Integration of Fuzzy GTMA and Logarithmic Fuzzy Preference Programming for Supplier Selection

Ali Mohaghar [1], Maryam Sadat Fagheyi [2], Mohsen Moradi-Moghadam [3], Saiedeh Sadat Ahangari [2]

[1] Associate Professor, Faculty of Management, University of Tehran, Tehran, Iran
[2] M.S. Candidate of Industrial Management, University of Tehran, Iran
[3] M.S. of Industrial Management, University of Tehran, Tehran, Iran
Email: s.ahangary@ut.ac.ir

Abstract: Supplier selection is one of the most important decision making problems including both qualitative and quantitative factors to identify Suppliers with the highest potential for meeting a firm's needs consistently and at an acceptable cost and plays a key role in supply chain management (SCM).The purpose of this paper is applying a new integrated method to Supplier selection. Proposed approach is based on Logarithmic Fuzzy Preference Programming and Fuzzy GTMA (graph theory and matrix approach) methods. LFPP method is used in determining the weights of the criteria by decision makers and then rankings of Suppliers are determined by Fuzzy GTMA method. We apply the integrated approach in a real case to demonstrate the application of the proposed method.

[Ali Mohaghar, Maryam Sadat Fagheyi, Mohsen Moradi-Moghadam, Saiedeh Sadat Ahangari. **Integration of Fuzzy GTMA and Logarithmic Fuzzy Preference Programming for Supplier Selection.** *Rep Opinion* 2013;5(5):9-16]. (ISSN: 1553-9873). http://www.sciencepub.net/report. 2

Keywords: Supplier selection, graph theory and matrix approach, Fuzzy set and Logarithmic fuzzy preference programming.

1. Introduction

In today's highly competitive and interrelated manufacturing environment, the performance of the Supplier becomes a key element in a company's success, or failure. Supplier selection decisions are an important component of production and logistics management for many companies. These decisions are typically complicated, for several reasons. First, potential options may need to be evaluated on more than one criterion. A second complication is the fact that individual vendors may have different performance characteristics for different criteria. A third complication arises from internal policy constraints, and externally imposed system constraints placed on the buying process. The nature of Supplier selection decision usually is complex, unstructured, and inherently a multiple criteria problem (Rao, 2007). Handfield et al (2002) illustrated the use of AHP as a decision support model to help managers understand the tradeoffs between environmental dimensions. Gunasekaran et al (2001) established a framework consisting of three-level indices: strategic performance, tactical performance, and operational performance. Feng et al (2001) presented a stochastic integer programming approach for simultaneous selection of tolerances and suppliers based on the quality loss function and process capability indices. Oliveria et al (2002) developed a multi criteria model for assigning new orders to service vendors. Kwang et al (2002) combined a scoring method and fuzzy expert systems for Supplier assessment, and presented a case study. Cebi et al (2003) structured Supplier selection problem in terms of an integrated lexicographic goal programming (LGP) and AHP model, including both quantitative and qualitative conflicting factors. Cengiz et al (2003) applied the fuzzy AHP method for solving the Supplier selection problem. Ibrahim et al (2003) used activity-based costing and fuzzy present-worth techniques for vendor selection. Kumar et al (2004) presented a fuzzy goal programming approach for the Supplier selection problem in a supply chain. Ge et al (2004) developed an integrated AHP and preemptive goal programming (PGP)-based multi criteria decision-making methodology to account both qualitative and quantitative factors in supplier selection. Pi et al (2005) presented a supplier evaluation and selection approach using Taguchi's loss function and AHP. Degraeve et al (2005) used total cost of ownership information for evaluating a firm's strategic procurement options. The approach was used to develop a decision support system at a European multinational steel company. Shyur et al (2006) proposed a hybrid MCDM model using ANP and TOPSIS methods for strategic Supplier selection. Sucky (2006) proposed a dynamic decision making approach based on the principle of hierarchical planning for strategic Supplier selection. Cao et al (2006) discussed the aspects of optimizing vendor selection in a two-stage outsourcing process. Wadhwa et al (2006) presented multi-objective optimization methods including goal programming and compromise programming. Amid et al (2006) proposed a multi-objective linear model for supplier selection in a supply chain. Rao (2007) proposed a combined AHP and genetic algorithm (GA) method for the Supplier selection problem. The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology. The application of the proposed method is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. Fuzzy Sets and Fuzzy Numbers

Fuzzy set theory, which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework. A fuzzy set $\tilde{A}$ can be defined mathematically by a membership function $\mu_{\tilde{A}}(X)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. A triangular fuzzy number $\tilde{A}$can be defined by a triplet (a, b, c) as illustrated in Fig 1.

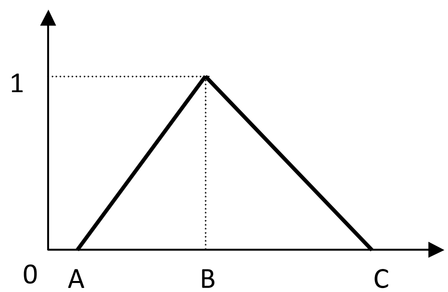

Fig 1.A triangular fuzzy number $\tilde{A}$

The membership function $\mu_{\tilde{A}}(X)$is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-c}{b-c} & b \le x \le c \\ 0 & oterwise \end{cases} \quad (1)$$

Basic arithmetic operations on triangular fuzzy numbers $A_1 = (a_1,b_1,c_1)$, where $a_1 \le b_1 \le c_1$, and $A_2 = (a_2,b_2,c_2)$, where $a_2 \le b_2 \le c_2$,can be shown as follows:

Addition: $A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$ (2)

Subtraction: $A_1 \ominus A_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$ (3)

Multiplication: if k is a scalar

$$K \otimes A_1 = \begin{cases} (ka_1, kb_1, kc_1), & k > 0 \\ (kc_1, kb_1, ka_1), & k < 0 \end{cases}$$

$A_1 \otimes A_2 \approx (a_1a_2, b_1b_2, c_1c_2)$, if $a_1 \ge 0$, $a_2 \ge 0$ (4)

Division: $A_1 \oslash A_2 \approx (\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2})$,

if $a_1 \ge 0$, $a_2 \ge 0$ (5)

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications (Kaufmann et al. 1988). Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including personnel selection (e.g. rating for creativity, personality, leadership, etc.). The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation (Karsak, 2002). A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill-defined to be described in conventional quantitative terms (Zadeh, 1975).

3. Research Methodology

In this paper, the weights of each criterion are calculated using LFPP method. After that, Fuzzy GTMA is utilized to rank the alternatives. Finally, we select the best Supplier based on these results.

3.1. The LFPP-based nonlinear priority method

In this method for the fuzzy pairwise comparison matrix, Wang et al (2011) took its logarithm by the following approximate equation:

$\ln \tilde{a} = (\ln l_{ij}, \ln m_{ij}, \ln lu_{ij})$, i,j = 1….,n (6)

That is, the logarithm of a triangular fuzzy judgment a_{ij} can still be seen as an approximate triangular fuzzy number, whose membership function can accordingly be defined as

$$\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) = \begin{cases} \frac{\ln\left(\frac{w_i}{w_j}\right) - \ln l_{ij}}{\ln m_{ij} - \ln l_{ij}}, & \ln\left(\frac{w_i}{w_j}\right) \le \ln m_{ij}, \\ \frac{\ln u_{ij} - \ln\left(\frac{w_i}{w_j}\right)}{\ln u_{ij} - \ln m_{ij}}, & \ln\left(\frac{w_i}{w_j}\right) \ge \ln m_{ij}, \end{cases}$$

(7)

Where $\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right)$ is the membership degree of $\ln\left(\frac{w_i}{w_j}\right)$ belonging to the approximate triangular fuzzy judgment $\ln \tilde{a} = (\ln l_{ij}, \ln m_{ij}, \ln lu_{ij})$. It is very natural that we hope to find a crisp priority vector to maximize the minimum membership degree λ= min $\{\mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) \mid$ i=1,…,n-1 ; j=i+1,…, n$\}$. The resultant model can be constructed (Wang et al, 2011) as

Maximize λ

Subject to

$$\begin{cases} \mu_{ij}\left(\ln\left(\frac{w_i}{w_j}\right)\right) \geq \lambda, i = 1, \ldots, n-1; j = i+1, \ldots, n, \\ w_i \geq 0, i = 1, \ldots, n, \end{cases} \quad (8)$$

Or as

Maximize 1- λ

Subject to

$$\begin{cases} \ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}, \\ i = 1, \ldots, n-1; j = i+1, \ldots, n, \\ -\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}, \\ i = 1, \ldots, n; j = i+1, \ldots, n, \end{cases} \quad (9)$$

It is seen that the normalization constraint $\sum_{i=1}^{n} w_i = 1$ is not included in the above two equivalent models. This is because the models will become computationally complicated if the normalization constraint is included. Before normalization, without loss of generality, we can assume $w_i \geq 1$ for all $i = 1, \ldots, n$ such that $\ln w_i \geq 0$ for $i = 1, \ldots, n$. Note that the nonnegative assumption for $\ln w_i \geq 0$ (i = 1,. . . ,n) is not essential. The reason for producing a negative value for λ is that there are no weights that can meet all the fuzzy judgments in $\widetilde{A}$ within their support intervals. That is to say, not all the inequalities $\ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}$ or $-\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}$ can hold at the same time. To avoid k from taking a negative value, Wang et al (2011) introduced nonnegative deviation variables δ_{ij} and η_{ij} for $i = 1, \ldots, n-1; j = i+1, \ldots, n$, such that they meet the following inequalities:

$$\ln w_i - \ln w_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) \geq \ln l_{ij}, i = 1, \ldots, n-1; j = i+1, \ldots, n$$
$$-\ln w_i + \ln w_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) \geq -\ln u_{ij}, i = 1, \ldots, n; j = i+1, \ldots, n \quad (10)$$

It is the most desirable that the values of the deviation variables are the smaller the better. Wang et al (2011) thus proposed the following LFPP-based nonlinear priority model for fuzzy AHP weight derivation:

Minimize $J = (1-\lambda)^2 + M.\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\delta_{ij}^2 + \eta_{ij}^2)$

Subject to

$$\begin{cases} x_i - x_j - \lambda \ln\left(\frac{m_{ij}}{l_{ij}}\right) + \delta_{ij} \geq \ln l_{ij}, \\ i = 1, \ldots, n-1; j = i+1, \ldots, n, \\ -x_i + x_j - \lambda \ln\left(\frac{u_{ij}}{m_{ij}}\right) + \eta_{ij} \geq -\ln u_{ij}, \\ i = 1, \ldots, n; j = i+1, \ldots, n, \\ \lambda, x_i \geq 0, i = 1, \ldots, n \\ \delta_{ij}, \eta_{ij} \geq 0, i = 1, \ldots, n-1; j = i+1, \ldots, n \end{cases} \quad (11)$$

Where $x_i = \ln w_i$ for i = 1,. . . , n and M is a specified sufficiently large constant such as $M = 10^3$. The main purpose of introducing a big constant M into the above model is to find the weights within the support intervals of fuzzy judgments without violations or with as little violations as possible.

3.2. The GTMA method

Graph theory is a logical and systematic approach. The advanced theory of graphs and its applications are very well documented. Rao (2007) in his book presents this methodology and shows some of its applications. Graph/digraph model representations have proved to be useful for modeling and analyzing various kinds of systems and problems in numerous fields of science and technology (Darvish et al, 2009). The matrix approach is useful in analyzing the graph/digraph models expeditiously to derive the system function and index to meet the objectives (Rao, 2007). The graph theory and matrix methods consist of the digraph representation, the matrix representation and the permanent function representation. The digraph is the visual representation of the variables and their inter dependencies. The matrix converts the digraph into mathematical form and the permanent function is a mathematical representation that helps to determine the numerical index (Faisal, 2007).

The step by step explanation of the methodology is as follows:

Step 1. Identifying equipment selection attributes. In this step all the criteria which affect the decision is determined. This can be done by using relevant criteria available in the literature or getting information from the decision maker.

Step 2. Determine equipment alternatives. All potential alternatives are identified.

Step 3. Graph representation of the criteria and their inter dependencies. Equipment selection criterion is defined as a factor that influences the selection of an alternative. The equipment selection criteria digraph models the alternative selection criteria and their inter relationship. This digraph consists of a set of nodes N = {n_i}, with i = 1, 2,...,M and a set of directed edges E = {e_{ij}}. A node n_i represents i-th alternative selection criterion and edges represent the relative importance among the criteria. The number of nodes M considered is equal to the number of alternative selection criteria considered. If a node 'i' has relative importance over another node 'j' in the alternative selection, then a directed edge or arrow is drawn from node i to node j (i.e. e_{ij}). If 'j' has relative importance over 'i' directed edge or arrow is drawn from node j to node i (e_{ji}) (Rao, 2007).

Step 4. Develop equipment selection criteria matrix of the graph. Matrix representation of the alternative selection criteria digraph gives one-to-one representation. A matrix called the equipment selection criteria matrix. This is an M in M matrix and considers all of the criteria (i.e. A_i) and their relative importance (i.e. a_{ij}). Where A_i is the value of the i-th criteria represented by node n_i and a_{ij} is the relative importance of the i-th criteria over the j-th represented by the edge e_{ij} (Rao, 2007 & Faisal et al, 2007).

The value of A_i should preferably be obtained from available or estimated data. When quantitative values of the criteria are available, normalized values of a criterion assigned to the alternatives are calculated by v_i/v_j, where v_i is the measure of the criterion for the i-th alternative and v_j is the measure of the criterion for the j-th alternative which has a higher measure of the criterion among the considered alternatives. This ratio is valid for beneficial criteria only. A beneficial criteria means its higher measures are more desirable for the given application. Whereas, the non-beneficial criterion is the one whose lower measures are desirable and the normalized values assigned to the alternatives are calculated by v_j/v_i.

$$\text{CS Matrix} = \begin{bmatrix} A_1 & a_{12} & a_{13} & a & a & a_{1.m} \\ a_{21} & A_2 & a_{23} & \cdots & \cdots & a_{2.m} \\ a_{31} & a_{32} & A_3 & \cdots & \cdots & a_{3.m} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \ddots & \cdots & \cdots & \cdots & \vdots \\ a_1 & a_1 & a_1 & \cdots & \cdots & A_m \end{bmatrix} \quad (12)$$

Step 5. Obtaining alternative selection criteria function for the matrix. The permanent of this matrix, is defined as the alternative selection criteria function. The permanent of a matrix was introduced by Cauchy in 1812. At that time, while developing the theory of determinants, he also defined a certain subclass of symmetric functions which later Muir named permanents (Nourani, 1999). The permanent is a standard matrix function and is used in combinatorial mathematics (Faisal, 2007 & Rao, 2006). The permanent function is obtained in a similar manner as the determinant but unlike in a determinant where a negative sign appears in the calculation, in a variable permanent function positive signs replace these negative signs (Faisal, 2007 & Rao, 2006). Application of the permanent concept will lead to a better appreciation of selection attributes. Moreover, using this no negative sign will appear in the expression (unlike determinant of a matrix in which a negative sign can appear) and hence no information will be lost (Rao, 2006).

The per (CS) contains terms arranged in (M + 1) groups, and these groups represent the measures of criteria and the relative importance loops. The first group represents the measures of M criteria. The second group is absent as there is no self-loop in the digraph. The third group contains 2- criterion relative importance loops and measures of (M-2) criteria. Each term of the fourth group represents a set of a 3-criterion relative importance loop, or its pair, and measures of (M-3) criteria. The fifth group contains two sub-groups. The terms of the first sub-group is a set of two 2-criterion relative importance loops and the measures of (M-4) criteria. Each term of second sub-group is a set of a 4-attribute relative importance loop, or its pair, and the measures of (M-4) criteria. The sixth group contains two subgroups. The terms of the first sub-group are a set of a 3-criterion relative importance loop, or its pair, and 2-criterion importance loop and the measures of (M-5) criteria. Each term of the second sub-group is a set of a 5-criterion relative importance loop, or its pair, and the measures of (M-5) criteria. Similarly other terms of the equation are defined. Thus, the CS fully characterizes the considered alternative selection evaluation problem, as it contains all possible structural components of the criteria and their relative importance. It may be mentioned that this equation is nothing but the determinant of an M-M matrix but considering all the terms as positive.

Step 6. Evaluation and ranking of the alternatives, in this step all alternatives are ranked according to their permanent values calculated in the previous step.

$$\text{per (Cs)} = \prod_{i=1}^{M} A_i + \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \dots \sum_{M=t+1}^{M} (a_{ij} a_{ji}) A_k A_l A_m A_n A_o \dots A_t A_M$$
$$+ \sum_{i=1}^{M-2} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^{M} \dots \sum_{M=t+!}^{M} (a_{ij} a_{jk} a_{ki} + a_{ik} a_{kj} a_{ji}) A_l A_m A_n A_o \dots A_t A_M$$
$$+ \sum_{i=1}^{M-3} \sum_{j=i+1}^{M} \sum_{k=i+1}^{M-1} \sum_{l=i+2}^{M} \dots \sum_{M=t+1}^{M} (a_{ij} a_{ji} + a_{kl} a_{lk}) A_m A_n A_o \dots A_t A_M$$
$$+ \sum_{i=1}^{M-3} \sum_{j=i+1}^{M} \sum_{k=i+1}^{M-1} \sum_{l=i+2}^{M} \dots \sum_{M=t+1}^{M} (a_{ij} a_{jk} a_{kl} a_{li} + a_{il} a_{lk} a_{kj} a_{ji}) A_m A_n A_o \dots A_t A_M +$$

$$\cdots\cdots$$

$$\sum_{i=1}^{M-2}\sum_{j=1}^{M-1}\sum_{j=i+1}^{M}\sum_{l=1}^{M-1}\sum_{m=l+1}^{M-2}\sum_{m=t+1}^{M}(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji})(a_{lm}a_{m})A_nA_o\dots A_tA_m +$$
$$\sum_{i=1}^{M-4}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M}\dots\dots\sum_{M=t+1}^{M}(a_{ij}a_{jk}a_{kl}a_{lm}a_{mi}+ a_{im}a_{mj}a_{lk}a_{kj}a_{ji})A_nA_o\dots\dots A_tA_m +$$
$$\sum_{i=1}^{M-3}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M-1}\sum_{n=m+1}^{M}\dots\dots\sum_{M=t+1}^{M}(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji})(a_{lm}a_{mn}a_{nl} + a_{ln}a_{nm}a_{ml})A_o\dots A_tA_m$$
$$+\sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M-2}\sum_{m=l+1}^{M-1}\sum_{n=m+1}^{M}\dots\dots\sum_{M=t+1}^{M}(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji}) + (a_{lm}a_{mn}a_{nl}+a_{ln}a_{nm}a_{ml})A_o\dots A_tA_M$$
$$+\sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M}\sum_{n=m+1}^{M}\dots\sum_{M=t+1}^{M}(a_{ij}+a_{jk}a_{kl}a_{lm}a_{mn}a_{nj}+a_{in}a_{nm}a_{ml}a_{lk}a_{kj}a_{ji})A_o\dots A_tA_M \quad (13)$$

4. A Numerical Application of Proposed Approach

In this section, we presented a case study to demonstrate the application of proposed method for a firm that manufactures Tire. The company had divided all purchased parts into 5 groups, including A_1, A_2, A_3, A_4, A_4, A_5, A_6, A_7, A_8, A_9 and A_{10}. Selecting the best Supplier has a great importance for this company. But it is hard to choose the most suitable one among the other Suppliers. In the application, firstly through the literature investigation and studying other papers that are related to Supplier selection, ten criteria are selected. These criteria include Capacity (C_1), Availability of Raw materials (C_2), Geographic Location (C_3), Shipment Accuracy (C_4), Cost (C_5), and Customer Service (C_6). In this paper, the weights of criteria are calculated using of LFPP, and these calculated weight values are used as Fuzzy GTMA inputs. Then, after Fuzzy GTMA calculations, evaluation of the alternatives and selection of best Supplier is realized.

Logarithmic Fuzzy Preference Programming:

In LFPP, firstly, we should determine the weights of each criterion by utilizing pair-wise comparison matrices. We compare each criterion with respect to other criteria. You can see the pair-wise comparison matrix for Supplier selection criteria in Table 1.

After forming the model (11) for the comparison matrix and solving this model using of Genetic algorithms, the weight vector is obtained as follow:

w^t = (0.01979, 0.0537, 0.1540, 0.2218, 0.2440, 0.3065) ^{T}

Fuzzy GTMA calculations

The weights of the criteria are calculated by LFPP up to now, and then these values can be used in Fuzzy GTMA. After calculating the weights, we formed the fuzzy decision matrix of GTMA and after that we normalized the Fuzzy decision matrix of GTMA that shows in Table 2.

Table 1. Inter-criteria comparison matrix

P	C_1			C_2			…	C_4			C_5			C_6		
	L	m	u	L	m	u	…	L	m	u	L	m	u	L	m	u
C_1	1.00	1.00	1.00	1.00	1.00	1.00	…	0.73	0.87	0.93	0.37	0.50	0.63	0.77	0.93	0.97
C_2	1.00	1.00	1.00	1.00	1.00	1.00	…	0.77	0.93	0.97	0.57	0.72	0.80	0.73	0.87	0.93
C_3	1.17	1.36	1.75	1.08	1.18	1.50	…	0.60	0.77	0.87	0.63	0.77	0.83	0.50	0.65	0.80
C_4	1.07	1.17	1.37	1.04	1.08	1.31	…	1.00	1.00	1.00	0.80	1.00	1.00	0.73	0.87	0.93
C_5	1.04	1.08	1.31	1.17	1.36	1.75	…	1.00	1.00	1.25	1.00	1.00	1.00	0.63	0.77	0.83
C_6	1.04	1.08	1.31	1.07	1.17	1.37	…	1.07	1.17	1.37	1.26	1.42	1.73	1.00	1.00	1.00

Table 2. Decision matrix of Fuzzy GTMA

	C_1			C_2			…	C_4			C_5			C_6		
	L	m	u	L	m	u	…	L	m	u	L	m	u	L	m	u
A_1	0.42	0.50	0.55	0.87	0.80	0.90	…	0.87	0.80	0.90	0.87	0.80	0.90	0.62	0.60	0.70
A_2	0.00	0.00	0.22	0.50	0.50	0.60	…	0.50	0.50	0.60	0.12	0.20	0.30	0.50	0.50	0.60
A_3	0.57	0.62	0.66	0.25	0.35	0.50	…	0.00	0.00	0.20	0.62	0.65	0.80	0.00	0.00	0.20
A_4	0.28	0.43	0.55	0.875	0.80	0.90	…	0.62	0.65	0.80	0.25	0.35	0.50	0.62	0.65	0.80
A_5	0.14	0.25	0.33	1.00	1.00	1.00	...	1.00	1.00	1.00	0.50	0.50	0.60	0.00	0.00	0.20
A_6	0.71	0.81	0.88	0.50	0.50	0.60	…	0.00	0.00	0.20	0.12	0.20	0.30	0.87	0.80	0.90
A_7	1.00	1.00	1.00	0.25	0.35	0.50	…	0.50	0.50	0.60	0.25	0.35	0.50	1.00	1.00	1.00
A_8	0.28	0.43	0.55	0.12	0.20	0.30	…	0.87	0.80	0.90	0.62	0.65	0.80	0.50	0.50	0.60
A_9	0.57	0.62	0.66	0.62	0.65	0.80	…	0.25	0.35	0.50	1.00	1.00	1.00	0.00	0.00	0.20
A_{10}	0.14	0.25	0.33	0.12	0.20	0.30	…	0.50	0.50	0.60	0.87	0.800	0.90	0.25	0.35	0.50

In Fuzzy GTMA method, we carry out pair-wise comparison with respect to their weight that shows in Table 3.

Table 3. Pair-wise comparison of criteria with respect to each other

	C_1	C_2	C_3	C_4	C_5	C_6
C_1		0.269	0.114	0.082	0.075	0.061
C_2	0.731		0.259	0.195	0.181	0.149
C_3	0.886	0.741		0.410	0.387	0.334
C_4	0.918	0.805	0.590		0.476	0.420
C_5	0.925	0.819	0.613	0.524		0.443
C_6	0.939	0.851	0.666	0.580	0.557	
w_j	0.020	0.054	0.154	0.222	0.244	0.306

Because in Fuzzy GTMA method our decision matrix is fuzzy, we should obtain the fuzzy permanent matrix for each criterion. For example, for calculating fuzzy permanent matrix for A_1, first we should obtain the permanent matrix with the lower bound of fuzzy decision matrix as well as we should obtain the permanent matrix with the mean bound and the upper bound that show from Table 4 to Table 6.

Table 4. Pair-wise comparison of criteria with respect to A_1 with the lower bound of fuzzy decision matrix

A_1-L	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.429	0.269	0.114	0.082	0.075	0.061
C_2	0.731	0.875	0.259	0.195	0.181	0.149
C_3	0.886	0.741	0.286	0.410	0.387	0.334
C_4	0.918	0.805	0.590	0.875	0.476	0.420
C_5	0.925	0.819	0.613	0.524	0.875	0.443
C_6	0.939	0.851	0.666	0.580	0.557	0.625

Table 5. Pair-wise comparison of criteria with respect to A_1 with the mean bound of fuzzy decision matrix

A_1-m	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.500	0.269	0.114	0.082	0.075	0.061
C_2	0.731	0.800	0.259	0.195	0.181	0.149
C_3	0.886	0.741	0.438	0.410	0.387	0.334
C_4	0.918	0.805	0.590	0.800	0.476	0.420
C_5	0.925	0.819	0.613	0.524	0.800	0.443
C_6	0.939	0.851	0.666	0.580	0.557	0.600

Table 6. Pair-wise comparison of criteria with respect to A_1 with the upper bound of fuzzy decision matrix

A_1-u	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.556	0.269	0.114	0.082	0.075	0.061
C_2	0.731	0.900	0.259	0.195	0.181	0.149
C_3	0.886	0.741	0.556	0.410	0.387	0.334
C_4	0.918	0.805	0.590	0.900	0.476	0.420
C_5	0.925	0.819	0.613	0.524	0.900	0.443
C_6	0.939	0.851	0.666	0.580	0.557	0.700

The permanent matrix for Table 4, Table 5 and Table 6 are 6.531, 7.21 and 9.187. According to this method the fuzzy permanent matrix for A_1 is (6.531, 7.21, 9.187). After that we obtain the fuzzy permanent matrix of all alternatives that shows in Table 7.

Table 7. The fuzzy permanent matrix

Alternative	Fuzzy permanent matrix
A_1	(6.531, 7.21, 9.17)
A_2	(4.23, 4.66, 6.72)
A_3	(4.44, 4.5, 6.89)
A_4	(4.69, 5.23, 7.11)
A_5	(6.18, 6.65, 7.89)
A_6	(5.45, 5.50, 7.66)
A_7	(6.18, 8.30, 8.99)
A_8	(4.72, 5.11, 6.83)
A_9	(7.12, 7.44, 8.99)
A_{10}	(7.76, 7.98, 8.23)

In the next step, by using of extent analysis method, we obtain the crisp permanent matrix and we rank Suppliers based on crisp permanent matrix. Finally, we rank all Suppliers with respect to their permanent matrix that shows in Table 8.

Table 8. Ranking of Suppliers

Alternative	Crisp Permanent matrix	rank
A_1	0.078	6
A_2	0.211	2
A_3	0.022	10
A_4	0.034	9
A_5	0.123	4
A_6	0.119	3
A_7	0.335	1
A_8	0.041	8
A_9	0.112	5
A_{10}	0.042	7

According to Table 8, A_7 is the best Supplier among other Suppliers and other Suppliers ranked as follow: $A_7 > A_2 > A_6 > A_5 > A_9 > A_1 > A_{10} > A_8 > A_4 > A_3$.

5. Conclusions

The objective of Supplier selection is to identify Suppliers with the highest potential for meeting a company's needs consistently and at an acceptable cost. Selection is a broad comparison of Suppliers based on a common set of criteria and measures. However, the level of details used for examining potential Suppliers may vary depending on a company's needs. The overall goal of selection is to identify high potential Suppliers and their quota allocations. An effective and appropriate Supplier assessment method is therefore crucial to the competitiveness of companies. In this paper, LFPP and Fuzzy GTMA are combined that Fuzzy GTMA uses LFPP result weights as input weights. Then a real case study is presented to show applicability and performance of the method.

Corresponding Author:
Saiedeh Sadat Ahangari
M.S. Candidate of Industrial Management, University of Tehran, Iran
Email: s.ahangary@ut.ac.ir

Acknowledgement
The authors wish to thank an anonymous referee for the valuable suggestions which considerably improve the quality of the paper.

References

1. Amid, A., Ghodsypour, SH. and O'Brien C. 2006.Fuzzy multi objective linear model for supplier selection in a supply chain. International Journal of Production Economics 104:394–407
2. Cao, Q., and Wang, Q. 2006. Optimizing vendor selection in a two-stage outsourcing process. Computers & Operations Research doi:10.1016/j.cor.2006.01.013
3. Cebi, F., Bayraktar, D .2003. An integrated approach for vendor selection. Logistics Information Management 16:395–400
4. Cengiz, K., Ufuk, C., and Ziya, U .2003. Multi-criteria vendor selection using fuzzy AHP. Logistics Information Management 16:382–394
5. Darvish, M., M. Yasaei, and A. Saeedi . (2009). Application of the graph theory and matrix methods to contractor ranking. International Journal of Project Management, 27(6): p. 610-619.
6. Degraeve1, Z., Roodhooft, F., and Doveren, VB, 2005. The use of total cost of ownership for strategic procurement, a company-wide management information system. Journal of the Operational Research Society 56:51–59
7. Faisal, M.N., D. Banwet, and R. Shankar. (2007). Quantification of risk mitigation environment of supply chains using graph theory and matrix methods. European Journal of Industrial Engineering, 1(1): p. 22-39.
8. Feng CX., Wang, J., and Wang, JS .2001.An optimization model for concurrent selection of tolerances and vendors. Computers & Industrial Engineering 40:15–33
9. Ge, W., Huang, SH., and Dismukesa, JP. 2004. Product-driven supply chain selection using integrated multi-criteria decision-making methodology. International Journal of Production Economics 91:1–15
10. Gunasekaran, A., Patel, C., Tirtirouglu, E. 2001. Performance measures and metrics in a supply chain environment. International Journal of Operations & Production Management 21:71–87
11. Handfield, R., Walton, SV. Sroufe, R., and Melnyk, SA. 2002. Applying environmental criteria to vendor assessment, a study in the application of the analytical hierarchy process. European Journal of Operational Research 141:70–87
12. Ibrahim, D., and Ugur, S. 2003. Vendor selection using activity-based costing and fuzzy present-worth techniques. Logistics Information Management 16:420–426
13. Kaufmann, A., and Gupta, M. M. 1988. Fuzzy mathematical models in engineering and management science. Amsterdam: North-Holland.

14. Karsak, E. E. 2002. Distance-based fuzzy MCDM approach for evaluating flexible manufacturing system alternatives. International Journal of Production Research 40(13), 3167–3181.
15. Kumar, M., Vrat, P., and Shankar, R .2004. A fuzzy goal programming approach for vendor selection problem in a supply chain. Computers & Industrial Engineering 46:69–85
16. Kwang, CK., Ip, WH., Chan, J.W.K .2002. Combining scoring method and fuzzy expert systems approach to vendor assessment: a case study. Integrated Manufacturing Systems 13:512–519
17. Nourani, Y. and B. Andresen. (1999). Exploration of NP-hard enumeration problems by simulated annealing--the spectrum values of permanents. Theoretical computer science, 215(1-2): p. 51-68.
18. Oliveria, RC., and Lourenco, JC .2002. A multi-criteria model for assigning new orders to service vendors. European Journal of Operations Research 139:390–399
19. Pi, WN. and Low, C .2005. Supplier evaluation and selection via Taguchi's loss function and an AHP. International Journal of Advanced Manufacturing Technology doi:10.1007/s00170-004-2227-z
20. Rao, R.V. (2006). A decision-making framework model for evaluating flexible manufacturing systems using digraph and matrix methods. The International Journal of Advanced Manufacturing Technology, 30(11): p. 1101-1110.
21. Rao, RV. 2007. Decision making in the manufacturing environment: using graph theory and fuzzy multiple attribute decision making methods. Springer, London.
22. Rao, RV .2007.Vendor selection in a supply chain using analytic hierarchy process and genetic algorithm methods. International Journal of Services and Operations Management 3:355–369
23. Shyur, HJ., Shih, HS .2006. A hybrid MCDM model for strategic vendor selection. Mathematical and Computer Modelling doi:10.1016/j.mcm.2005.04.018
24. Sucky, E .2006. A model for dynamic strategic vendor selection. Computers & Operations
25. Wadhwa, V., and Ravindran, AR .2006. Vendor selection in outsourcing. Computers & Operations Research doi:10.1016/j.cor.2006.01.009
26. Wang, Y., Chin.,K. 2011, Fuzzy analytic hierarchy process: A logarithmic fuzzy preference Programming methodology, International Journal of Approximate Reasoning 52, 541–553.
27. Zadeh, L. A. 1965. Fuzzy sets. Information and Control, 8(3), 338–353.
28. Zadeh, L. A. 1975. The concept of a linguistic variable and its application to approximate reasoning-I. Information Sciences, 8(3), 199–249.

5/10/2013