Z y + B_Z u \end{aligned}\right\} \qquad (10)$$

In this paper, the state space model of plant with disturbance (A_Z,B_Z,C_Z) described by equation (7) will be called the extended plant model, and the state space model of DO described by equation (10) will be called the extended observer.

Subtract equation (7) by equation (10), letting e = $z - \hat{z}$, which is the error between actual and estimated variables and disturbances, so that:

Noticing that y = Czz in equation (8), the equation above can be rewritten as:

$$\dot{e}_z = A_Z z - L_Z C_Z z - \left(A_Z - L_Z C_Z\right)\hat{z} = \left(A_Z - L_Z C_Z\right)e \qquad (12)$$

Thus, as long as $A_Z - L_Z C_Z = \begin{bmatrix} A - L_p C & E \\ -L_I C & 0 \end{bmatrix}$ is Routh-Hurwitz stable, the error between actual and estimated variables will become zero as $t \to \infty$

Recall equation (8) and (10):

$$\left.\begin{aligned} \dot{\hat{z}} &= \left(A_Z - L_Z C_Z\right) z + L_Z y + B_Z u \\ y &= C_Z z \end{aligned}\right\} \qquad (13)$$

Comparing the equations above with the state-space model of proportional observer described by equation (2) and (3):

$$\left.\begin{aligned}\dot{\hat{x}} &= (A-LC)x + Ly + Bu \\ y &= Cx\end{aligned}\right\} \qquad (14)$$

It is obvious that the extended PIO model has the same formation with PO, by changing (*A, B, C, L*) into (A_Z, B_Z, C_Z, L_Z). Therefore, disturbance observer and proportional integral observer both can be regarded as a higher order proportional observer for the extended model, with $\hat{z} = \begin{bmatrix}\hat{x} \\ \hat{d}\end{bmatrix}$. Thus, design methods for proportional observers can be applied to observer gain *L* calculation for proportional integral observers with this extended observer model.

3. Illustrative Examples

3.1 State and Output Estimation with PO

A 3rd order single-input, single-output (SISO) system, the state space model is given by:

$$A = \begin{pmatrix} -4 & 2 & 2 \\ 20 & -20 & 5 \\ -16 & 8 & -8 \end{pmatrix},\ B = \begin{pmatrix} 10 \\ 2 \\ 0.1 \end{pmatrix},\ C = \begin{pmatrix} -1 & 0 & 1 \end{pmatrix},\ E = \begin{pmatrix} 1 \\ 2.828 \\ 3.464 \end{pmatrix}$$

The initial state of the system is $x_o = \begin{pmatrix} 3.142 & 2.718 & 0.618 \end{pmatrix}^T$, and a constant disturbance d = 10 is added to the plant at t = 2 sec. The PO state space representation is

$$A_{PO} = \begin{pmatrix} -51.31 & 27.15 & 50.31 \\ -136.6 & 51.3 & 144.6 \\ 48.99 & -28.49 & -80.99 \end{pmatrix},\ B_{PO} = \begin{pmatrix} -4.631 & 6 \\ -15.46 & 5 \\ 7.299 & 0.9 \end{pmatrix}$$

$$C_{PO} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ D_{PO} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The Simulink model for the Estimated and actual state variables and Estimated and actual system output using PO is shown in Figure 3 below.

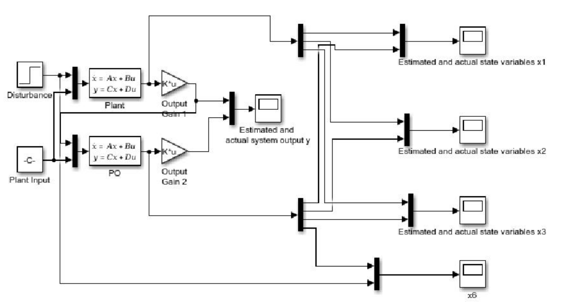

Figure 3 Simulink model for the Estimated and actual state variables and Estimated and actual system output using PO

The estimated and actual state variables and estimated and actual system output using PO simulation result is shown in Figure 4 and Figure 5 respectively.

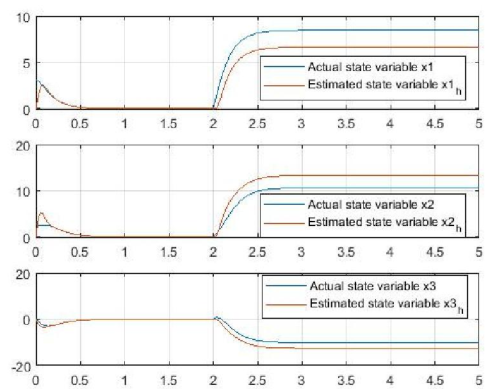

Figure 4 Estimated and actual state variables

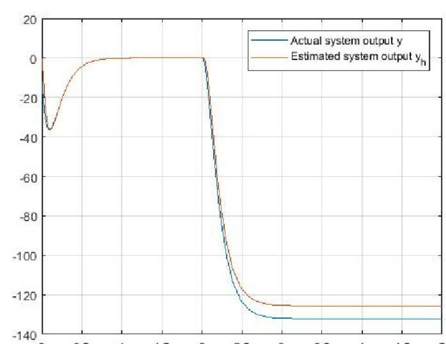

Figure 5 Estimated and actual system output

The simulation result given by Figure 4 and 5 shows that proportional observer estimates the state variables and output correctly when there is no disturbance in the plant for t < 2 sec. However, there will be a constant steady-state error in estimation after leading a constant disturbance into the plant at t = 2 sec, for both state variables and plant output.

3.2 State, Output and Disturbance Estimation with PIO

A 3rd order single-input, single-output (SISO) system, the state space model is given by:

$$A = \begin{pmatrix} -4 & 4 & 6 \\ 25 & -10 & 15 \\ -16 & 18 & -8 \end{pmatrix},\ B = \begin{pmatrix} 10 \\ 21 \\ 0.98 \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & 0 & 1 \end{pmatrix},\ E = \begin{pmatrix} 4 \\ 1.43 \\ 4.34 \end{pmatrix}$$

The initial state of the system is $x_o = \begin{pmatrix} 2.82 & 4.6 & 0.8 \end{pmatrix}^T$, and a constant disturbance d = 10 is added to the plant at t = 2 sec. The PIO state space representation is

$$A_{PIO} = \begin{pmatrix} -91.43 & 2 & 89.43 & 1 \\ -91.4 & -20 & 105.4 & 1.414 \\ -40.73 & 4 & 24.73 & 1.732 \\ -359.3 & 0 & 359.3 & 0 \end{pmatrix},\ B_{PIO} = \begin{pmatrix} -87.43 & 10 \\ -111.4 & 1 \\ -28.73 & 0.1 \\ -359.3 & 0 \end{pmatrix}$$

$$C_{PIO} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},\ D_{PIO} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The Simulink model for the estimated and actual state variables and estimated and actual system output using PIO is shown in Figure 6 below.

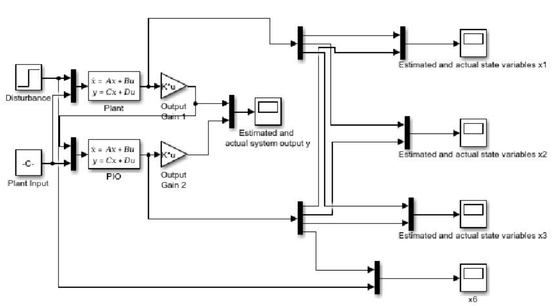

Figure 6 Simulink model for the Estimated and actual state variables and Estimated and actual system output using PIO

The estimated and actual state variables, estimated and actual system output and estimated disturbance using PIO simulation result is shown in Figure 7, Figure 8 and Figure 9 respectively.

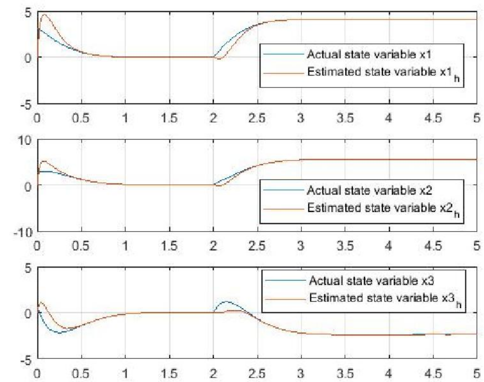

Figure 7 Estimated and actual state variables

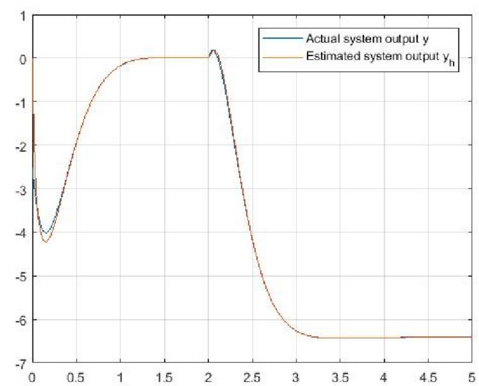

Figure 8 Estimated and actual system output

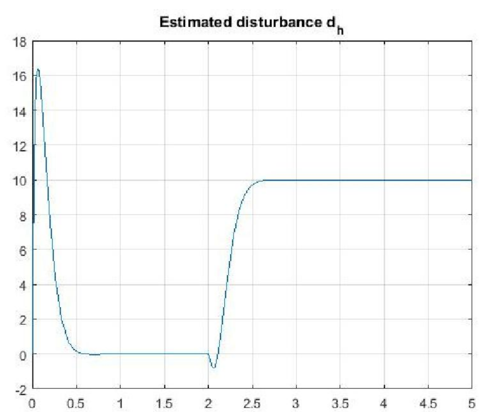

Figure 9 Estimated disturbance

As is shown in Figure 7, 8 and 9, simulation result shows that PIO has the ability to estimate state variables, disturbance and system output correctly within 0.6 sec, with or without the disturbance in plant.

4. Conclusion

Observers are systems that estimate the values of unmeasured state variables from input-output measurements. The envisioned state variables produced by means of proportional observers converge to the values of the actual ones if there are no disturbances appearing on the plant. In the presence of disturbances, a proportional integral observer is wanted to gain correct estimations. For the proportional observer, the simulation result shows that estimates the state variables and output correctly when there is no disturbance in the plant and there is a constant steady-state error in estimation after leading a constant disturbance into the plant for both state variables and plant output. For the proportional integral observer, the simulation result shows that the ability to estimate state variables, disturbance and system output correctly with or without the disturbance in plant.

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