



CLASS 1 OF MATRICES: A CRITICAL STUDY

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Abstract: We represent each column of a certain matrix (order $m \times n$) in a well defined pattern of a n^{th} degree polynomial in a single variable; say x . This, in turn can help determine each column as being a m^{th} degree curve for each column. This particular convention casts the specified matrix of order $N \times n$ matrix as a set of n curves each of order N in a single adjustable. We determine that the square matrices could be classified according to several unique properties; right here it's the amount of each column entry. This particular amount - known as the Libra worth, plays a crucial role and remains at par in connection to the algebraic properties of its. Additional distinction is completed on introducing the attributes connected to algebraic sum to row entries. As we go on introducing the 'sum property, generally called Libra value', we get narrow down but get much better refinement and hence better convergence to the ultimate goal of ours of commutativity.

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Introduction

Hurley, Ted. (2006) It's shown that the number band RG of a team G of order n with a band R is actually isomorphic to a specific band of $n \times n$ matrices over R . If the band R comes with an identity component as well as no zero divisors, this particular representation allows us to describe the devices as well as zero-divisors of the team band in phrases of attributes of the matrices and exactly where appropriate in terms of the determinant of the matrices. The isomorphism extends to cluster rings of infinite organizations once the components of the team could be listed. The rings of matrices that turn up as isomorphic to particular team rings include things like Toeplitz, circulant, Walsh-Toeplitz matrices, Toeplitz matrices, or circulant matrices coupled with Block-Type circulant as well as hankel matrices matrices. Group rings hence could be regarded as to become a generalization of the rings of matrices, which happen in communications, signal processes, time-sequence analysis and anywhere else. When G is actually limited and R is actually an area, it makes sense out of the representation that $U(RG)$, the number of devices of RG , satisfies the Tits' alternative and consequently the generalized Burnside issue has a good solution for $U(RG)$. Zappa et al., (2014) We check out the subgroup system of the hyper octahedral cluster of 6 dimensions. Particularly, we examine the

subgroups isomorphic to the icosahedral team. We classify the orthogonal crystallographic representations of the icosahedral team and analyse their subgroups and intersections, using outcomes from graph theory as well as the spectra of theirs. Manoj Kumar et al., (2017) In this particular paper, we introduce Kannan, Chatterjea, Rhodes and Zamfirscu sort expansive mappings in the setting of generalized metric areas (js metric spaces). The outcomes proved in the setting of generalized metric areas generalizes the outcomes in metric spaces, dislocated metric spaces, b metric spaces, as well as modular spaces. We likewise illustrate the outcomes of ours with the assistance of particular examples. Bijendra Singh et al., (2006) The goal of this particular paper is actually introducing the notion of semi compatible maps in D metric areas and deduce fixed point theorems via semi compatibility by using orbital idea, which improve extend as well as generalize the outcomes of Ume as well as Kim [eight], Rhoades [seven Dhage and] et. al [six]. All of the outcomes of this particular paper are modern. Ahmed et al., (2016) The goal of this report is actually studying JS contractions also to build new frequent fixed point theorems for these contractions in the setup of total metric areas. Presented theorems are actually generalizations of the newest fixed point theorems as a result of Hussain et al. [Fixed Point Theory as well as Applications (2015) 2015:185]. A good

example is provided supporting our generalized result. Philip Saltenberger (2020) In that work some outcomes on the structure preserving diagonalization of person adjoint and skew adjoint matrices in indefinite inner merchandise areas are actually offered. Particularly, sufficient and necessary conditions on the symplecticdiagonalizability of (skew) Hamiltonian matrices as well as the perplecticdiagonalizability of per(skew) Hermitian matrices are supplied. To assume the organized matrix at hand is in addition natural, it's shown that any perplectic or symplectic diagonalization could constantly be built to be unitary. As a consequence of this reality, the presence of a unitary, structure preserving diagonalization is actually equivalent to the existence of a specially organized preservative decomposition of such matrices. The implications of this particular decomposition are illustrated by a number of examples.

MATRICES OF CLASS1

We classify square matrices in over 5 sessions based upon some pre-defined attributes in the components of provided leading, non-leading, row, and column diagonal component.

With this chapter, first property and hence top class is actually talked about. Lots of significant outcomes were derived focusing on matrix algebra. Member matrices of class1 form a mathematical framework of Ring underneath the standard operation matrix inclusion as well as matrix multiplication. Additionally, member matrices of class1 matrix represent as a set of polynomial and most of this polynomial intersects at one frequent point (1, L (A)).

Property1 (=P1)

Distinction of square matrices is completed on the foundation of constant sum' property in the entries of all columns / rows. Together with this we might think about the entries of top and / or maybe non leading diagonals.

We consider a square matrix $A = (a_{ij})_{n \times n}$ be a matrix on $R, \forall i = 1$ to n and $j = 1$ to n . where $n \in N$. If $\sum_{i=1}^n a_{ij} = \text{Constant}$ for each $j = 1, 2 \dots n$.

i.e. in case the amount of all of the entries of a column for every one of the columns of the specified matrix A, remains exactly the same genuine constant as opposed to the matrix is actually believed to fulfill the property P1.

LibraValue

Libra value of a certain category of matrices is the actual constant that is related with the property of a category. This's the top property

that is the determining property to a certain category on hand and which can help determine a class. Libra value is going to be denoted by the symbol L (A);

Where A is the given matrix, $L(A) \in R$.

$$E. G. A = \begin{pmatrix} 2 & 1 & -1 \\ -3 & 2 & 3 \\ 5 & 1 & 2 \end{pmatrix}$$

This specific matrix A satisfies property P1. [The amount of each column continues to be constant (= 4)]; which we call its libra worth. i.e. $L(A) = 4$

INTRODUCTION TO CLASSCJ1

A set of matrices which observe the property P1 constitutes class1; denoted as CJ1.

CJ1 = {A | A = (a_{ij})_{n×n}, A satisfies P1 and L (A) = p; p ∈ R for a given matrix A}

We denote, for the specified matrix A, the notation A CJ1 (m×n, p), the very first notation m×n' in the parenthesis denotes the order of the given matrix and also the 2nd notation shows the Libra worth of the given matrix.

[In the situation of square matrix, the order might be proven by a single letter i.e. rather than writing mm one can write 'm' only.]

The general format of matrix A of CJ1 is,

$$A = \left\{ \sum_{i=1}^n a_{ij} = p : j = 1, 2 \dots n \right\}$$

$$i. e A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{nn} \end{bmatrix}$$

And $\sum_{i=1}^n a_{ij} = p$ for each $j = 1$ to n

Here, n (n - 1) entries are arbitrary and can be set independently.

$$E. G. A = \begin{pmatrix} 2 & 1 & 1 \\ -3 & 2 & 3 \\ 4 & 0 & -1 \end{pmatrix}$$

The above mentioned matrix A is actually a square matrix for which the amount of all entries for every column is actually the same constant; this's the key property of the matrix. The root matrix A satisfies property P1.

The actual importance of the regular sum = 3 = L (A) is called the Libra value of the matrix. We denote this as $A \in CJ1 (3, L(A) = 3)$ or $A \in CJ1 (3, 3)$

Note: Following the definition of the class CJ1, we can write that the identity matrix

$= I_{m \times m}$, the null matrix $= O_{m \times m}$, the scalar matrix $A(a); a \in R$, are as a virtue of property P1, member of CJ1.

$$E. G. I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in CJ1 (3, L(I_3) = 1)$$

$$O_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in \text{CJ1} (3, L(O_3) = 1)$$

$$A_\alpha = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix} \in \text{CJ1} (3, L(A) = \alpha), \alpha \in \mathbb{R}$$

[What we require for the member matrix of class 1 i.e. CJ1, is that the matrix satisfies property P1.]

According to the general format of class 1 matrix given by defining property (1), for each fixed value of $p \in \mathbb{R}$ there exists a class of square matrices denoted $\text{aCJ1}(m, L(A)=p)$.

I.e. for each $p \in \mathbb{R}$, there corresponds infinite square matrices to a given class.

GENERAL STRUCTURE OF CLASSCJ1

Within this section we provide typical framework of class CJ1; this helps understand additional algebraic operation and deriving specific instances also.

General structure of 3×3 matrix of class CJ1 is as follows.

$$A = \begin{pmatrix} a & c & e \\ b & d & f \\ p - (a + b) & p - (c + d) & p - (e + f) \end{pmatrix}$$

Where all the letters used are real numbers.

We write $A \in \text{CJ1}(3, L(A) = p)$; where $p \in \mathbb{R}$.

There are lots of ways of expressing exactly the same matrix but with no loss of generality, we are going to follow the above mentioned like.

Also, we note that the columns of $A \in \text{CJ1}$ are Linearly Independent.

We also note that for a given fixed value of 'p – a real value' a given set of values either $\{a, c, e\}$ or $\{b, d, f\}$ --- components of any one set have to be noted for the actual worth of its.

i.e. the method, for a certain valuation of 'p' has 6 degree of freedom. (dof)

INTRODUCTION TO CLASSZ_L

Around this junction, we present a crucial point that leads to division of C

J1 ($m, L(A) = p$) where $p \in \mathbb{R}$.

We consider two cases; $L(A) = p = 0$ and $p \neq 0$

This makes a dichotomous classification of the class CJ1 ($m \times n, L(A) = p$)

For Libra value $L(A) = p = 0$, we've Zero Libra class as an infinite sub class of class CJ1 ($m \times n, p = 0$); we denote this class by the notation $Z_L(m \times n, p = 0)$

We denote an infinite sub-class when $L(A) = p \neq 0$. We shall denote this class by the notation **CJ11**.

Recalling here the class $Z_L(m \times n, p = 0)$; we have

$$\text{CJ1}(m \times n, p) = \text{CJ11}(m \times n, p \neq 0) \cup Z_L(m \times n, p = 0)$$

In case the matrices under consideration are actually square matrices well then, with no ambiguity as well as loss of generality we create; $\text{CJ1}(m, p) = \text{CJ11}(m, p \neq 0) \cup Z_L(m, p = 0)$

It is important to mention at this point that these classes CJ11 and Z_L are mutually disjoint.

$$\text{i.e. } \text{CJ11}(m \times n, p \neq 0) \cap Z_L(m \times n, p = 0) = \emptyset$$

These classes **CJ11** ($m \times n, p \neq 0$) and $Z_L(m \times n, p = 0)$ possess different structural properties with respect to different algebraic operations.

Zero Libra classZ_L

As a specific situation to the above mentioned note there's a situation when $L(A) = \text{zero}$ where A is actually a matrix fulfilling property P1. In the common form Eq. (1.2) when 'a + b = p' then the class is denoted as $Z_L(m \times n, 0)$ or simply $Z_L(m)$ if there is no ambiguity for the matrix A being a square one of order $m \times m$ and $L(A) = 0$.

E.g. consider a case of 3×3 square matrices.

$$A_1 = \begin{pmatrix} 3 & 4 & -1 \\ -4 & 1 & 1 \\ 1 & -5 & 0 \end{pmatrix} \in ZL(3, 0)$$

It is an important point to note that the null matrix O_3 is a member of this class Z_L .

$$\text{Thenullmatrix } O_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in Z_1(3, 0) \subset CJ_1(3, 0)$$

FUNDAMENTAL ALGEBRA OF CLASS 1

So as to hold out essential algebraic operations require present some definitions that will in tune with matrix algebra.

We think about 3 matrices of class 1 as follows.

$A = A(a_{ij}) \in CJ_1(n, L(A) = \alpha_1)$, $B = B(b_{ij}) \in CJ_1(n, L(B) = \alpha_2)$ and

$C = C(c_{ij}) \in CJ_1(n, L(C) = \alpha_3) \forall i$, and j from 1 to n . $n \in \mathbb{N}$.

We present some fundamental definitions.

- (1) Equality of 2 matrices of the class One
- (2) Matrix Addition
- (3) Multiplication of a matrix by a scalar

Equality of twomatrices:

2 matrices of the identical order as well as exact same class are actually identical in case and just when their corresponding entries are actually identical.

Let $A \in CJ_1(n, L(A) = \alpha)$ and $B \in CJ_1(n, L(B) = \alpha)$; $\alpha \in \mathbb{R}$

$A = B \implies a_{ij} = b_{ij} \forall i$ and $j = 1$ to n

This could hold true when both the matrices under consideration have exactly the same Libra worth.

$L(A) = L(B) = \alpha$.

[We remember that 2 matrices having identical libra values might not be the identical matrices; i.e. the converse might not be accurate in basic.]

Addition of TwoMatrices:

Let $A \in CJ_1(n, L(A) = \alpha_1)$ and $B \in CJ_1(n, L(B) = \alpha_2)$, $\alpha_1, \alpha_2 \in \mathbb{R}$ be the two matrices of the same class CJ_1 .

The result of Addition of matrices A and B denoted as $A + B$ is also a matrix,

Let $C = A + B$;

$C = A + B$ with $c_{ij} = a_{ij} + b_{ij} \forall i$, and j from 1 to n , $n \in \mathbb{N}$

$C(c_{ij}) = C = A + B \in CJ_1(n, L(C) = L(A) + L(B) = \alpha_1 + \alpha_2)$; which can be easily verified.

We conclude that 'Addition of two matrices'-- this property preserves the class.

$$\text{E.g. Let } A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 1 \\ 0 & 1 & 1 \end{pmatrix} \in CJ_1(3, 4) \text{ and } B = \begin{pmatrix} 5 & -1 & 6 \\ -1 & -1 & -3 \\ -2 & 4 & -1 \end{pmatrix} \in CJ_1(3, 2) \text{ then}$$

$C = A + B \in CJ_1(n, L(C) = L(A) + L(B))$

$$C = \begin{pmatrix} 8 & -2 & 8 \\ 0 & 3 & -2 \\ -2 & 5 & 0 \end{pmatrix} \in CJ_1(3, 6)$$

We have established that $L(A + B) = L(A) + L(B)$. (Refer to the Fig. 1.3)

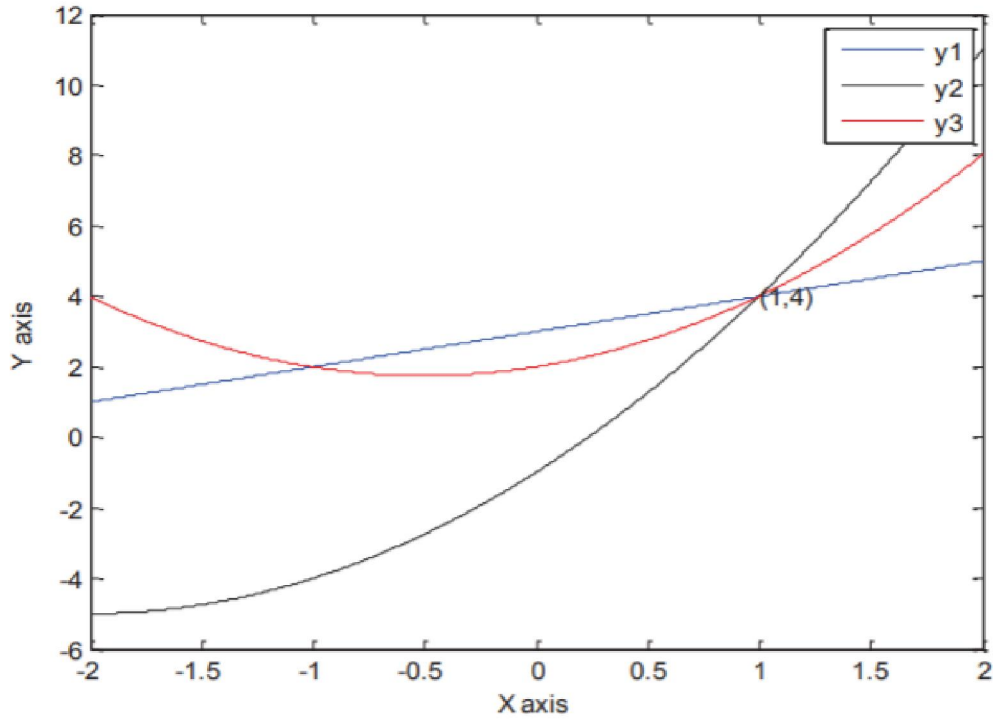


Fig. 1.1 Polynomials of Column vectors of Matrix A [Point of Concurrency is (1, 4) which is (1, L (A))]

It has a reference of device in which we think about every column entries representing a quadratic polynomial. This's reviewed below.

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 1 \\ 0 & 1 & 1 \end{pmatrix} \in \text{CJ1} (3, 4) \text{ then each}$$

column vector is a polynomial in real variable x .
 $Y_1 = 3 + x + 0x^2$, $Y_2 = -1 + 4x + 1x^2$, $Y_3 = 2 + x + 1x^2$
 For $x = 1$, all these quadratic curves have a common point of intersection (1, 4) i.e. (1, L (A)).

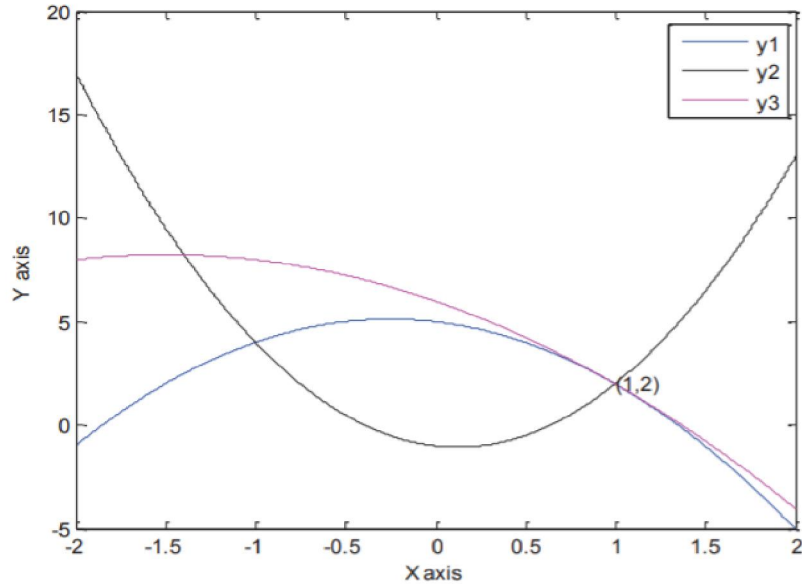
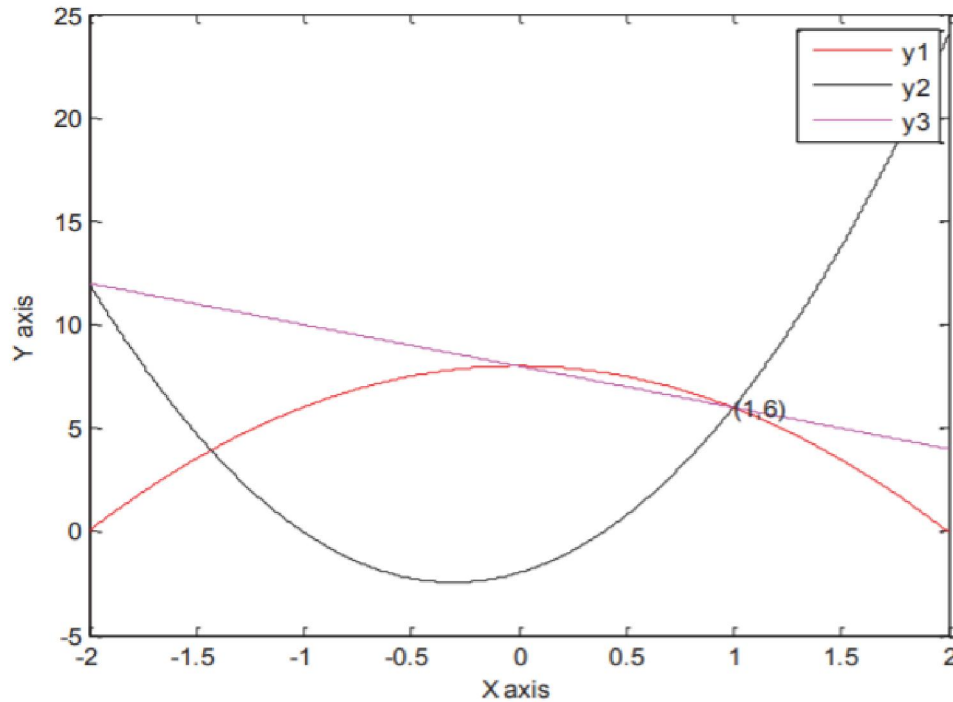


Fig. 1.2 Polynomials of Column vectors of Matrix B [Point of Concurrency is (1, 2) which is (1, L (B))]

Whenever we carry out the matrix addition, we find the next consequence.
 With the addition of corresponding column components of matrix A and matrix B, the

resulting quadratic polynomial is actually the amount of 2 corresponding quadratic polynomials with reference to the matrix A as well as the matrix B.



**Fig. 1.3 Addition of Two Matrices A+B polynomial
 [Point of intersection is at (1, 6) which is (1, L (A) + L (B))]**

From this graph and above shown two graphs, we conclude that

$$L(A) + L(B) = L(A + B).$$

Multiplication by Scalar:

Let $A \in CJ1(n, L(A)=\alpha)$ then for some $k \in \mathbb{R}$; the product of A by the scalar k denoted as kA is defined as follows.

Let $A = (a_{ij})$ then $kA = (ka_{ij}) \forall i$ and j be a matrix of class CJ1.

It can be seen that $L(kA) = k.L(A)$ and hence $kA = (n, L(k\alpha)) \in CJ1$

We note that multiplication by a scalar is a class preserving property.

If $k = -1$ then the matrix $-1A$ will be denoted as $-A$; which is also a member of class CJ1; $-A \in CJ1(n)$.

$$\text{Let } A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 1 \\ 0 & 1 & 1 \end{pmatrix} \in CJ1(3, 4) \text{ and } 2A = \begin{pmatrix} 6 & -2 & 4 \\ 2 & 8 & 2 \\ 0 & 2 & 2 \end{pmatrix} \in CJ1(3, 2 \times 4 = 8)$$

THE CLASS CJ1 -- AN ABELIAN GROUP

With this section, we verify all of the

properties which are essential to establish that the set (G,) is actually an Abelian team under the functioning matrix inclusion. We think about 3 matrices A, B, and C as outlined in the prior device.

Binary Operation:

As we've seen and could be quickly verified that the outcome of inclusion of 2 matrices of exactly the same order of class CJ1 is once again a matrix of the very same category.

i.e. For the 2 member matrices A, and B of class CJ1, the matrix derived on addition that's the matrix $A + B \in CJ1$.

Hence we conclude that '+' is a binary operation on the member of the set CJ1.

Associative Property:

Now we check for associative property on the member matrices of the class CJ1.

Let $A = A(a_{ij}) \in CJ1(n, L(A)=\alpha_1)$, $B = B(b_{ij}) \in CJ1(n, L(B)=\alpha_2)$ and

$C = C(c_{ij}) \in CJ1(n, L(C)=\alpha_3)$ for $\forall i$, and j from 1 to $n \in \mathbb{N}$.

As the entries a_{ij} , b_{ij} , and c_{ij} are real numbers and

associative property for the operation + holds true in the set R;

$$i.e. a_{ij} + (b_{ij} + c_{ij}) = (a_{ij} + b_{ij}) + c_{ij}$$

Which holds true for all natural numbers i and j; as a consequence, to this we can write $A + (B + C) = (A + B) + C$;

Thus associative property for binary operation addition holds true.

In addition to the above property we have a strong result about the libra value property.

$$L(A + (B + C)) = L((A + B) + C) = L(A) + L(B) + L(C)$$

Existence of Identity:

Let A_1 be a given matrix $A_1 \in CJ1(3, p)$ then there exists a matrix

$$O_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in CJ1(3 \times 3, L(+,) = 0)$$

so that $A_1 + O_3 = O_3 + A_1 = A_1$;

The matrix O_3 is a null matrix. We call this null matrix O_3 as identity matrix for the class CJ1.

Existence of Additive Inverse:

As a virtue of the outcome of the property 'multiplication by a scalar' mentioned previously, we determine this for every $A \in CJ1(n, L(A) = p)$, there exists exactly one matrix $B \in CJ1(n, L(B) = -p)$ such that $A + B = B + A = O$ and $O \in CJ1(m \times n, L(O) = 0)$

We call this matrix as an additive inverse of the matrix A and denote it as $-A$.

i.e. $A + (-A) = -A + A = O$ ----- a null matrix.

For the matrix $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 1 \\ 0 & 1 & 1 \end{pmatrix} \in CJ1(3, 4)$,

there exists $-A = -1A = \begin{pmatrix} -3 & 1 & -2 \\ -1 & -4 & -1 \\ 0 & -1 & -1 \end{pmatrix} \in$

CJ1(3, -4), such that

$A + (-A) = -A + A = O$; these matrices are additive inverses of each other.

** We conclude from all points discussed above unit 1.6.1 to 1.6.4 that the set—the class CJ1 with the binary operation '+'; i.e. **(CJ1, '+') is a group.**

Commutative Property:

We have already established that **(CJ1, '+') is a group** and now we check the group for commutative property.

Let $A = A(a_{ij}) \in CJ1(n, L(A) = \alpha_1)$, $B = B(b_{ij}) \in CJ1(n, L(B) = \alpha_2)$ with a_{ij} and b_{ij} real numbers.

We realize that the set R of numbers that are real observes commutative property for regular operation '+', i.e. $a_{ij} + b_{ij} = b_{ij} + a_{ij}$ holds true.

By the virtue of this property we write that for the matrices A, and B of class CJ1,

$$A + B = B + A$$

We conclude that **(CJ1, '+') is a Commutative Group An Abelian Group.

Important Notes:

We have already defined the **Zero Libra** class— Z_L ; it's a set of all the square matrices for which libra great is actually 0. It's a noteworthy thing that the null Matrix O_3 is as well a part of this particular category.

Furthermore, there are matrices with a few non zero entries likewise which fall in this particular category.

We remember the normal type of class 1.

$$A = \begin{pmatrix} a & c & e \\ b & d & f \\ p - (a + b) & p - (c + d) & p - (e + f) \end{pmatrix}$$

For this matrix to be a member of the class Z_L , we need to have the libra value $p = 0$

For any real values a and b, for $p = 0$ we have $p - (a + b)$ such that

$a + b + (p - (a + b)) = 0$ [Precisely the same arguments stand correct for the next as well as the third column.] With exactly the same kind of rational arguments, it could be started that the category Z_L is a commutative team.

We provide numerical illustrations and escape its abstract strategy.

Let $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 1 \\ -4 & -3 & -3 \end{pmatrix} \in Z_L(3, 0)$, $B = \begin{pmatrix} 5 & -1 & 6 \\ -1 & -1 & -3 \\ -4 & 2 & -3 \end{pmatrix} \in Z_L(3, 0)$,

It can be seen that the resultant matrix $A + B = \begin{pmatrix} 8 & -2 & 8 \\ 0 & 3 & -2 \\ -8 & -1 & -6 \end{pmatrix} \in \text{CJ1}(3, 0)$,

Also on the set of sequential arguments that,

- (1) The associative property holds true in Z_L
- (2) The Null Matrix is an Identity matrix
- (3) For a given matrix A, the matrix $-A = -1A$ is its additive inverse.
- (4) On the same lines we can establish that commutative property for addition operation also holds true.

All the above points guide us to conclude that ----
The Zero Libra Class Z_L is an Abelian group.

Around this junction, it's crucial that you be aware the following areas.

It's noteworthy that the set of matrices of class I under average binary operation '+' forms an Abelian group.

i.e. $\text{CJ1}(m \times n, p)$ is a team under the binary operation inclusion of matrices.

[We have extended the idea out of square matrices to rectangular matrices and

followed with exactly the same operation'; it preserves the setup of its for an Abelian team]

The infinite category Z_L , which is currently developed to become an Abelian group, is actually a subset of the class CJ1 .

i.e. $Z_L \subset \text{CJ1}$

This is a very important property.

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