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EMPIRICAL STUDY OF NEWTON'S LAW OF COOLING BY ABOODH TRANSFORM

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Abstract: Nweton's Law of cooling explains the rate of cooling of a body. The rate at which an object cool down is directly proportional to the temperature difference between the object and its surroundings. Nweton's Law of cooling are generally analyzed by adopting Laplace transform method. The paper inquires the Nweton's Law of cooling by Aboodh transform technique. The purpose of paper is to prove the applicability of Aboodh transform to analyze Newton's Law of cooling.
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Introduction: The Aboodh Transform has been applied in different areas of science, engineering and technology. The Aboodh Transform is applicable in so many fields and effectively solving linear differential equations. Ordinary linear differential equation with constant coefficient and variable coefficient can be easily solved by the Aboodh Transform without finding their general solutions [1], [2], [3].Aboodh Transform has been applied in solving boundary value problems in most of the science and engineering disciplines [4], [5], [6], [7], [8], [9]. It also comes out to be very effective tool to analyze the network circuits with delta potential. The Newton's Law of cooling are generally solved by adopting Laplace transform method. In this paper, we present a new technique called Aboodh transform to analyze the Nweton's Law of cooling.[10], [11], [12], [13].

Basic Definition of Aboodh Transform:

If the function $\hbar(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Aboodh transform of $\hbar(y)$ is given by

$$A\{\hbar(y)\} = \bar{\hbar}(p) = \frac{1}{p}\int_0^{\infty} e^{-py}\,\hbar(y)dy.$$

The Aboodh Transform [1, 2, 3] of some of the functions are given by

- $A\{y^n\} = n!/p^{n+2}, where\ n = 0,1,2,..$
- $A\{e^{ay}\} = \frac{1}{p(p-a)},$
- $A\{sinay\} = \frac{a}{p(a^2+p^2)},$
- $A\{cosay\} = \frac{1}{a^2+p^2},$
- $A\{sinhay\} = \frac{a}{p(p^2-a^2)},$
- $A\{coshay\} = \frac{1}{p^2-a^2}.$
- $A\{\delta(t)\} = 1/p$

The Inverse Aboodh Transform of some of the functions are given by

- $A^{-1}\{p^{n+2}\} = n!/y^n$
  $n = 0,1,2,3,4\ldots$
- $A^{-1}\{\frac{1}{p(p-a)}\} = e^{ay}$
- $A^{-1}\{\frac{1}{p(a^2+p^2)}\} = \frac{1}{a}\sin ay$
- $A^{-1}\{cosay\} = \cos ay$
- $A^{-1}\{\frac{1}{p(p^2-a^2)}\} = \frac{1}{a}\sin hay$
- $A^{-1}\{\frac{1}{p^2-a^2}\} = \cos hay$

Aboodh Transform of Derivatives

The Aboodh Transform [1, 2, 3] of some of the Derivatives of h(y) are given by

$$A\{h'(y)\} = pA\{h(y)\} - h(0)/p$$
$$or\ A\{\hbar'(y)\} = p\bar{\hbar}(p) - \hbar(0)/p,$$
$$\{\hbar''(y)\} = p^2\bar{\hbar}(p) - \frac{\hbar'(0)}{p} - \hbar(0), \quad and\ so\ on$$

Material and Method:

Newton's Law of Cooling is called an ordinary differential equation that expects the cooling of a warm body sited in a cold environment. According to Newton's law of cooling, the rate of loss of heat from a body is directly proportional to the difference in the temperature of the body and its surroundings. [14],.In this paper, we present Transform transform technique to Newton's Law of cooling.

$$T' = -k(T - T_s) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathrm{I})$$

with initial condition as $\mathrm{T}(t_0) = T_0$ ………(II)

Where, T is the temperature of the object

T_s is the constant temperature of the surrounding,

k is Positive constant that depends on the area and nature of the surface of the body under consideration,

T_0 is the initial temperature of the object at time t_0

The negative sign of RHS in (1), indicate temperature of the body is decreasing with time and so the derivative $\frac{dT}{dt}$ must be negative.

From (I),

$$\dot{T} = -k(T - T_s)$$

Taking Aboodh Transform on both sides,

$$A\{\dot{T}\} = -A\{k(T - T_s)\}$$

$$A\{\dot{T}\} = -kA\{T(t)\} + kA\{T_s\}$$

$$pA\{T(t)\} - \frac{T(0)}{p} = -kA\{T(t)\} + kT_sA\{1\}$$

From (II), As $\mathrm{T}\ (t_0) = T_0$

Now, $pA\{T(t)\} - \frac{T_0}{p} = -kA\{T(t)\} + kT_s\frac{1}{p^2}$

$$(p + k)A\{T(t)\} = \frac{T_0}{p} + kT_s\frac{1}{p^2}$$

$$A\{T(t)\} = \frac{T_0}{p(p + k)} + kT_s\frac{1}{p^2(p + k)}$$

Taking inverse Aboodh Transform

$$\{T(t)\} = T_0e^{-kt} + kT_s - kT_se^{-kt}$$

$$T(t) = (T_0 - kT_e)e^{-kt} + kT_s$$

$$T(t) = Ce^{-kt} + kT_s \ldots\ldots\ldots\ldots\ldots\ldots (\mathrm{III})$$

$$where, C = (T_0 - kT_s)$$

While this function decreases exponentially, it approaches T_e as t⟶ ∞ instead of zero.

Application:

An apple pie with an initial temperature of 170^0 C is removed from the oven and left to cool with an air temperature 20^0C. Given that the temperature of the pie initially decreases at a rate of 3.0^0 C/min. How long will it take for the pie to cool to a temperature of 30^0 C? . [22]

Suppose the pie is in compliance with newton's cooling law; we have the following information

$$\dot{T} = -k(T - 20), T(0) = 170\ , \dot{T}(0) = -3.0$$

Where, T is the temperature of the pie in degree Celsius, T' is the time in minutes and k is an unknown constant.

Now, we will find the value of k by putting the given information we know about t = 0 directly into the differential equation:

$$-3 = -k(170 - 20)$$

$$k = 0.02$$

So, the differential equation can be written as

$$\dot{T} = -\frac{1}{50}(T - 20)$$

Taking Aboodh on both sides,

$$A\{\dot{T}\} = -\frac{1}{50}A\{(T(t) - 20)\}$$

$$A\{\dot{T}\} = -\frac{1}{50}A\{T(t)\} + \frac{2}{5}A\{1\}$$

$$pA\{T(t)\} - \frac{T(0)}{p} = -\frac{1}{50}A\{T(t)\} + \frac{2}{5p^2}$$

$$pA\{T(t)\} - \frac{T_0}{p} = -\frac{1}{50}A\{T(t)\} + \frac{2}{5p^2}$$

$$\left(p + \frac{1}{50}\right)A\{T(t)\} = \frac{170}{p} + \frac{2}{5p^2}$$

$$\left(p + \frac{1}{50}\right)A\{T(t)\} = \frac{170}{p} + \frac{2}{5p^2}$$

$$A\{T(t)\} = \frac{170}{p\left(p + \frac{1}{50}\right)} + \frac{2}{5p^2\left(p + \frac{1}{50}\right)}$$

$$A\{T(t)\} = \frac{170}{p\left(p + \frac{1}{50}\right)} - \frac{20}{p\left(p + \frac{1}{50}\right)} + \frac{20}{p^2}$$

$$M\{T(t)\} = \frac{150p^2}{\left(p + \frac{1}{50}\right)} + \frac{20}{p^2}$$

Taking inverse Aboodh on both sides, we get,

$$T(t) = 150e^{-\frac{1}{50}t} + 20 \ \ldots\ldots\ldots\ldots\ldots\ldots.(IV)$$

Putting T=30 in (IV), $30 = 150e^{-\frac{1}{50}t} + 20$

$$e^{-\frac{1}{50}t} = \frac{1}{15}$$

$$e^{\frac{1}{50}t} = 15$$

$$\frac{1}{50}t = \ln 15$$

$$t = 50\ \ln 15$$

$$t = 50 * 2.7080502011$$

$$t = 135.4\ minute$$

Hence, this will require 135.4 minutes for the pie to cool to a temperature of 30^0 C.

Conclusion: In this paper, we have successfully analyzed the Newton's Law of Cooling by Aboodh Transform. It may be finished that the technique is accomplished in analyzing the Newton's Law of Cooling. The applications presented demonstrate effectiveness of Aboodh Transform in the problems of Newton's Law of Cooling. The proposed scheme is widely in various field of Physics, Electrical engineering.

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