New York Science Journal

Websites:
http://www.sciencepub.net/newyork
http://www.sciencepub.net

Emails:
newyorksci@gmail.com
editor@sciencepub.net

MARSLAND PRESS
Multidisciplinary Academic Journal Publisher

Stability Analysis of LS Estimator for Comb-Type and Block-Type Channel Estimation in OFDM System

Latif Ullah Khan, M. Irfan Khattak, Naeem Khan, Faheem Khan and M. Shafi

University of Engineering & Technology, Peshawar, Pakistan
latifullahkhan@nwfpuet.edu.pk

Abstract: This paper deals with the comparative analysis of the performance of LS estimator for comb-type and block-type channel estimation in Orthogonal Frequency Division Multiplexing (OFDM). The effect of increase in channel order on the performance of the LS estimator for both comb-type and block-type channel estimation is studied through simulation and mean square error expression (MSE). MSE expression for both comb-type and block-type channel estimation, incorporating the effect of the channel order on the performance, is derived and it is found that the performance of the LS estimator for block-type channel estimation remains stable for an increase in channel order which is indicated by the MSE expression and simulation results. On the other hand, the performance of LS estimator for comb-type channel estimation degrades with an increase in channel order.

[Chih-Cheng Chen , Yung-Tan Lee, Chung-Ming Tsai. **Professional Baseball Team Starting Pitcher Selection Using AHP and TOPSIS Methods.** *N Y Sci J* 2021;14(10):41-50] ISSN 1554-0200 (print); ISSN 2375-723X (online) http://www.sciencepub.net/newyork. 7. doi:10.7537/marsnys141021.07.

Keywords: OFDM, LS Estimator, comb-type channel estimation, block-type channel estimation

1. Introduction

Modern communication standards are using Orthogonal Frequency Division Multiplexing (OFDM) because of its inherent qualities: the spectral efficiency and effective cancellation of the Inter Symbol Interference (ISI) using simply the guard interval. OFDM is adopted by 802.11 based wireless LANs, Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB) [1-3]. In OFDM, conversion of the available bandwidth into smaller sub channels is carried out in order to transform the frequency selective fading channel into a number of flat fading channels. The better spectral efficiency of the OFDM is due to the overlapping nature of the orthogonal subcarriers in the frequency domain. Single carrier communication systems employ complex equalization techniques for ISI cancellation; however, in OFDM guard interval is used for ISI mitigation [4].

Estimating the channel state information effectively with low pilot overhead is a challenging task in OFDM. Pilots can be inserted into OFDM symbols according to two dimensional (2-D) channel estimation or one dimensional (1-D) channel estimation. 2-D channel estimation outperforms 1-D channel estimation at the cost of high computational complexity [5]. The 1-D pilot insertion has two categories: (a) Block-type (b) Comb-type. Comb-type channel estimation is used for fast fading channel where the channel impulse response variations are fast while block-type channel estimation is used for slow fading channels where the channel impulse response variations are slow and it remains constant for a block period of time [5]. Blind, semi-blind and pilot assisted channel estimation strategies are three categories of channel estimation based on the density of pilots. In Blind channel estimation, the channel frequency response at the pilot subcarriers is estimated using the received signal statistics. The performance of blind channel estimation can be enhanced using pilots. In pilot assisted channel estimation, pilot symbols are used for estimation of the channel frequency response at pilot subcarriers.

In [6], a robust modified Minimum Mean Square Error (M-MMSE) estimator is proposed which is based on the alterations in the observation matrix. The performance of the M-MMSE estimator is compared with MMSE estimated over the fast fading Rayleigh channel for different channel orders. The performance of the M-MMSE estimator remains stable for an increase in channel order; however, the performance of MMSE estimator degrades with an increase in channel order. The performance of M-MMSE is satisfactory; however, the computational complexity associated with M-MMSE estimator makes it unattractive for practical implementation. In [7], the performance of LS estimator is studied for comb-type and block-type channel estimation. The estimation of the channel frequency response at data tones is performed using one dimensional interpolation techniques such as low pass, spline, linear, second order and time domain interpolation.

The performance of LS estimator for comb-type channel estimation has been evaluated over Rayleigh fading channel for one dimensional interpolation techniques. The low pass interpolation technique has the optimum performance of the other one dimensional interpolation techniques.

Mahmoud *et al* compared the performance of LS and Kalman Filter for comb-type pilot arrangement over fast fading Rayleigh channel [8]. The performance of both estimators for different one dimensional interpolation is also studied. Low pass has better performance than other one dimensional interpolation techniques for both estimators. Comparative analysis of LS and Kalman filter shows the performance improvement of Kalman filter over LS estimator. In [9], the MMSE estimator based on the Discrete Fourier Transform (DFT) channel estimation is proposed which uses the time domain channel impulse response for computation of noise variance. The channel auto correlation matrix is computed using the time domain channel impulse response after the suppression of the channel noise. The performance comparison between the proposed MMSE, ideal MMSE and LS estimator illustrates the performance improvement of proposed MMSE over LS and degraded performance than the ideal MMSE estimator.

In this paper, the comparative analysis of LS estimator for block-type and comb-type channel estimation has been carried out. The effect of increase in channel order on the performance of LS estimator is investigated for block-type and comb-type channel estimation. Mean Square Error (MSE) expression is developed for the LS estimator for both comb-type and block-type channel estimation. The derived MSE expression also validates the simulation results carried out for the LS estimator for both block-type and comb-type pilot insertion techniques. The channel frequency response at data tones in case of comb-type pilot arrangement has been estimated using one dimensional interpolation techniques i.e. low pass, linear and spline. The performance of LS estimator for comb-type pilot arrangement with different 1-Done dimensional interpolation techniques degrades with an increase in channel order. However, the performance of LS estimator for block-type pilot arrangement remains stable for an increase in channel order.

The rest of the paper is organized as follows: section II introduces the OFDM system model, Channel estimation strategies are discussed in section III, MSE analyses are presented in section IV, Simulation results and analysis are presented in section V, and section VI concludes the paper.

Notations: Matrices are denoted by bold face italics upper case letters and vectors are denoted by italics lower case letters. Superscripts ^{H} and ^{T} are used denote Hetmitian transpose and Transpose respectively.

2. OFDM System Overview

In an OFDM system (illustrated in Fig. 1), the input serial sequence of G(where G is considered integer multiple of subcarriers per OFDM symbol in order to avoid bits padding) bits $\boldsymbol{x} = [x(0), x(1), x(2), x(3), \dots, x(G-1)]$ is mapped to produce the frequency domain sequence of symbols. The serialized frequency domain sequence of symbols is parallelized and divided into groups of size N (where N is the number of subcarriers per OFDM symbol) for block-type channel estimation. The size of groups of comb-type channel estimation is N_d(where $N_d$is the number of subcarriers reserved for data per OFDM symbol). The i^{th} group of symbols for block-type channel estimation is $\boldsymbol{s_B}^{(i)} = [s_B{}^{(i)}(0), s_B{}^{(i)}(1), s_B{}^{(i)}(2), \dots, s_B{}^{(i)}(N-1)]^T$ and $\boldsymbol{s_C}^{(i)} = [s_C{}^{(i)}(0), s_C{}^{(i)}(1), s_C{}^{(i)}(2), \dots, s_C{}^{(i)}(N_d-1)]^T$ for comb-type channel estimation. The pilot symbols are inserted into the dedicated subcarriers in every OFDM symbol for comb-type channel estimation to yield $\boldsymbol{d_C}^{(i)} = [d_C{}^{(i)}(0), d_C{}^{(i)}(1), d_C{}^{(i)}(2), \dots, d_C{}^{(i)}(N-1)]^T$and at all subcarriers in the first OFDM symbol of the block for block-type channel estimation to yield $\boldsymbol{d_B}^{(i)} = [d_B{}^{(i)}(0), d_B{}^{(i)}(1), d_B{}^{(i)}(2), \dots, d_B{}^{(i)}(N-1)]^T$. The signal is transformed into the time domain after pilot insertion to yield $\boldsymbol{u_C}^{(i)} = [u_C{}^{(i)}(0), u_C{}^{(i)}(1), u_C{}^{(i)}(2), \dots, u_C{}^{(i)}(N-1)]^T$ and $\boldsymbol{u_B}^{(i)} = [u_B{}^{(i)}(0), u_B{}^{(i)}(1), u_B{}^{(i)}(2), \dots, u_B{}^{(i)}(N-1)]^T$ for comb-type and block-type channel estimation respectively:

$$\boldsymbol{u_C}^{(i)} = \boldsymbol{F^H} \boldsymbol{d_C}^{(i)} \qquad (1)$$

$$\boldsymbol{u_B}^{(i)} = \boldsymbol{F^H} \boldsymbol{d_B}^{(i)} \qquad (2)$$

Where $[\boldsymbol{F}]n, n = \frac{1}{\sqrt{N}} e^{-j2\pi(n-1)(n-1)/N}$ for n=0,1,2,…, N-1is the Discrete Fourier Transform (DFT) Matrix. Cyclic prefix of length $L_G$greater than or equal to the channel order is inserted into the OFDM symbol for effective cancellation of ISI. The cyclic prefix is added by the insertion of last $L_G$symbols at the start of the OFDM symbol. The cyclic prefix added OFDM symbol for block-type and comb-type channel estimation is $\boldsymbol{m_B}^{(i)} = [m_B{}^{(i)}(-L_G + N), m_B{}^{(i)}(-L_G +$

$N+1),\dots,m_B{}^{(i)}((N-1)),$ $m_B{}^{(i)}(0),m_B{}^{(i)}(1),\dots,m_B{}^{(i)}(N-1)]^T$ and $\boldsymbol{m_C}{}^{(i)} =$ $[m_C{}^{(i)}$ $(-L_G+N),m_C{}^{(i)}(-L_G+N+1),\dots,m_C{}^{(i)}((N-1)),$ $m_C{}^{(i)}(0),m_C{}^{(i)}(1),\dots,m_C{}^{(i)}(N-1)]^T$ respectively. Finally, the OFDM signal passes through the multipath fading channel in the existence of AWGN. The received signal is transformed into frequency domain after the removal of the cyclic prefix.

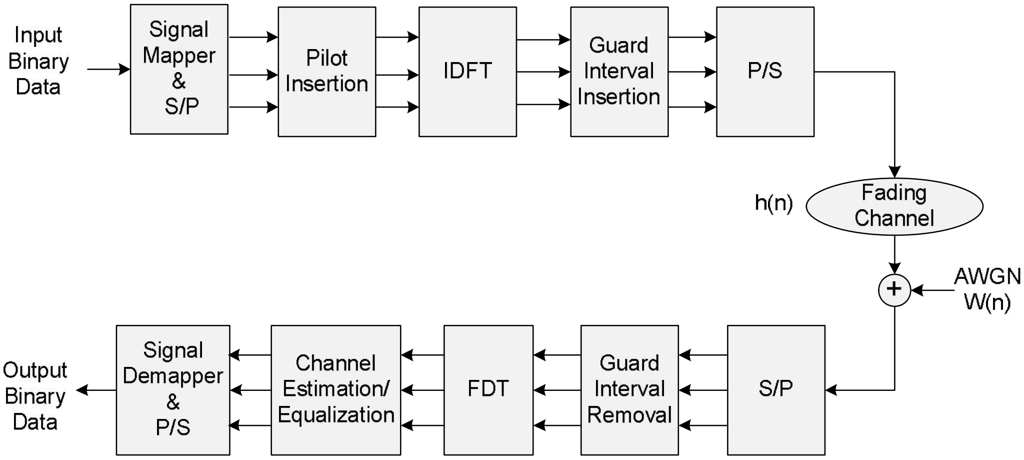


Figure 1. OFDM system Model

$$\boldsymbol{r_C}{}^{(i)} = \boldsymbol{F^H}\boldsymbol{\hat{u}_C}{}^{(i)} \tag{3}$$

$$\boldsymbol{r_B}{}^{(i)} = \boldsymbol{F^H}\boldsymbol{\hat{u}_B}{}^{(i)} \tag{4}$$

The channel frequency response is estimated and equalization is performed. The signal is then demapped to produce the output bits. The performance of the system is evaluated in terms of Bit Error Rate (BER) Vs Signal to Noise Ratio (SNR) plot.

3. Channel Estimation Algorithm

3.1 Channel Estimation based on Block-Type Pilot Arrangement

In block-type channel estimation, the pilot symbols are inserted into all subcarriers in the first symbol of the block for which the channel frequency response remains constant. The arrangement of pilots in comb-type and block-type channel estimation is shown in Fig. 2. The estimated channel frequency response for the first symbol is then used for equalization of the subsequent symbols. Estimation of the channel frequency response at pilot tones is performed by LS estimator. The LS estimator is based on minimization of the square of the difference between detection and estimation. The LS estimate of the channel is given by [10]:

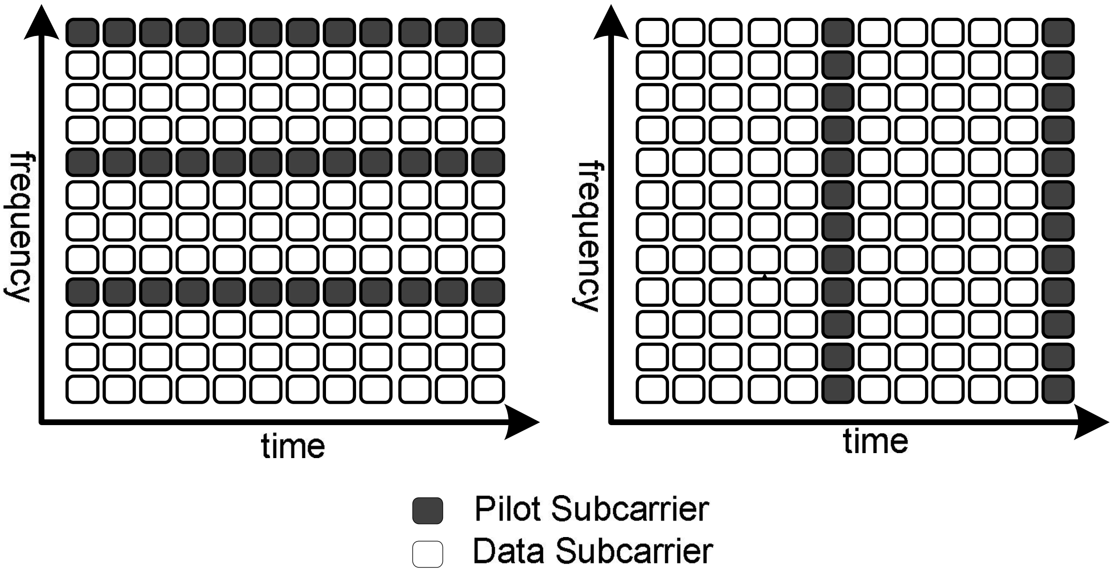
$$\boldsymbol{h_{LS}} = \boldsymbol{y}\boldsymbol{x}^{-1} \tag{5}$$

Where $\boldsymbol{x}$ and $\boldsymbol{y}$ are the vectors containing the transmitted and received pilots respectively. In block-type channel estimation, the pilot symbols are transmitted at all subcarriers only in the first symbol because the channel impulse response remains constant for a block period of time in case of slow fading channels. The overhead associated with block-type channel estimation in the case when the block consists of seven OFDM symbols with the first symbol consisting entirely of pilots is 14.28%. The overhead associated with comb-type channel estimation for fast fading channels with pilot ratio of 1/8 is also 14.28% which is equal to the block-type case previously considered. The performance of the block-type for slow fading channels is better because of the avoidance of the interpolation step which tends to add interpolation error [11].

3.2 Channel Estimation based on Comb-Type Pilot Arrangement

In comb-type channel estimation, pilot tones are inserted into the dedicated subcarriers as illustrated in Fig. 2. Optimum performance is achieved by equally spaced pilot insertion [12]. For equally spaced pilots, the number of subcarriers divided by the total number of pilots per OFDM symbols should result in an integer value. The channel at pilot tone is estimated using the LS estimator. At data tones, the channel frequency response is computed using one dimensional interpolation techniques due to their low computational complexity as compared to two dimensional interpolation techniques [5]. Low pass, Linear and Spline interpolation techniques are considered for estimation of the channel frequency response at data subcarriers. In low pass interpolation, the resulting sequence after zero padding is passed through the Low pass Finite Impulse Response (FIR) filter. Drawing of smooth curves using several points is the basis of spline cubic interpolation. Linear interpolation uses two consecutive pilots for computation of the channel impulse response at data tones located between the considered pilot tones. Linear interpolation outperforms piece wise interpolation [13]. The channel frequency response estimated at K subcarrierwhere, $dL \le K < (d+1)L$, (where L is the number of subcarriers divided by the total pilots) using linear interpolation is given by [7]:

$$\boldsymbol{h}(k) = \boldsymbol{h}(dL+l) = \left(1-\frac{l}{L}\right)\boldsymbol{h}_{LS}(d) + \frac{l}{L}\boldsymbol{h}_{LS}(d+1)$$
$$= \boldsymbol{h}_{LS}(d) + \frac{l}{L}\left(\boldsymbol{h}_{LS}(d+1) - \boldsymbol{h}_{LS}(d)\right), 0 \le l < L \quad (6)$$



(a) Comb-Type Channel Estimation (b) Block-Type Channel Estimation

Figure 2. Pilot Arrangement in comb-type and block-type channel estimation

4. Mean Square Error Analysis

In this section, the Mean square Error expression is derived for LS estimation based on the comb-type pilot arrangement. The mean square error (MSE) at a particular subcarrier K is given by:

$$MSE[k] = E[|FFT\{h[n] - h_{LS}[n]\}|^2] \quad (7)$$

Where $h[n]$and $h_{LS}[n]$are the actual channel impulse response and estimated channel impulse response in the time domain. As the LS estimator estimates the channel impulse response in such a way that it does not consider the channel noise in the estimation process. Therefore, the estimated channel impulse response by LS estimator consists of the channel impulse response and noise:

$$h_{LS}[n] = h[n] + \tilde{v}[n] \quad (8)$$

Where $\tilde{v}[n] = \frac{v[n]}{c[n]}$is the noise vector divided by the transmitted pilots vector. (7) after putting the value of $h_{LS}[n]$becomes:

$$MSE[k] = E[|FFT\{h[n] - h[n] - \tilde{v}[n]\}|^2] \qquad (9)$$

$$MSE[k] = E[|FFT\{\tilde{v}[n]\}|^2] \qquad (10)$$

$$.MSE[k] = E[|\{E[| \textstyle\sum_{w=0}^{L} \tilde{v}[w]\, e^{-\frac{j2Kw}{P}} |^2] \qquad (11)$$

$$MSE[k] = E[|\{\textstyle\sum_{w=0}^{L} \sum_{f=0}^{L} \tilde{v}[w]\, \tilde{v}^H[f]\, e^{-\frac{j2K(w-f)}{P}} \qquad (12)$$

$$MSE[k] = \textstyle\sum_{w=0}^{L} \sum_{f=0}^{L} \tilde{v}[w]\, \tilde{v}^H[f]\, e^{-\frac{j2\pi K(w-f)}{P}} \qquad (13)$$

$$MSE[k] = \textstyle\sum_{w=0}^{L} \sum_{f=0}^{L} E[\tilde{v}[w]\, \tilde{v}^H[f]]\, e^{-\frac{j\pi 2K(w-f)}{P}} \qquad (14)$$

$$MSE[k] = P * \ E[\tilde{v}^H[f]\tilde{v}[w]] \qquad (15)$$

$$MSE[k] = P * \left(\frac{1}{P}\textstyle\sum_{K_1}^{P-1} E\left[\frac{V[k_1]}{X[k_1]}\right] e^{-\frac{j2\pi k_1 n}{P}}\right) \mathrm{x} \left(\frac{1}{P}\textstyle\sum_{K_2}^{P-1} E\left[\frac{V[k_2]}{X[k_2]}\right] e^{-\frac{j2\pi k_2 n}{P}}\right)^H \qquad (16)$$

$$MSE[k] = L * \frac{P}{P^2} {\sigma_p}^2 E\left[\left|\frac{1}{X[k_1]}\right|^2\right] \qquad (17)$$

$$MSE[k] = \frac{L}{P}\frac{\beta}{SNR} \qquad (18)$$

Where β is the constellation factor.

5. Simulation Results and Analysis

This section presents the MATLAB® simulation results for performance evaluation of the LS estimator for comb-type and block-type pilot arrangement. Simulation parameters are given in Table 1. The channel used for performance evaluation consists of L independent taps with zero mean and Gaussian distribution. Constant and exponential power delay profiles are used. The variance of each tap for an exponential power delay profile is given by:

$${\sigma_l}^2 = e^{-\frac{l}{35}} \quad l = 0,1,2 \ldots L - 1 \qquad (19)$$

Table (1): Simulation Parameters

Parameters	Specifications
Number of subcarriers, N	512
FFT size	512
Pilot ratio	1/8
Channels	Rayleigh Fading Channel
Cyclic Prefix Length	1/8 i.e.32
Channel Estimation	Comb-Type , Block-Type
Digital Modulation	BPSK, QPSK, 16-QAM, 64-QAM
Power Delay Profiles	Exponential, Constant

Fig. 3 shows the frame structure used for simulation of the OFDM system in case of block-type channel estimation. Each frame consists of seven symbols with the first symbol containing the pilots. The channel frequency response estimated for the first symbols is then used for equalization of the subsequent symbols in a block.

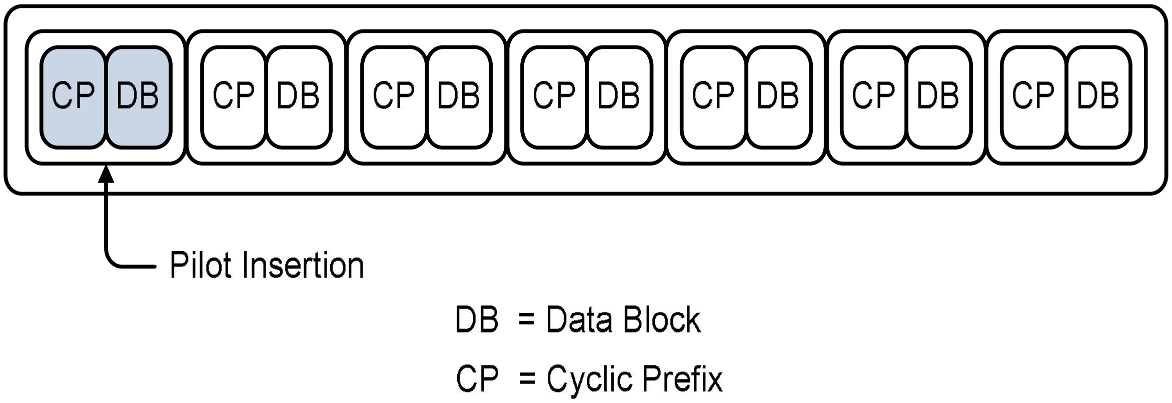


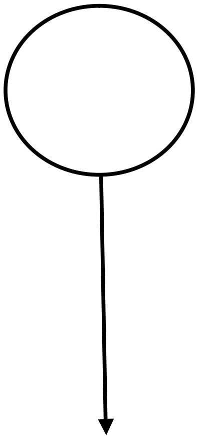
Figure 3. Simulated frame structure [14]

5.1 Performance Evaluation for comb-type Channel Estimation

The performance of the OFDM system for comb-type pilot arrangement with LS estimator used for estimation of the channel frequency response at the pilot subcarriers is illustrated in Fig. 4 for channel order L= 15. The performance of the LS estimator for one-dimensional interpolation techniques follows the following order from best to worst: Low pass, Spline and linear. The performance order of this paper also matches with the performance order in [7][8]. The performance of the OFDM system for comb-type channel estimation for channel order L=20 and L=30 is illustrated in Fig. 6 and Fig. 7. Comparison of the Fig. 6 and Fig. 7 clearly shows the performance degradation caused by the increase in channel order on the performance of the OFDM system with comb-type channel estimation. The reason for this performance degradation is due to increase in the distortion introduced by the channel with an increase in channel order. These results are also clear from the MSE expression. Clearly, the increase in the channel order L results in increase in MSE for the particular subcarrier. The channel frequency response is estimated at the data subcarriers using an interpolation technique. The interpolation error also increases with an increase in channel order as the channel order causes an increase in MSE at the pilot subcarrier which subsequently increases the interpolation error.

5.2 Performance Evaluation for Block-type Channel Estimation

The legends ‘LS-L=20’ and ‘LS-L=80’ are used to denote the performance of the LS estimator for channel order 20 and 80 respectively. The performance of the OFDM system with block-type channel estimation for different channel orders is shown in Fig. 8. The performance is shown for both constant and exponential power delay profiles. It is clear from Fig. 8 that the performance of the OFDM system is same for both constant and exponential power delay profiles for block-type channel estimation.



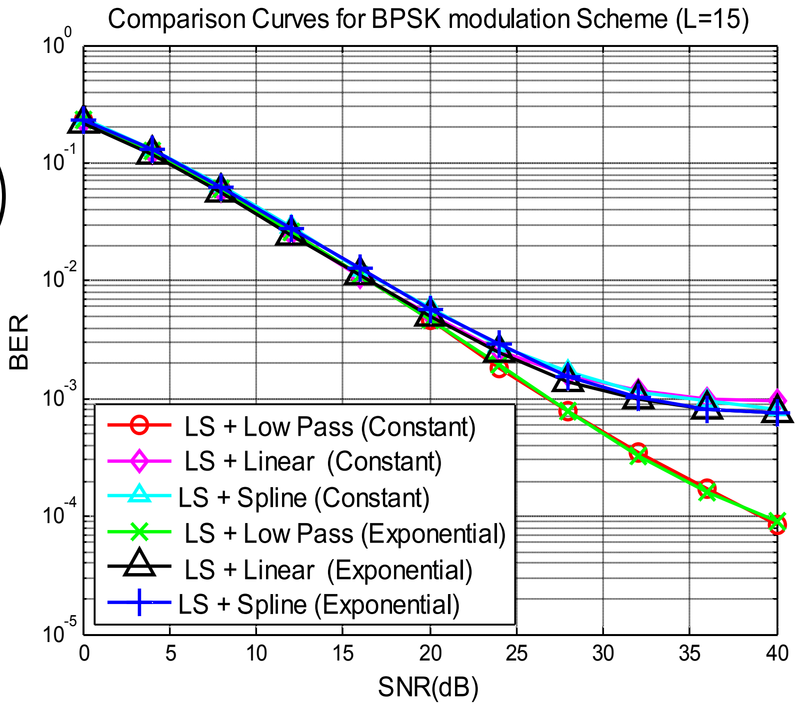


Figure 4.Performance of the OFDM System for comb-type channel estimation over Rayleigh fading channel with L=15

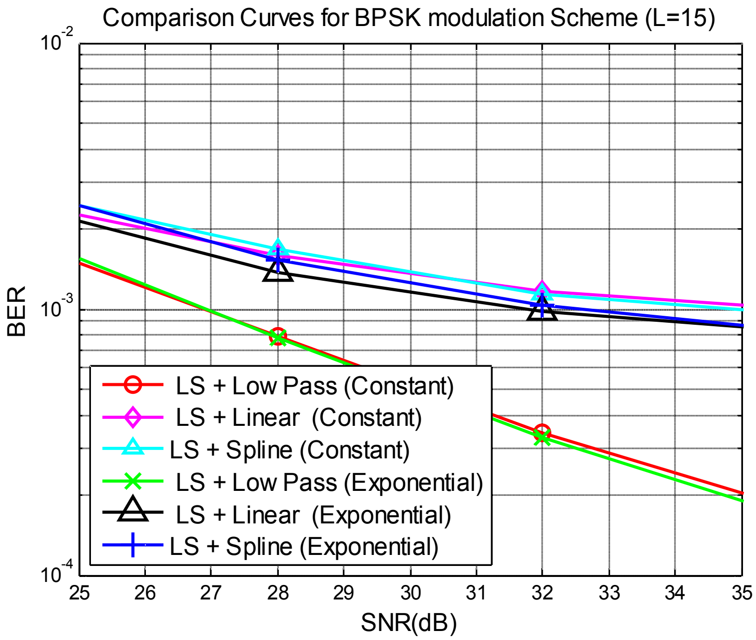


Figure 5. Performance of the OFDM System for comb-type channel estimation over Rayleigh fading channel with L=15

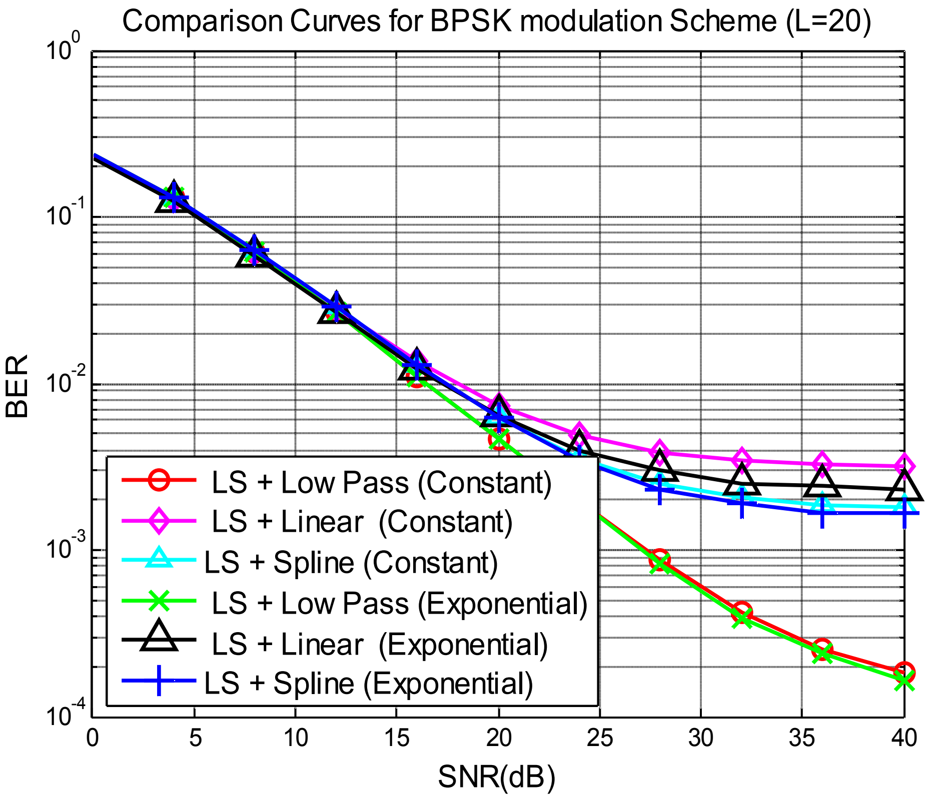


Figure 6. Performance of the OFDM System for comb-type channel estimation over Rayleigh fading channel with L=20

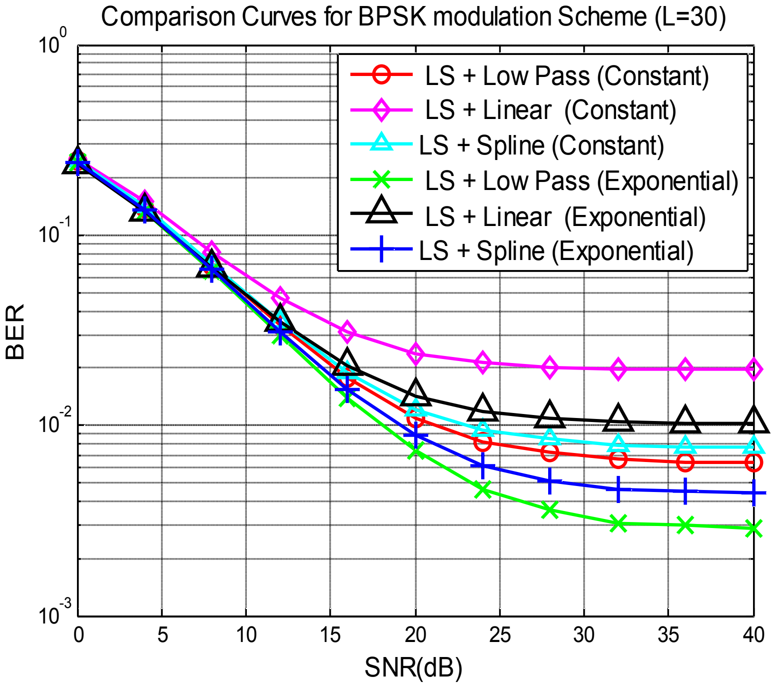


Figure 7. Performance of the OFDM System for comb-type channel estimation over Rayleigh fading channel with L=30

The MSE expression for LS estimator in case of block-type pilot arrangement is given by [15]:

$$MSE = \frac{\beta}{SNR} \qquad (20)$$

(20) clearly shows that the MSE of the LS estimator for block-type channel estimation is independent of the channel order and thus, has the performance stability against the increase in channel order.

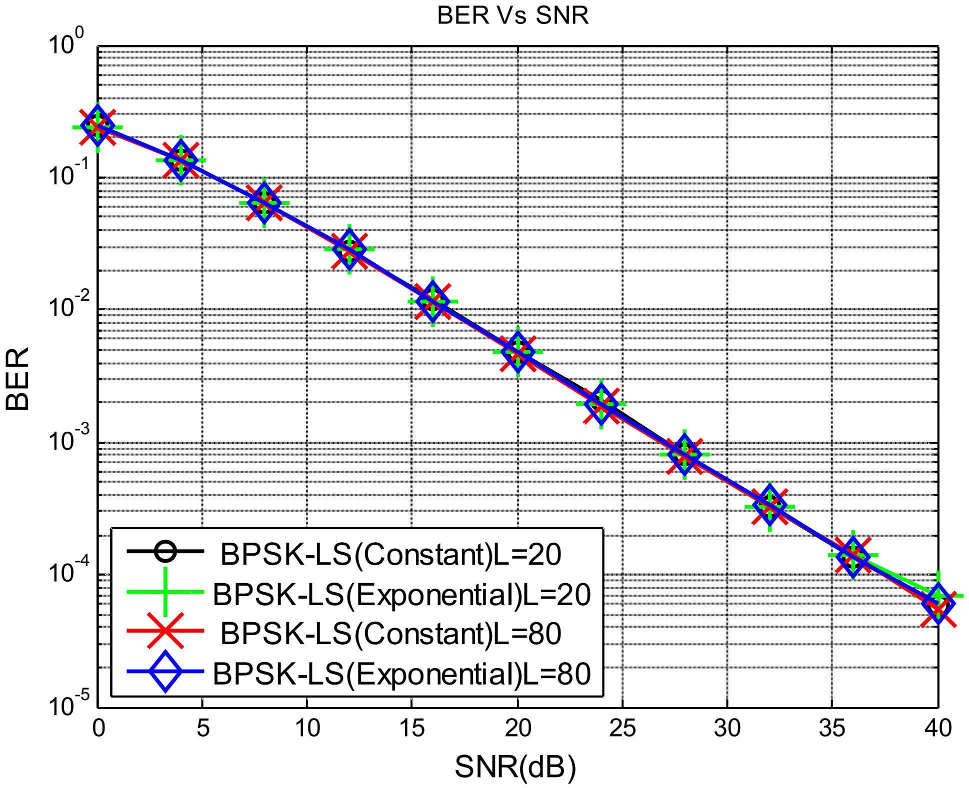


Figure 8. Performance of the OFDM System for block-type channel estimation over Rayleigh fading channel

6. Conclusions

A comprehensive comparative analysis has been carried out for performance comparison of LS estimator for comb-type and block-type channel estimation. The performance of OFDM is evaluated over Rayleigh fading channels for constant and exponential power delay profiles. The effect of increase in channel order on the performance of LS estimator for comb-type and block-type channel estimation is evaluated. The performance of the OFDM system for LS estimator with comb-type pilot arrangement degrades with an increase in channel order as indicated by both simulation results and MSE expression; however, the performance of OFDM system with block-type pilot arrangement remains stable for an increase in channel order.

References

[1]. Hui Liu & Guoqing Li, (2005). OFDM-based Broadband Wireless Networks, Design and optimization. New jersey: John Wiley & sons.

[2]. Adarsh B. Narasimhamurthy, Mahesh K. Banavar and Cihan Tepedelenioglу, 2010. OFDM Systems for Wireless Communications, Morgan and Claypool publishers.

[3]. Armstrong, Jean, (2012). Tutorial on optical OFDM, in the Proceedings of the 14th International Conference on Transparent Optical Networks (ICTON), 2-5.

[4]. Yang, Z. Bai, W. Liu, Z., (2006). A Decision-Aided Residual ISI Cancellation Algorithm for OFDM Systems," in the Proceedings of the

8th IEEE International Conference on Signal Processing , 16-20

[5]. Yushi Shen and Ed Martinez, (2008). Channel Estimation in OFDM Systems, Free scale Semiconductor, AN3059 Inc., 2006, www.freescale.com,

[6]. L.U Khan, M.I Babar and Zeeshan Sabir, (2013). Robust modified MMSE estimator for comb-type channel estimation in OFDM systems, in the Proceedings of the 15th International Conference on Advanced Communication Technology (ICACT), 924-928.

[7]. Sinem Coleri, Mustafa Ergen, AnujPuri, and Ahmad Bahai, (2002). Channel Estimation Techniques Based on Pilot Arrangement in OFDM Systems, IEEE trans. Broadcasting, 48(1), 223-229.

[8]. Hala M. Mahmoud, Allam S. Mousa and Rashid Saleem, (2010). Channel Estimation Based in Comb-Type Pilots Arrangement for OFDM System over Time Varying Channel, Journal of networks, 5(7), 766-772.

[9]. Jie Ma; Hua Yu; Shouyin Liu; , (2009). The MMSE Channel Estimation Based on DFT for OFDM System, in the Proceedings of the 5th International Conference on Wireless Communications, Networking and Mobile Computing, 1-4.

[10]. Meng-Han Hsieh; Che-Ho Wei;, (1998). Channel estimation for OFDM systems based on comb-type pilot arrangement in frequency selective fading channels, IEEE Transactions on Consumer Electronics, 44(1), 217-225.

[11]. Zeeshan Sabir, M. Inayatullah Babar, Syed Waqar Shah, (2012). Performance enhancement of wireless mobile ad-hoc networks through improved error correction and ICI cancellation, EURASIP Journal on Advances in Signal Processing.

[12]. Xiaodong Cai; Giannakis, G.B.;, (2004). Error probability minimizing pilots for OFDM with M-PSK modulation over Rayleigh-fading channels, IEEE Transactions on Vehicular Technology, 53(1), 146- 155.

[13]. L. J. Cimini, (1985). Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing, IEEE Trans. Commun., 33(7), 665–675.

[14]. Ramjee Prasad, (2004). OFDM for wireless communication systems, Universal Personal Communication Series.

[15]. Ove Edfors, Magnus Sandell, Jan-Jaap Van De Beek, Sarah Kate Wilson, Per Ola Borjesson, (2000). Analysis of DFT-Based Channel Estimators for OFDM, Wireless Personal Communications, 12, 55-70.

2/3/2021