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Analytical Interpretation Of Inverse Laplace Transformation

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Abstract: The Laplace transformation is a mathematical tool which is used in the solving of differential equations by converting it from one form into another form. The Laplace Transform has also its inverse. During the process of solving physical problems it is necessary to epiclesis the inverse transform of the Laplace transform. The Laplace transformation is used in solving the time domain function by converting it into frequency domain function. In this paper we will discuss the Analytically Interpretation of inverse Laplace Transformation.
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Sub area: Laplace transformation
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Introduction:

The Laplace transformation is applied in different areas of science, engineering and technology. The Laplace transformation is applicable in so many fields [1, 2, 3, 4, 5, 6]. The Laplace Transform has also its inverse. During the process of solving physical problems it is necessary to epiclesis the inverse transform of the Laplace transform [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]**.** The Laplace Transform was primary used and named after by Pierre Simon Laplace Pierre Simon Laplace was a French Mathematician an Astronomer, who had a lot of control in the growth of several theories in mathematics, statistics, physics, and astronomy [13, 14, 15, 16, 17]. He contributed seriously to physical mechanics, by converting the previous geometrical analysis to one based on calculus, which opened up application of his formulas to a wider range of problems. It is effective in solving linear differential equation either ordinary or partial [18, 19, 20, 21, 22].

Basic Definition:

Let F (t) is a well defined function of t for all t ≥ 0. Then the Inverse Laplace transformation of $f(p)$, denoted by F (t) or $L^{-1}\{f(p)\}$, is defined as

$$L^{-1}\{f(p)\} = \int_0^\infty e^{-pt} F(t)dt = F(t)$$

Provided that the integral exists, i.e. convergent. Where p the parameter which may be real or complex number and L^{-1} is the Inverse Laplace transformation operator.

Methodology:

Some Properties of inverse Laplace Transform

(1) *Linearty Property*:

If c_1 and c_2 are constants and if

$L^{-1}\{f(p)\} = F(t)$ and $L\{G(t)\} = g(p)$

then,

$$L^{-1}[c_1 f(p) + c_2 g(p)] = c_1 L^{-1}[f(p)] + c_2 L^{-1}[g(p)]$$

(2) ***First Shifting Property***:

if $L^{-1}\{f(p)\} = F(t)$, then

$$L^{-1}\{f(p-a)\} = e^{at}F(t),$$

(3) ***Change of Scale Property***:

if $L^{-1}\{f(p)\} = F(t)$, then

$$L^{-1}\{f(ap)\} = \frac{1}{a}F\left(\frac{t}{a}\right),$$

(4) ***Inverse Laplace Transformation of derivative***:

if $L^{-1}\{f(p)\} = F(t)$, then

$$L^{-1}\left[-\frac{d\{f(p)\}}{dp}\right] = tF(t)$$

$$L^{-1}\left[-\frac{d^n\{f(p)\}}{dp^n}\right] = (-1)^n t^n F(t),$$

$whrer\ n = 123 \ldots\ldots\ldots$

(A) Inverse Laplace Transform by using Convolution theorem:

If $H_1(t)$ and $H_2(t)$ be two functions of class A and if

$$L^{-1}\{h_1(p)\} = H_1(t), L^{-1}\{h_2(p)\} = H_2(t).$$

Then the convolution of these two

$$L^{-1}\{h_1(p).h_2(p)\} = \{H_1 * H_2\}(t) = \int_0^t H_1(t)H_2(t-y)dy$$

Which of course exists if $H_1(t)$ and $H_2(t)$ are piecewise continuous. the relation is called the convolution or falting of $H_1(t)$ and $H_2(t)$.

Using Convolution Theorem, Find the Inverse Laplace Transform of the Function

$$L^{-1}\left\{\frac{1}{p(p^2+\beta^2)}\right\}$$

***Solution*:**

$$let,\ h_1(p) = \frac{1}{p}\ and\ \ h_2(p) = \frac{1}{p(p^2+\beta^2)}$$

then,

$$H_1(t) = L^{-1}\{h_1(p)\} = L^{-1}\left\{\frac{1}{p}\right\} = 1$$

and

$$H_2(t) = L^{-1}\{h_2(p)\} = L^{-1}\left\{\frac{1}{(p^2+\beta^2)}\right\} = \frac{1}{\beta} sin\beta t$$

By, convolution theorem,

$$L^{-1}\{h_1(p).h_2(p)\} = \{H_1 * H_2\}(t) = \int_0^t H_1(t)H_2(t-y)dy$$

$$= \frac{1}{\beta} sin\beta t * 1$$

$$= \frac{1}{\beta}\int_0^t sin\beta u\ du$$

$$= \frac{1}{\beta^2}\{1 - cos\beta t\}$$

(B) Inverse Laplace Transform by using Heaviside's Expansion Theorem:

$if\ \ \bar{f}(p) = \frac{R(p)}{S(p)}$, Where $R(p)$ & $S(p)$ do not keep common factor and the degree of Numerator is less than the degree of denominator. If $S(p)$ has different roots, than we find by this theorem, inverse Laplace Transformations.

Heaviside's Expansion Formula:

If $R(p)$ & $S(p)$ be polynomials in p where the degree of Numerator is less than the degree of denominator. If $S(p)$ has n different roots $c_1, c_2, c_3, \ldots\ldots\ldots c_n$, *Than,*

$$L^{-1}\left\{\frac{R(p)}{S(p)}\right\} = \sum_{i=1}^{n}\frac{R(c_i)}{S'(c_i)}e^{tc_i}$$

Using Heaviside's Expansion Formula, Find the Inverse Laplace Transform of the Function

$$L^{-1}\left\{\frac{1}{p(p^2+\beta^2)}\right\}$$

***Solution*:**

Let,

$$R(p) = 1$$

$$S(p) = p(p^2+\beta^2)$$

$$S'(p) = 3p^2+\beta^2$$

To find roots of

$$S(p) = p(p^2+\beta^2) = 0$$

Then roots are,

$$c_1 = 0, c_2 = i\beta, c_3 = -i\beta$$

Now,

$$R(p) = 1$$

Then,

$$R(0) = 213, R(i\beta) = 1, R(-i\beta) = 1$$

And,

Since, $S'(p) = 3p^2+\beta^2$

$$S'(0) = \beta^2, S'(i\beta) = -2\beta^2, S'(-i\beta) = -2\beta^2$$

Now, By Heaviside's Expansion formula.

$$L^{-1}\left\{\frac{R(p)}{S(p)}\right\} = \sum_{i=1}^{n}\frac{R(c_i)}{S'(c_i)}e^{tc_i}$$

Or,

$$L^{-1}\left\{\frac{R(p)}{S(p)}\right\} = \frac{R(c_1)}{S'(c_1)}e^{tc_1} + \frac{R(c_2)}{S'(c_2)}e^{tc_2} + \frac{R(c_3)}{S'(c_3)}e^{tc_3}$$

$$L^{-1}\left\{\frac{R(p)}{S(p)}\right\} = \frac{R(0)}{S'(0)}e^{0t} + \frac{R(i\beta)}{S'(i\beta)}e^{i\beta t} + \frac{R(-i\beta)}{S'(-i\beta)}e^{-i\beta t}$$

$$L^{-1}\left\{\frac{R(p)}{S(p)}\right\} = \frac{1}{\beta^2} - \frac{1}{2\beta^2}e^{i\beta t} - \frac{1}{2\beta^2}e^{-i\beta t}$$

$$L^{-1}\left\{\frac{R(p)}{S(p)}\right\} = \frac{1}{\beta^2}[1 - \frac{e^{i\beta t}+e^{-i\beta t}}{2}]$$

Hence,

$$L^{-1}\left\{\frac{R(p)}{S(p)}\right\} = \frac{1}{\beta^2}[1 - cos\beta t]$$

(C) Inverse Laplace Transform by using Inversion Formula:

Complex Inversion Formula

If $F(t)$ is continuous function and is of exponential order and if the Laplace transform of $F(t)\ i.e.\ L\{F(t)\}\ is\ f(s)$, then the inverse Laplace transform of $f(s)$ i.e. $L^{-1}\{f(s)\}$ is given by

$$F(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{pt} f(p)dp\ \ for\ t > 0$$

This result is known as complex inversion formula.

Using Complex Inversion Formula, Find the Inverse Laplace Transform of the Function

$$L^{-1}\left\{\frac{1}{p(p^2+\beta^2)}\right\}$$

Solution:

We have,

$$f(p) = \frac{1}{p(p^2+\beta^2)}$$

$$or,$$

$$f(p) = \frac{1}{p(p+\beta i)(p-\beta i)}$$

$Thus, f(p)\ has\ three\ simple\ pole\ at\ p = 0\ and\ p = \pm i\beta$

Now,

$$Res\ at\ p = 0,$$

$$\lim_{p\to 0} pe^{pt}f(p) = \lim_{p\to 0}\frac{e^{pt}}{(p^2+\beta^2)} = \frac{1}{\beta^2}$$

$$Res\ at\ p = i\beta,$$

$$\lim_{p\to i\beta}(p-i\beta)e^{pt}f(p) = \lim_{p\to 0}\frac{e^{pt}}{p(p+\beta i)} = -\frac{e^{i\beta t}}{2\beta^2}$$

$$Res\ at\ p = -i\beta,$$

$$\lim_{p\to -i\beta}(p+i\beta)e^{pt}f(p) = \lim_{p\to 0}\frac{e^{pt}}{p(p-\beta i)} = -\frac{-e^{i\beta t}}{2\beta^2}$$

Hence,

$$F(t) = \frac{1}{\beta^2} - \frac{e^{i\beta t}}{2\beta^2} - \frac{-e^{-i\beta t}}{2\beta^2}$$

$$F(t) = \frac{1}{\beta^2}[1 - \frac{e^{i\beta t} + -e^{-i\beta t}}{2}]$$

Hence,

$$F(t) = \frac{1}{\beta^2}[1 - cos\beta t]$$

(D) Inverse Laplace Transform by using Partial Fraction:

Using Partial Fraction Method, Find the Inverse Laplace Transform of the Function

$$L^{-1}\left\{\frac{1}{p(p^2+\beta^2)}\right\}$$

Solution:

$$Let,$$

$$f(p) = \frac{1}{p(p+\beta i)(p-\beta i)} \ldots.(1)$$

$$\frac{1}{p(p+\beta i)(p-\beta i)} = \frac{A}{p} + \frac{B}{(p+\beta i)} + \frac{C}{(p-\beta i)}$$

$$Or,$$

$$1 = A(p^2+\beta^2) + Bp(p-\beta i) + Cp(p+\beta i)$$

$$Or,$$

By solving this,

$$A = \frac{1}{\beta^2}, B = -\frac{1}{2\beta^2}\ and\ C = -\frac{1}{2\beta^2}$$

$$Now, From\ (1),$$

$$f(p) = \frac{1}{p(p+\beta i)(p-\beta i)}$$

Or,

$$f(p) = \frac{A}{p} + \frac{B}{(p+\beta i)} + \frac{C}{(p-\beta i)}$$

Or,

$$f(p) = \frac{1}{\beta^2 p} - \frac{1}{2\beta^2(p+\beta i)} - \frac{1}{2\beta^2(p-\beta i)}$$

Taking inverse Laplace Transform on both sides,

$$F(t) = \frac{1}{\beta^2}L^{-1}\left\{\frac{1}{p}\right\} - \frac{1}{2\beta^2}L^{-1}\left\{\frac{1}{(p+\beta i)}\right\} - \frac{1}{2\beta^2}L^{-1}\left\{\frac{1}{(p-\beta i)}\right\}$$

$$F(t) = \frac{1}{\beta^2} - \frac{1}{2\beta^2}e^{-i\beta t} - \frac{1}{2\beta^2}e^{i\beta t}$$

$$F(t) = \frac{1}{\beta^2}[1 - \frac{e^{i\beta t} + e^{-i\beta t}}{2}]$$

Hence,

$$F(t) = \frac{1}{\beta^2}[1 - cos\beta t]$$

(E) Inverse Laplace Transform by using Multiplication by t-Property:

Multiplication by t − Property:

$$if L^{-1}\{f(p)\} = F(t), then$$

$$L^{-1}\left[\frac{d\{f(p)\}}{dp}\right] = -tF(t)$$

$$L^{-1}\left[-\frac{d^n\{f(p)\}}{dp^n}\right] = (-1)^n t^n F(t),$$

$$whrer\ n = 123 \ldots\ldots\ldots$$

Using Partial Fraction Method, Find the Inverse Laplace Transform of the Function

$$L^{-1}\left\{\frac{1}{p(p^2+\beta^2)}\right\}$$

Solution:

$$Let,$$

$$f(p) = \frac{1}{p(p^2+\beta^2)}$$

By Multiplication by t − Property,

$$L^{-1}\left[\frac{d\{f(p)\}}{dp}\right] = -tF(t)$$

Or,

$$L^{-1}\left[\frac{d}{dp}\left\{\frac{1}{p(p^2+\beta^2)}\right\}\right] = -tF(t)$$

Or,

$$L^{-1}\left[\frac{d}{dp}\left\{\frac{1}{\beta^2}\left\{\frac{1}{p}\right\} - \frac{1}{2\beta^2}\left\{\frac{1}{(p+\beta i)}\right\} - \frac{1}{2\beta^2}\left\{\frac{1}{(p-\beta i)}\right\}\right\}\right] = -tF(t)$$

Or,

$$L^{-1}\left[\left\{\frac{1}{\beta^2}\left\{-\frac{1}{p^2}\right\} + \frac{1}{2\beta^2}\left\{\frac{1}{(p+\beta i)^2}\right\} + \frac{1}{2\beta^2}\left\{\frac{1}{(p-\beta i)^2}\right\}\right\}\right] = -tF(t)$$

Or,

$$\left[\left\{-\frac{1}{\beta^2}\{t\} + \frac{1}{2\beta^2}\{te^{-i\beta t}\} + \frac{1}{2\beta^2}\{te^{i\beta t}\}\right\}\right] = -tF(t)$$

Or,

$$\left[\left\{-\frac{1}{\beta^2}\{t\} + \frac{t}{2\beta^2}\frac{e^{i\beta t}+e^{-i\beta t}}{2}\right\}\right] = -tF(t)$$

Or,

$$\left[-\frac{t}{\beta^2}\left\{1 - \frac{e^{i\beta t}+e^{-i\beta t}}{2}\right\}\right] = -tF(t)$$

Or,

$$\left[-\frac{t}{\beta^2}\{1 - cos\beta t\}\right] = -tF(t)$$

Hence,

$$F(t) = \left[\frac{1}{\beta^2}\{1 - cos\beta t\}\right]$$

(f) Inverse Laplace Transform by using Change of scale property:

Change of Scale Property:

$$if L^{-1}\{f(p)\} = F(t), then$$

$$L^{-1}\{f(ap)\} = \frac{1}{a}F\left(\frac{t}{a}\right)$$

Using Change of scale property,

If

$$L^{-1}\left\{\frac{p}{(p^2+1)^2}\right\} = tsint$$

$$Find$$

$$L^{-1}\left\{\frac{50p}{(25p^2+1)^2}\right\}$$

Solution:

Given,

$$L^{-1}\left\{\frac{p}{(p^2+1)^2}\right\} = tsint = f(t)$$

Written as for p

$$L^{-1}\left\{\frac{bp}{\{(bp)^2+1)\}^2}\right\} = \frac{1}{2}.\frac{1}{b}f(\frac{t}{b})$$

Or,

$$L^{-1}\left\{\frac{bp}{\{(bp)^2+1)\}^2}\right\} = \frac{1}{2}.\frac{1}{b}\frac{t}{b}\,sin\,\frac{t}{b}$$

Or,

$$L^{-1}\left\{\frac{bp}{\{(bp)^2+1)\}^2}\right\} = \frac{t}{2b^2}\,sin\,\frac{t}{b}$$

putting as $b = 5$

$$L^{-1}\left\{\frac{5p}{\{(5p)^2+1)\}^2}\right\} = \frac{t}{50}sin\,\frac{t}{5}$$

Hence,

$$L^{-1}\left\{\frac{50p}{\{(5p)^2+1)\}^2}\right\} = \frac{t}{5}sin\,\frac{t}{5}$$

(g) Inverse Laplace Transform by using Second shifting property:

Second shifting property:

$$If\ L^{-1}\{f(p)\} = f(t)then$$

$$L^{-1}\{e^{-bp}f(p)\} = u(t-b)f(t-b)$$

Or,

$$L^{-1}\{e^{-bp}f(p)\} = \begin{cases} f(t-b), & t > b \\ 0, & 0 < t < b \end{cases}$$

Using Second shifting property, Find the Inverse Laplace Transform of the Function

$$L^{-1}\left\{\frac{pe^{-5p\pi/6}}{(p^2+81)}\right\}$$

Solution:

$$f(p) = \frac{p}{(p^2+81)}$$

Then,

$$f(t) = cos9t$$

By Second shifting property

$$L^{-1}\left\{e^{-5p\pi/6}.\frac{p}{(p^2+81)}\right\}$$

$$= \begin{matrix} f(t-5\pi/6), & t > 5\pi/6 \\ 0, & 0 < t < 5\pi/6 \end{matrix}$$

Or,

$$= \begin{matrix} cos9(t-5\pi/6), & t > 5\pi/6 \\ 0, & 0 < t < 5\pi/6 \end{matrix}$$

Or,

$$= \begin{matrix} cos(9t-15\pi/2), & t > 5\pi/6 \\ 0, & 0 < t < 5\pi/6 \end{matrix}$$

Hence,

$$cos(3t-15\pi/2)\,u(t-5\pi/6)$$

(h) Inverse Laplace Transform by using First shifting property:

***First Shifting Property*:**

$$if L^{-1}\{f(p)\} = F(t), then$$

$$L^{-1}\{f(p-a)\} = e^{at}F(t),$$

Using First shifting property, Find the Inverse Laplace Transform of the Function

$$L^{-1}\left\{\frac{3p+7}{(p^2-2p-3)}\right\}$$

Solution:

$$We\ have\ , L^{-1}\left\{\frac{3p+7}{(p^2-2p-3)}\right\}$$

$$Numerator = A\frac{d}{dp}\{Denominator\} + B$$

$$3p+7 = A\frac{d}{dp}\{p^2-2p-3\} + B$$

$$3p+7 = A\frac{d}{dp}\{2p-2\} + B$$

Comparing and solving,

$$A = \frac{3}{2}\quad and \quad B = 10$$

Therefore,

$$L^{-1}\left\{\frac{3p+7}{(p^2-2p-3)}\right\}$$

Or,

$$L^{-1}\left\{\frac{\frac{3}{2}\{2p-2\}+10}{(p-1)^2-4}\right\}$$

Or,

$$L^{-1}\left\{\frac{3\{p-1\}+10}{(p-1)^2-4}\right\}$$

Or,

$$3L^{-1}\left\{\frac{\{p-1\}}{(p-1)^2-4}\right\} + 10L^{-1}\left\{\frac{1}{(p-1)^2-4}\right\}$$

Or,

$$3e^{t}L^{-1}\left\{\frac{p}{p^2-4}\right\}+5e^{t}L^{-1}\left\{\frac{2}{p^2-4}\right\}$$

Hence,

$$e^{t}\{3cosh2t+5sinh2t\}$$

Conclusion:

The main purpose of this paper is the Analytically Interpretation of inverse Laplace Transform. The primary use of Laplace transformation is converting a time domain functions into frequency domain function. Laplace transformation is a very useful mathematical tool to make simpler complex problems in the area of stability and control.

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