Analysis of the Hydrodynamic Forces in the Abrupt Roughened Bed of the Stilling Basins

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Abstract: In many hydraulic structures it is required to dissipate flow energy to prevent erosion or degradation of the structure. In practice, there are several methods for this, including hydraulic jumps. Hydraulic jump can occur in different modes of channel geometry and conditions. Due to the sudden fall of the channel bed which is also referred to as the abrupt drop, the length of the hydraulic jump, conjugate depths of jump, hydrodynamic pressures on the bed and other characteristics are changed. Also, the use of roughness in the bed of the stilling basins of the hydraulic jump type can reduce the jump length. In this research, the abrupt drop conditions with a rough bed are jointly applied in hydraulic jumps and changes in hydrodynamic pressures at the bed of the stilling basin is studied. The experimental results can be useful in designing an abrupt roughened bed of a stilling basin based on the thickness of concrete slabs needed to ensure the stability of the bed covers.

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Keywords: stilling basin, abrupt drop, rough bed

1. Introduction:

Abrupt drop stilling basin is one of the energy dissipater structures. The purpose of dropping the floor in a hydraulic jump is to improve the jump conditions and to ensure that hydraulic jumps are considered locally and will be controlled. The first research on these types of basins was carried out by Moore and Morgan (1958). They concluded there are four different modes of jump that can be seen in Fig.1.

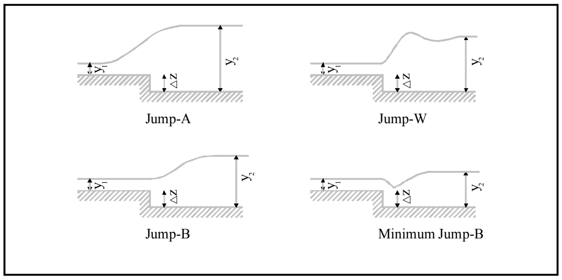


Figure 1. Four types of jumps in an abrupt drop stilling basin

Moore and Morgan (1958) investigated the conjugate depths and velocity near the bed during a jump in an abrupt drop and showed that during jump the bed velocity to downstream velocity (vb /v2) ratio will show variations in terms of Froude numbers and jump types. This in fact indicated that the hydrodynamic pressures on the flume bed and along it would show a different behavior of hydrostatic pressures (water surface profiles). JJ Sharp (1974) studied the hydraulic conditions on a round-edged abrupt drop and compared it with sharp-edged abrupt drop. He showed that the amount of downstream depth is related to the amount of abrupt drop's roundness and this roundness decreases the value of y2, but at high values of Froude number, this roundness increases the depth of the water at downstream and consequently reduces the jump control. Hager and Kawagoshi1990) continued research on a hydraulic jump in an abrupt drop. They defined the ΔY parameter (Depth difference between abrupt drop jump and classic jump) as $Y=Y^{*}+\Delta Y$. Where, Y and Y * are the secondary depth of the hydraulic jump in abrupt drop and classic jumps. The also provided a number of relationships for it. Regarding the effect of roughness on the characteristics of jump 17, Rajaratnam (1968) was one of the first ones to carry out extensive studies. He introduced the parameter $k = k_e/y_1$ as a roughness parameter in which k_e is the equivalent roughness and y1 is the depth of the input current at the top of the roughness and showed that the roller length (L_r) and jump length (L_J) on the rough beds are greatly reduced compared to the classic jumps. In his research, he showed that for a roughness parameter of about 0.4, the value of y_2 could be reduced to $0.8y_2$. Subsequent studies such as Gill (1980), Hughes and Flack (1984), and Rajaratnam (2002), 6. Carollo and Ferro (2004), Izadjoo and Shafai Bejestan (2007), Carollo and Ferro (2007), et al. (2008) and Nissi and Shafai Bejestan (2009) showed that roughness has a significant effect on the reduction of jump length. Research on hydrodynamic pressures on hydraulic jumps also expanded with Rajaratnam (1968) studies. In the research on hydrodynamic forces in a classic jump, he showed that the range of 30 percent rollover length is the area in which the maximum turbulent flow and maximum air intake occur. Therefore, maximum hydrodynamic pressure of the bed is produced in this area. Several authors have analyzed the statistical properties of turbulent pressures under a classical hydraulic jump experimentally, including Vasiliev and Bukreyev (1967), Abdul Kader and Elango (1974), Lopardo and Henning (1985), Toso and Bowers (1988) and Fiorotto and Rinaldo (1992). Toso and Bowers (1988) in their research formed a hydraulic jump in a flat bed and showed that the positive and negative coefficients of pressure (Cp$^+$ and Cp$^-$) along the jump length and for a given Froude number at the upstream jump increased first and then decreased. They also examined the conditions in the USBR type 2 and 3 stilling basin and plotted the Cp$^+$ and Cp$^-$ values for them. They showed that these coefficients are influenced by the presence of the blocks. Fiorotto and Rinaldo (1992) in their research to determine the maximum thickness of the hydraulic jump in the bed of stilling basins provided the following relation.

$$s=\Omega\left(\frac{L_x}{y_1},\frac{L_x}{I_x},\frac{L_y}{I_y}\right)\left(Cp^{+}+Cp^{-}\right)\frac{v_1^2}{2g}\frac{\gamma}{\gamma_c-\gamma}$$
1.

Where, s is the thickness of the bed slab, Ω is equal to the dimensionless coefficient of the force, Cp$^+$ and Cp$^-$ are the positive and negative coefficients of pressure, $v^2/2g$ is the velocity energy head and γ and cγ are the specific gravity of the water and concrete. L_x and L_y are the horizontal and vertical lengths in the bed sheet, and I_x and I_y are the pressure fluctuations in the horizontal and vertical directions. C_p values are calculated from the following relationships:

2.
$$\frac{\Delta p^{-}_{\max}}{\gamma}=C_p^{-}\frac{v_1^2}{2g}$$

3.
$$\frac{\Delta p^{+}_{\max}}{\gamma}=C_p^{+}\frac{v_1^2}{2g}$$

Where, $\Delta p^{-}_{\max}$ and $\Delta p^{+}_{\max}$ are the maximum values of the positive and negative pressures relative to the average pressure.

Aremenio et al. (2000) also carried out research on pressure fluctuations in an abrupt drop with flat bed. They conducted their studies on two types of abrupt drops with and without roundness (sudden fall) for two types of jumps on the abrupt drop as Jump-B and Jump-W, and compared them with a classic jump. The dimensionless pressure coefficient (C_p') is used in pressure variations which is calculated by equation (4).

4.
$$C_p'=\frac{\sigma p/\gamma}{v_1^2/2g}$$

Where, $\frac{v_1^2}{2g}$ is the velocity equivalent height before jump and σp is the standard deviation of the measured pressure which is calculated by equation 5.

5.
$$\sigma_P=\frac{1}{N^{0.5}}\left[\sum_{n=1}^{N}\left[p(x,y,n\Delta t)-\bar{P}(x,y)\right]^{2}\right]^{0.5}$$

In equation 5, $\bar{P}(x,y)$ is the mean time of change, p (x, y, nΔt) is the amount of pressure at each moment, N is the number of information taken in the discrete time series and Δt is the time interval between the recorded data. They that in an abrupt drop the values of the dimensionless pressure coefficient (C_p') is first increased and then decreased after reaching the maximum value. Also, by measuring the positive and negative dimensionless pressure values (C_p^+ and C_p^-) that their values in Jump-B mode were clearly higher than the classic jump, they concluded that if an abrupt drop stilling basin, it is required to increase the thickness of the bed plate compared to the classic jump mode. Hassounizadeh and ShafaiBejestan (2001) on the hydrodynamic pressures of the classical jump found that the positive and negative dimensionless pressure values (C_p^+ and C_p^-) increased initially for the specified flows and during the jump and after reaching the maximum value, they have a downward trend. They also showed that this maximum value always occurs in $x/y1<15$. This study was carried out taking into account that so far pressure fluctuation parameters have not been studied in jumps with abrupt drop and rough bed.

Materials and methods: In order to do this research, a flume was constructed in Shahid Chamran University's Hydraulic Laboratory, which has a length of 15 meters, a width of 0.8 meters and a depth of 0.7 meters (Fig. 2).

In the upper part of the flume as sliding valve is used to control the flow. The jump position is controlled along the test area by a valve located at the bottom of the flume. A number of copper connections are installed to read the pressure of the bed with a diameter of 0.006 meters along the center of the flume in the flow path. The turbulent pressure of the bed is calibrated by pressure transducers in the range of -10 to +10 kPa with Motorola's M5010S measuring device. Considering that previous spectroscopic analyses with similar signals by Bendat and Piersol (1971) indicate the dominant frequency of pressure fluctuations of about 30 Hz, the intensity of sampling is set as 40 Hz. Pressure transducers are connected to copper interfaces with non-soft tubes with a maximum length of 2 meters and an internal diameter of 0.004 meters. A computer is connected to the pressure transducer by a data converter including 12 channels. Sampling is done by DM5010S Data Loger software with the possibility of converting data. Subsequently, data is stored on a hard disk and statistical calculation is started. Flow rate in the inlet flow tube is evaluated using an Easy Flow magnetic flowmeter. Also, to verify the accuracy of the flow rate the flowmeter is calibrated with a rectangular overflow. The experiments are performed using cubic roughness and curved corners (Fig. 3).

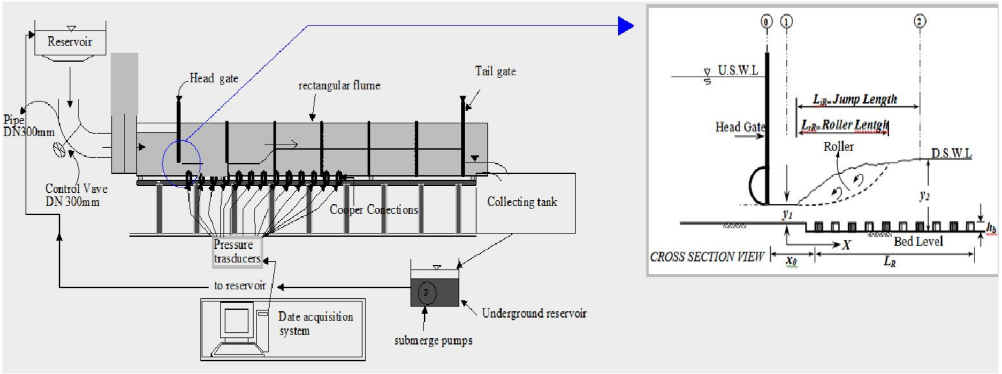


Figure 2. Schematic view of the flume used for testing

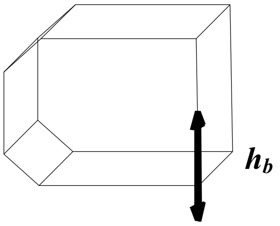
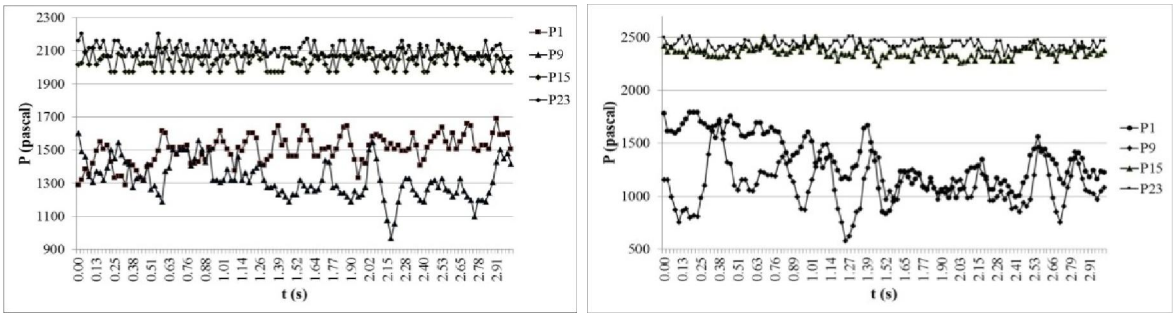


Figure 3. 3D view of the roughness used in the experiments

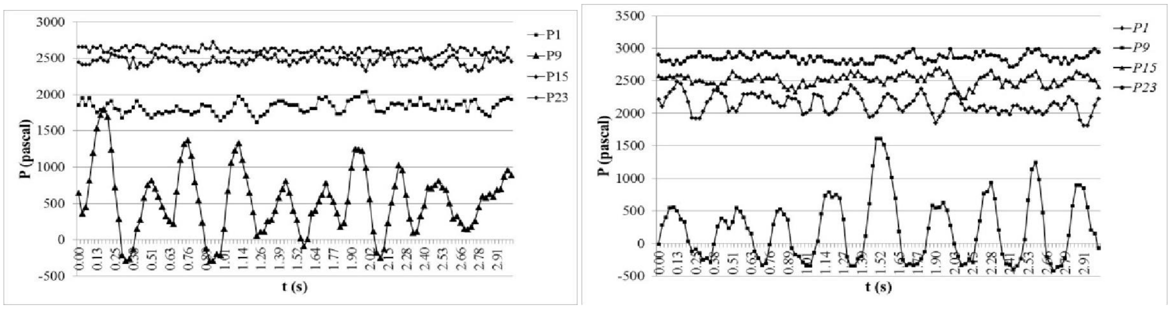
The roughness height is considered as high as the drop as h_b=4.5cm. The roughness is arranged in a zigzag 7-6-7 arrangement along the flume in 17 rows. To eliminate possible sources of distortion and disturbance of the transducer output and especially the effect of the air contained in the tubes, as well as uneven distribution of flow within the flume the necessary actions are taken into account. It should be noted that all jumps are Jump-B type and the hydrodynamic pressures in their bed was measured.

2. Results and discussion:



a. Pressure variations in Fr1=3.43

b. Pressure variations in Fr1= 4.33



c. Pressure variations in Fr_1= 5.04

d. Pressure variations in Fr_1= 5.85

Figure 4. Pressure variations relative to the time for different piezometers in the Froude numbers 3.43, 4.33, 5.04 and 5.85

In the experiments data are extracted for each of the piezometers in the flume floor by the barometer. Then, the data are analyzed statistically and the average, maximum, minimum, and standard deviation values for each of the piezometers are obtained. In Fig. 4, variations in pressure versus time in the Froude numbers 3, 5-43, 85 are presented for a number of piezoelectric transducers during jump.

As seen in them, piezometer No. 9 is significantly lower than all other piezometers in all Froude numbers. This distance increases with the increase of the Froude number so that in the Froude numbers5.04 and 5.85, the negative values are also included. These negative values indicate the probability of a cavitation phenomenon in this part of the stilling basin. The placement of the piezometer 9 is in x/y_1=10.44 (x is the distance from the beginning of drop and y1 is the height of the upstream water).

In order to study the hydrodynamic conditions the dimensionless pressure coefficient (C'p) is determined and compared with the data of Toso and Bowers (1988) and Aremenio et al. (2000). Aremenio et al data including classical hydraulic jump on the flat bed and the abrupt drop hydraulic jump on the flat bed have the drop ratio $\Delta z/y_1$=1 (see Fig. 1).

In Figure 5, C'p variations relative to x/y_1 are presented for the above-mentioned studies compared with this study. Aremenio et al. (2000) stated that abrupt drop jump shows a good match in terms of variation with undeveloped classic jump. Therefore, the numbers shown in Figure 5 of Toso and Bowers (1988) are for undeveloped classic jump.

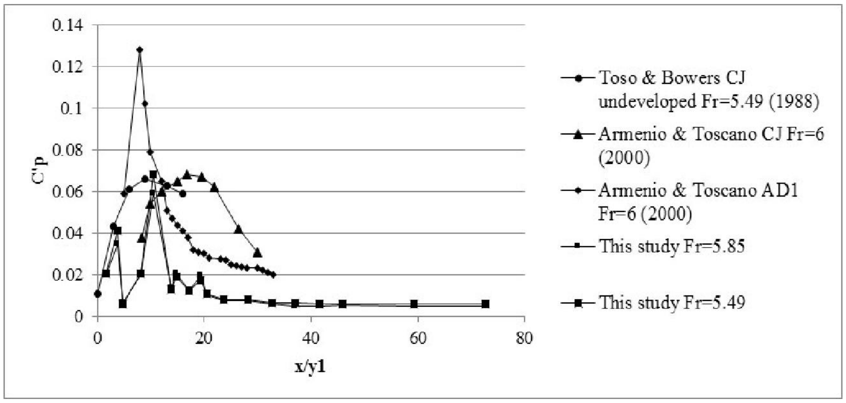


Figure 5. Comparison of dimensionless pressure coefficient with similar works

In Figure 5, the research data are presented for the drop ratio1. The following results are extracted based on this chart:

1) Maximum C'p values in abrupt drop with a flatbed are higher than the classic jump with the same Froude number.

2) Maximum C'p values in classic jump are equal with abrupt drop with a rough bed with the same Froude number.

3) The location of maximum C'p value in abrupt drop jump with a rough bed is ahead of the abrupt drop with the flatbed. This value in the abrupt drop with a flatbed is in x/y_1=8 and in abrupt drop with a rough bed is in x/y_1=10.44.

4) The fluctuations created in the abrupt drop with a rough bed appear in the presence of roughness. This means that roughness plays a role in reducing the C'p values and generates fluctuations in them. These fluctuations are reduced by moving away from the beginning of the jump (increasing the depth of water).

5) The maximum value of C'p in the abrupt drop with a rough bed occurs at x/y_1=10.44 which is the as piezometer No. 9.

6) C'p changes in two Froude numbers of 5.49 and 5.85 occur in similar amounts and locations, which indicates the accuracy of the data and confirms these changes.

In the explanation of Case 4 it is noted that this is due to the presence of secondary currents observed in the laboratory. These vortex flows pass along the flume in the first row before roughness then they are formed vertically in the first row and after each roughness. The intensity of these vortices is weaker with the advancement along the flume and taking distance from the beginning of the jump (increasing the depth of water). This concept is presented in Fig. 4.

It can be said that the presence of vertical vortices after roughness actually reduces C'p after roughness, which appears in Fig. 5 as decreasing oscillations after increasing C'p. The conditions are shown in Fig. 6. Table 1 lists the values for C'p for different jump states. According to the table, it can be seen that the values of C'p in abrupt drop with a rough bed are consistently lower than that of an abrupt drop with a flatbed about 35-80%. Also, the values of C'p in the abrupt drop with a flatbed relative to the classical jump are initially incremental (negative) and then decreasing. At the beginning of the abrupt drop jump with a flatbed, the dimensionless pressure coefficient is greater than the classic jump which becomes equal by taking distance from the beginning of the jump in x/y_1 about 13 and then decreases.

Table 1. Comparison of dimensionless pressure coefficients (C'p)

Jump type	Researchers	Fr_1	x/y_1	*C'p*	*C'p reduction %*	Compared to
Classic jump	Toso and Bowers	5.49	8	0.064	-	-
	Toso and Bowers	5.49	10	0.064	-	-
	Toso and Bowers	5.49	12	0.064	-	-
	Toso and Bowers	5.49	15	0.06	-	-
	Toso and Bowers	5.49	16	0.059	-	-
	Aremenio et al.	6	8.4	0.038	-	-
	Aremenio et al.	6	10	0.054	-	-
	Aremenio et al.	6	12	0.06	-	-
	Aremenio et al.	6	15	0.065	-	-
	Aremenio et al.	6	16	0.067	-	-
Abrupt drop with flatbed	Aremenio et al.	6	8	0.128	-116.95	Classic jump by Aremenio et al.
	Aremenio et al.	6	10	0.079	-107.89	
	Aremenio et al.	6	12	0.065	-20.37	
	Aremenio et al.	6	15	0.044	26.67	
	Aremenio et al.	6	16	0.041	36.94	
	Aremenio et al.	6	20	0.031	54.25	
Abrupt drop with rough bed	Present study	5.85	8.22	0.02	84.29	Classic jump with flatbed by Aremenio et al.
	Present study	5.85	10	0.051	34.97	
	Present study	5.85	12	0.038	41.95	
	Present study	5.85	15	0.02	58.06	
	Present study	5.85	16	0.017	59.68	
	Present study	5.85	20	0.014	54.30	

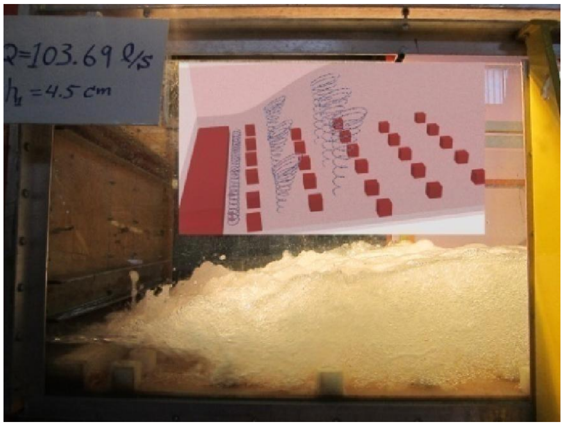


Figure 6- Horizontal and vertical vortex conditions in the test flume

Since the hydrodynamic forces cannot be fully investigated without examining the positive and negative coefficients (Cp + and Cp-), this research investigates these coefficients in reverse drop in the rough bed and then the results are compared with the reverse drop in flatbed. In Figures 7 and 8, the variations in the positive and negative coefficients of pressure along the basin with the drop ratio of 1.29 ($\Delta z/y_1 = 1.29$) are given. Therefore, these changes are severe at the beginning of the jump but their intensity is reduced by taking distance from the beginning of the jump and increasing the depth of water.

To determine the maximum and minimum values of the positive and negative pressure coefficients (Cp^{+}_{max}, Cp^{+}_{min}, Cp^{-}_{max}, Cp^{-}_{min}), their variations are plotted against the Froude number and the curve is fitted for each positive and negative coefficient which has a good fit (Figures 9 and 10).

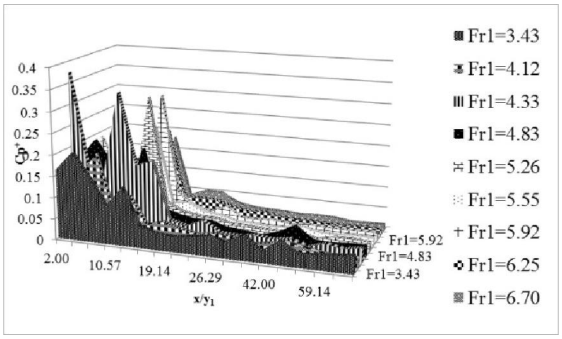


Figure 7- Variations in positive pressure coefficient (Cp^+) along the jump for the Froude numbers between 3.43 and 6.70

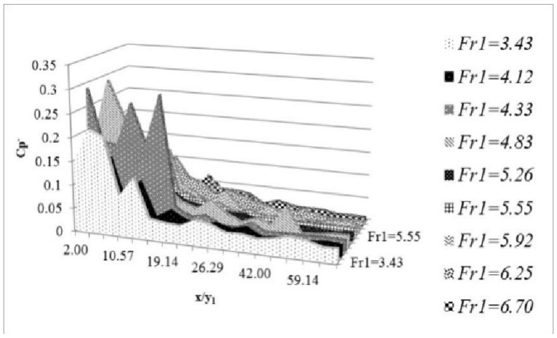


Figure 8- Variations in negative pressure coefficient (Cp^{-}) along the jump for the Froude numbers between 3.43 and 6.70

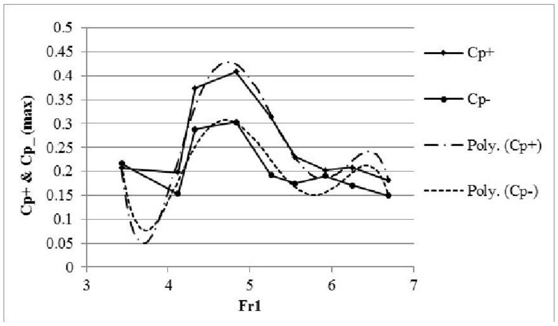


Figure 9- Variations in maximum positive and negative pressure coefficient for the upstream Froude numbers

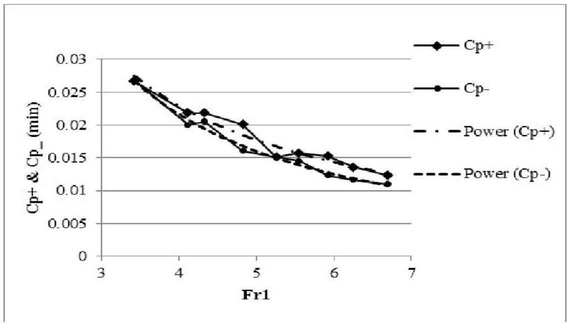


Figure 10- Variations in minimum positive and negative pressure coefficient for the upstream Froude numbers

Based on Figures 9 and 10, with the increase in Froude number the maximum values of the positive and negative pressure coefficients (Cp^{+}_{max} and Cp^{-}_{max}) are first increased and then decreased. This increase reaches its maximum up to Fr1 = 4.83. The reason for this increase can be attributed to the unstable hydraulic jump conditions in Froude numbers between 2.5 and 4.5. Also, the values of minimum positive and negative pressure coefficients (Cp^{+}_{max} and Cp^{-}_{max}) are decreased with the increase in Froude number. This decrease indicates a decrease in the deviation of the maximum and minimum of the mean value. In other words, the ratio of increased dynamic pressures is higher than the ratio of increasing pressure fluctuations. In addition, in both graphs the maximum and minimum values of the positive and negative coefficients of pressure the Cp^{+} value is always higher than Cp^{-}.

In order to determine the location of creating maximum and minimum pressure coefficients during the jump, the dimensionless coefficient of length (x / y1) versus Froude number (Fr1) are plotted as shown in Figures 11 and 12.

According to Figures 11 and 12, the values of the maximum positive and negative coefficients of pressure (Cp^{+}_{max} and Cp^{-}_{max}) initially and in Fr1≤4.83 occur at the same places. Then, after the Froude number Fr1 = 4.83, these places are separated and the Cp^{-}_{max} value becomes fixed after a change in $x/y_1=4.86$. Also, the minimum positive and negative pressure coefficients (Cp^{+}_{min} and Cp^{-}_{min}) have variable trend versus the Froude number but they occur within the range of $40<x/y_1<80$.

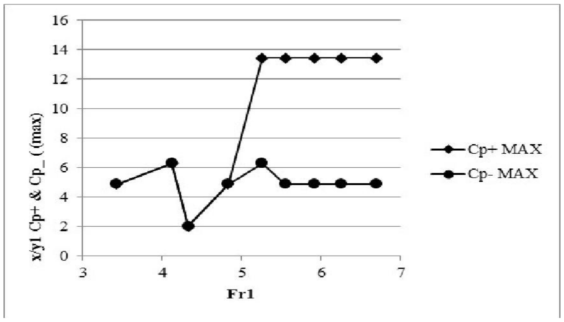


Figure 11- Locations for creating the maximum positive and negative pressure coefficients relative to the Froude number

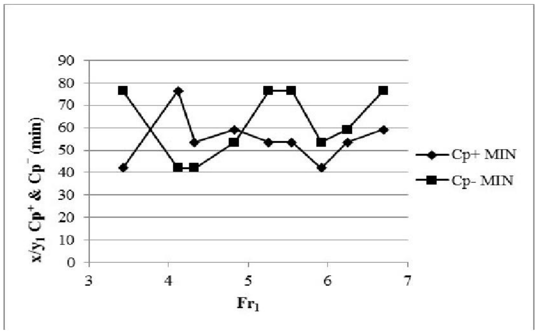


Figure 12- Locations for creating the minimum positive and negative pressure coefficients relative to the Froude number

Then the positive and negative pressure coefficients in abrupt drop with a rough bed are compared with that of the abrupt drop with a flatbed. Table (2) shows the variations of these coefficients in relation to the dimensionless length (x/y_1) in two types of flat and rough beds with a drop ratio of (Δz / y1 = 1).

Table 2. Comparison of positive and negative pressure coefficients in abrupt drop with rough and flat beds

Jump type	Researchers	Fr_1	x/y_1	Cp^-	Cp^+	Cp^- reduction	Cp^+ reduction
Abrupt drop with flat bed	Aremenio et al.	6	4.92	0.702	0.891	-	-
		6	6.15	0.765	0.808	-	-
		6	7.39	0.751	1.02	-	-
Abrupt drop with rough bed	The present study	5.85	4.9	0.018	0.018	97.37	98
		5.85	6.15	0.022	0.02	97.16	97.57
		5.85	7.38	0.043	0.039	94.29	96.20
		5.85	8.22	0.076	0.07	-	-
		5.85	10.44	0.083	0.07	-	-
		5.85	13.78	0.056	0.045	-	-

According to Table 2 the following cases can be deduced

1. At a constant distance from the start of the jump, the positive and negative pressure coefficients (Cp^+ and Cp^-) in the abrupt drop with rough bed are always less than the flatbed by more than 90%.
2. The maximum Cp^+ and Cp^- values occur in jumping over flatbed relative to a rough bed a location closer to the drop.
3. By roughing the jump bed in the abrupt drop, the reduced Cp^+ is always greater than the reduced Cp^- .

3. Conclusion:

According to the experiments it can be mentioned that roughness is quite effective on the hydraulic jump with rough bed and these effects are to increase the energy inhibition in jumping and reducing the hydrodynamic forces of the bed. In sum, the issues raised in the previous section can be summarized as follows:

1. The most critical jump zone in terms of the formation of maximum hydrodynamic forces and cavitation in in abrupt drop with a rough bed is in the area with the dimensionless length of $10.44 < x/y_1 < 13.43$.
2. Increasing the Froude number worsens the cavitation situations in this area.
3. The dimensionless pressure coefficient (C'p) in abrupt drop with a rough bed is less than abrupt drop with flatbed due to the formation of secondary currents and vertical vortices in the back of roughness.
4. The maximum C'p value occurs in the critical jump zone $x/y_1=10.44$.
5. By taking distance from the edge of the jump in abrupt drop jump the pressure fluctuations are first increased and after reaching the maximum level they take a descending trend. These changes are due to increased depth of water and a decrease in the effect of secondary flows. In fact, rising water level overcomes the hydrostatic forces by the hydrodynamic forces.
6. Maximum positive and negative pressure coefficients (Cp^+_{max} and Cp^-_{max}) are increased up to the $Fr_1=4.83$ in the abrupt drop with a rough bed and then begin to decrease.
7. Minimumpositive and negative pressure coefficients (Cp^+_{min} and Cp^-_{min}) are reduced in the abrupt drop with a rough bed by increasing the Froude number.
8. The location of maximum Cp^+ and Cp^- values in $Fr1 \leq 4.83$ occur at the same places and then it is fixed for Cp^+_{max} Cp^-_{max} at x/y1=13.43 and x/y1= 4.86 respectively. Also Cp^+ and Cp^- values occur within the range of $40 < x/y1 < 80$.
9. The positive and negative pressure coefficients (Cp^+ and Cp^-) in abrupt drop jump with rough bed have a significant decrease (more than 90%) compared to the abrupt drop jump with flat bed.

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