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Introductory Study On Various Statistical Convergence

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Abstract: The investigation of extreme events is extremely relevant for a range of disciplines in mathematical, natural, and social sciences and engineering. Understanding the large fluctuations of the system of interest is of great importance from a theoretical point of view, but also when it comes to assessing the risk associated with low probability and high impact events. In many cases, in order to gauge preparedness and resilience properly, one would like to be able to quantify the return times for events of different intensity and take suitable measures for preventing the expected impacts. Prominent examples are weather and climate extremes, which can have a huge impact on human society and natural ecosystems. The present uncertainty in the future projections of extremes makes their study even more urgent and crucial.
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Introduction

Statistics is concerned with the collection and analysis of data and with making estimations and predictions from the data. Typically two branches of statistics are discerned: descriptive and inferential. Inferential statistics is usually used for two tasks: to estimate properties of a population given sample characteristics and to predict properties of a system given its past and current properties. To do this, specific statistical constructions were invented. The most popular and useful of them are the average or mean (or more exactly, arithmetic mean) m and standard deviation s (variance s 2). To make predictions for future, statistics accumulates data for some period of time. To know about the whole population, samples are used. Normally such inferences (for future or for population) are based on some assumptions on limit processes and their convergence. Iterative processes are used widely in statistics. For instance the empirical approach to probability is based on the law (or better to say, conjecture) of big numbers, states that a procedure repeated again and again, the relative frequency probability tends to approach the actual probability. The foundation for estimating population parameters and hypothesis testing is formed by the central limit theorem, which tells us how sample means change when the sample size grows. In experiments, scientists measure how statistical characteristics (e.g., means or standard deviations) converge (cf., for example, [23, 31]). Convergence of means/averages and standard deviations have been studied by many authors and applied to different problems (cf. [1-4, 17, 19, 20, 24-28, 35]). Convergence of statistical characteristics such as the average/mean and standard deviation are related to statistical convergence as we show in this section.

Let m and c be the spaces of all bounded and convergent real sequences x = (xk) normed by x = supn |xn|, respectively. Let B be the class of (necessarily continuous) linear functional β on m which are nonnegative and regular, that is, if x ≥ 0, (i.e., xk ≥ 0 for all k ∈ N:= {1, 2,...}) then β(x) ≥ 0, and β(x) = limk xk, for each x ∈ c. If β has the additional property that β(σ(x)) = β(x) for all x ∈ m, where σ is the left shift operator, defined by σ(x1, x2,...)=(x2, x3,...) then β is called a Banach limit. The existence of Banach limits has been shown by Banach [2,17,19], and another proof may be found in [3]. It is well known [21] that the space of all almost convergent sequences can be represented as the set of all x ∈ m which have the same value under any Banach limit. In the research, we study some generalized limits so that the space of all bounded statistically convergent sequences can be represented as the set of all bounded sequences which have the same value under any such limit. It is proved that the set of such limits and the set of Banach limits are distinct but their intersection is not empty.

Review of Literature:

The Fibonacci sequence was firstly used in the theory of sequence spaces by Kara and Başarır [5]. Afterward, Kara [6] defined the Fibonacci difference matrix $\hat{F}$ by using the Fibonacci sequence (f_n) for $n \in \{0, 1, \ldots\}$ and introduced the new sequence spaces related to the matrix domain of $\hat{F}$.

Following [7] and [8], high quality papers have been produced on the Fibonacci matrix by many mathematicians [9].

In this paper, by combining the definitions of Fibonacci sequence and statistical convergence, we obtain a new concept of statistical convergence, which will be called Fibonacci type statistical convergence. We examine some basic properties of new statistical convergence defined by Fibonacci sequences. Henceforth, we get an analogue of the classical Korovkin theorem by using the concept of Fibonacci type statistical convergence.

Estimation frequently requires iterative procedures: the more iterations, the more accurate estimates. But when are estimates accurate enough? When can iteration cease? My the rule has become "Convergence is reached when more iterations do not change my interpretation of the estimates".

There is a trade-off between accuracy and speed. Greater accuracy requires more iterations - more time and computer resources. The specification of estimation accuracy is a compromise. Frequently, squeezing that last bit of inaccuracy out of estimates only affects the least significant digits of printed output, has no noticeable effect on model-data fit, and does not alter interpretation. Three numerical convergence rules are often employed:

1.1. Convergence of Random Variables

Convergence of random variables (sometimes called stochastic convergence) is where a set of numbers settle on a particular number. It works the same way as convergence anywhere else; For example, cars on a 5-line highway might converge to one specific lane if there's an accident closing down four of the other lanes. In the same way, a sequence of numbers (which could represent cars or anything else) can converge (mathematically, this time) on a single, specific number. Certain processes, distributions and events can result in convergence— which basically mean the values will get closer and closer together.

When Random variables converge on a single number, they may not settle exactly that number, but they come very, very close. In notation, x (xn → x) tells us that a sequence of random variables (xn) converges to the value x. This is only true if the absolute value of the differences approaches zero as n becomes infinitely larger. In notation, that's:

$|xn - x| \to 0$ as $n \to \infty$.

What happens to these variables as they converge can't be crunched into a single definition. Instead, several different ways of describing the behavior are used.

1.2. Types of Convergence of Random Variables

Convergence of Random Variables can be broken down into many types. The ones you'll most often come across:

Convergence in probability,
Convergence in distribution,
Almost sure convergence,
Convergence in mean.

Each of these definitions is quite different from the others. However, for an infinite series of independent random variables: convergence in probability, convergence in distribution, and almost sure convergence are equivalent (Fristedt & Gray, 2013, p.272).

1.3.1. Convergence in probability

If you toss a coin n times, you would expect heads around 50% of the time. However, let's say you toss the coin 10 times. You might get 7 tails and 3 heads (70%), 2 tails and 8 heads (20%), or a wide variety of other possible combinations. Eventually though, if you toss the coin enough times (say, 1,000), you'll probably end up with about 50% tails. In other words, the percentage of heads will converge to the expected probability.

More formally, convergence in probability can be stated as the following formula:

$$\lim_{n \to \infty} P(|X_n - c| < \varepsilon) = 1.$$

Where:
P = probability,
Xn = number of observed successes (e.g. tails) in n trials (e.g. tosses of the coin),
Lim (n→∞) = the limit at infinity — a number where the distribution converges to after an infinite number of trials (e.g. tosses of the coin),
c = a constant where the sequence of random variables converge in probability to,
ε = a positive number representing the distance between the expected value and the observed value.

The concept of a limit is important here; in the limiting process, elements of a sequence become closer to each other as n increases. In simple terms, you can say that they converge to a single number.

1.3.2. Convergence in distribution

Convergence in distribution (sometimes called convergence in law) is based on the distribution of random variables, rather than the individual variables themselves. It is the convergence of a sequence of cumulative distribution functions (CDF). As it's the CDFs, and not the individual variables that converge, the variables can have different probability spaces.

In more formal terms, a sequence of random variables converges in distribution if the CDFs for that sequence converge into a single CDF. Let's say you had a series of random variables, Xn. Each of these variables X1, X2,…Xn has a CDF FXn (x), which gives us a series of CDFs {FXn (x)}. Convergence in distribution implies that the CDFs converge to a single CDF, Fx (x) (Kapadia et. al, 2017).

Several methods are available for proving convergence in distribution. For example, Slutsky's Theorem and the Delta Method can both help to establish convergence. Convergence of moment generating functions can prove convergence in distribution, but the converse isn't true: lack of converging MGFs does not indicate lack of convergence in distribution. Scheffe's Theorem is another alternative, which is stated as follows (Knight, 1999, p.126).

In undergraduate courses we often teach the following version of the central limit theorem: if X1,..., Xn are an iid sample from a population with mean µ and standard deviation σ then n 1/2 (X¯ − µ)/σ has approximately a standard normal distribution. Also we say that a Binomial (n, p) random variable has approximately a N (np, np (1 − p)) distribution. What is the precise meaning of statements like "X and Y have approximately the same distribution"? The desired meaning is that X and Y have nearly the same cdf. But care is needed. Here are some questions designed to try to highlight why care is needed.

Q1) If n is a large number is the N (0, 1/n) distribution close to the distribution of X ≡ 0?

Q2) Is N (0, 1/n) close to the N (1/n, 1/n) distribution?

Q3) Is N (0, 1/n) close to N (1/ √ n, 1/n) distribution?

Q4) If Xn ≡ 2 −n is the distribution of Xn close to that of X ≡ 0?

Answers depend on how close close needs to be so it's a matter of definition. In practice the usual sort of approximation we want to make is to say that some random variable X, say, has nearly some continuous distribution, like N (0, 1). So: we want to know probabilities like P (X > x) are nearly P (N (0, 1) > x). The real difficulties arise in the case of discrete random variables or in infinite dimensions: the latter is not done in this course. For discrete variables the following discussion highlights some of the problems. See the homework for an example of the so-called local central limit theorem. Mathematicians mean one of two things by "close": Either they can provide an upper bound on the distance between the two things or they are talking about taking a limit. In this course we take limits.

Statistical Convergence, was published almost fifty years ago, has flatter the domain of recent research. Unlike mathematicians studied characteristics of statistical convergence and applied this notion in numerous extent such as measure theory, trigonometric series, approximation theory, locally compact spaces, and Banach spaces, etc. The present thesis emphasis on certain results studied by Ferenc Mo´ricz in his two research researches i.e., "Statistical Convergence of Sequences and Series of Complex Numbers with applications in Fourier Analysis and summability " and in "Statistical Limit of Lebesgue Measurable functions with ∞ with applications in Fourier Analysis and summability". The perception of conjunction has been generalized in various ways through different methods such as summability and also a method in which one moves from a sequence to functions. In 1932 earlier, Banach coined the first generalization of it and named as "almost convergence". Later it was studied by Lorentz in 1948 [1].

The most recent generalization of the classical convergence i.e., a new type of conjunction named as Statistical Convergence had been originated first via Henry Fast [3] in 1951. He characterizes this hypothesis to Hugo Steinhaus [19]. Actually, it was Antoni Zygmund [20] who evince the results, prepositions and assertion on Statistical Convergence in a Monograph in 1935. Antoni Zygmund in 1935 demonstrated in his book "Trigonometric Series" where instead of Statistical convergence he proposes the term "almost convergence" which was later proved by Steinhaus and Fast ([19] and [3]).

Then, Henry Fast [3] in 1951 developed the notion analogous to Statistical Convergence, Lacunary Statistical Convergence and λ Statistical Convergence and it was reintroduced by Schoenberg [18] in 1959. Since then the several research research related to the concept have been published explaining the notion of convergence and is applications. The objective of the study is to discuss the fundamentals and results along with various extensions which have been subsequently formulated [2].

A sequence (xn) in a Banach space X is said to be statistically convergent to a vector L if for any ε > 0 the subset {n: kxn − Lk > ε} has density 0. Statistical convergence is a summability method introduced by Zygmund [1] in the context of Fourier series convergence. Since then, a theory has been developed with deep and beautiful results [2] by different authors, and moreover at the present time this theory does not present any symptoms of abatement. The theory has important applications in several branches of Applied Mathematics (see the recent monograph by Mursaleen [3]). It is well known that there are results that characterize properties of Banach spaces through convergence types. For instance, Kolk [4] was one of the pioneering contributors. Connor, Ganichev and Kadets [5] obtained important results that relate the statistical convergence to classical properties of Banach spaces.

In this research we aim to unify some known results. In the process we pull together much of what is known about this topic and we will simplify some of their existing proofs. As a consequence we provide an unified point of view which allows us to solve several unsolved questions. In fact, we will obtain results in the context of ideal convergence. We will show that under reasonable conditions on a given non-trivial ideal, the studied properties do not depend on the ideal that we use to define the convergence spaces associated to the wuc series. This allows us to extend our results for an arbitrary summability method that shares some kind of ideal-convergence on the realm of all bounded sequences. This will allow us to unify the known results and obtain answers to some unresolved questions. The research is organized as follows. In Section 2, we will study the convergence induced by an ideal I ⊂ P (N), (that is, the I-convergence), which will provide the general framework of our results in Section 3. Next we will review some basic properties and some preliminary results about I-convergence that we will use later. Section 3 deals with the space of I-summability (which we will denote by SI (∑i xi)) associated to a weakly unconditionally Cauchy series ∑i xi. It is shown that for any non-trivial regular ideal I, a series ∑i xi is weakly unconditionally Cauchy if and only if SI (∑i xi) is complete. Moreover, if this equivalence is true for each series in a normed space X, then the space X must be complete. There is a counterpart of the above results for the weak topology, and moreover, we were able to extend these results for certain general summability methods. Finally, for the w -topology of X we will characterize when a series ∑i fi in the dual space X is wuc, and this characterization incorporates general summability methods. Moreover, this result is sharpened when the space X is barrelled. The research concludes with a brief section on applications [31].

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