



An analysis of the infinite existence of twin prime numbers

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Abstract: Guided by the principle of natural philosophy and combined with the gene knowledge of biological science, this paper makes an in-depth qualitative and quantitative analysis of the initial segment of prime infinite number sequence with gene significance. It is found that the percentages of prime infinite set and composite infinite set in odd number are 31.9% and 68.1% respectively. So the prime composite ratio of odd number is 31.9 : 68.1. On the basis of this prime composite ratio, this paper makes a qualitative and quantitative analysis of the distribution of prime numbers in the infinite sequence of positive integers, and draws a logical conclusion about the infinity of twin prime numbers: in the infinite sequence of positive integers, for any twin prime point $6m$ ($m \geq 1$, $6m-1$ and $6m+1$ are prime numbers), there must be a twin prime point $6(m+n)$ adjacent to $6m$ in the interval of $0 < n/m \leq 1$. Let $6(m+n)-1$ and $6(m+n)+1$ be prime numbers. Therefore, the existence of twin prime points must be infinite, and the average distance $6n$ of all twin prime points on the positive integer number axis is $n < 5$.

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1. Introduction

In the study of number theory, the infinity of twin prime number is still a mathematical problem which has not been proved. This paper holds that, like the proof of Goldbach's conjecture, the key to success or failure lies in the correct way of thinking and solving problems. The correct way of thinking must go beyond the professional knowledge of mathematics, especially the guidance of natural philosophy and the rational application of biological gene knowledge.

From the philosophical point of view, prime number is the leading factor in the formation of composite number, and the product of two or more prime numbers becomes composite number qualitatively. Therefore, prime numbers and composite numbers are qualitatively different from each other at the philosophical level, forming a causal relationship. Because the distribution of multiplication operation of prime factor on odd number axis presents jump, density and irregular fluctuation cycle, this result determines the corresponding irregularity of the distribution of prime itself. In this paper, it is considered that it is impossible to obtain the percentage of prime number in odd number simply and directly by quantitative analysis of prime infinite number sequence. In the same way, it is impossible to prove the infinity of twin primes directly from primes.

The odd infinite number sequence composed of prime number and composite number ≥ 3 is an

arithmetic number sequence. Any prime factor p can transform the odd number sequence into the composite arithmetic number sequence with P factor, so as to simply determine the percentage content of the composite infinite set with p factor in the odd number and realize the transformation from difficult to easy. Although the prime number is infinite, the sum of the percentages of the infinite sets of the infinite composite numbers is also computable. This is the solution to the quantitative analysis of the ratio of prime to total.

The result of finding prime composite ratio in odd numbers not only provides a new intuitive method to prove the infinity of prime numbers, but also provides a solid foundation for quantitative analysis to prove the infinity of twin prime numbers.

2. Analysis of the percentage and composite numbers in odd numbers

In the positive integer infinite series, there are two parts: the odd equal difference infinite series and the even equal difference infinite series. The tolerance of these two equal difference series is the same as 2. Therefore, the percentage content of odd infinite set and even infinite set in positive integer is 50 and the parity ratio is 1:1.

In the infinite column of odd and infinite numbers, it consists of two infinite sets of primes and infinite combinations, and there is necessarily a prime

combination ratio as well as the parity ratio. In order to obtain the prime prime composite ratio in odd numbers, according to the analysis of the previous section, the percentage content of the sum, and the prime composite ratio is a: b.

Homogenization transformation of odd equal difference infinite series can easily obtain the

composite equal difference infinite series with any prime factor p . The argumentation and analysis of this paper is based on this basic platform. In order to obtain the more reliable prime-to-synthesis ratio, the three continuous prime numbers 3, 5 and 7 in the first segment of the most meaningful prime-infinite series are analyzed.

A list of odd differential ≥ 3 is presented as follows:

3, 5, 7, 9, 11, 13, 15, 17, 19, $2m+1$ ($m > 1$)

By multiplying the prime factor 3, 5 and 7 with the odd number column above, the following three equal difference series are obtained:

9, 15, 21, 27, 33, 39, 45, $6m+3$ (1)

15, 25, 35, 45, 55, 65, 75, $10m+5$ (2)

21, 35, 49, 63, 77, 91, 105, $14m+7$ (3)

Analysis of the above three series shows that:

1) The tolerances of the three series of numbers are 6, 10, 14, and the general formula expressed p prime symbols is $2p$;

2) The percentage of the three combined infinite sets is: $1/3 \times 100\%$, $1/5 \times 100\%$, $1/7 \times 100\%$, and the prime symbol p is $1/p \times 100\%$;

3) The repeated part of the sequence (2) containing prime factor 3 is sifted out to obtain the unequal difference sequence containing prime factor 5:

25, 35, 55, 65, 85, $10m+5$

Calculation formula for percentage content in odd numbers:

$(1/5 - 1/15) \times 100\% = 1/5(1 - 1/3) \times 100\% = 2/15 \times 100\%$

4) The repeated parts of the series (3) containing prime factors 3 and 5 are sifted out to obtain the series of unequal difference:

49, 77, 91, 119, $14m+7$

Calculation formula for percentage content in odd numbers:

$(1/7 - 1/21 - 1/35) \times 100\% = 1/7(1 - 1/3 - 1/5) \times 100\% = 1/15 \times 100\%$.

The analysis of the above prime gene segments shows that:

1) The joint equal infinite column with the minimum prime factor 3 is the fundamental and constant joint equal infinite column, which is the basic platform for qualitative and quantitative analysis and demonstration of this paper;

2) For an infinite set of compounds containing any prime factor p , its initial term must be no $< p$ factor in the p^2 , column;

3) For a composite infinite set with prime factor $\geq p$, When calculating the percentage content of

combinations in odd numbers, A general formula can be expressed as $1/p(1 - 1/3 - 1/5 - 1/7 - \dots)$.

The values in brackets in the above formula are expressed η_p algebraic symbols, the meaning of which is the density coefficient of the infinite set of numbers containing $\geq p$ factors distributed on the odd number axis. When $p=3, \eta_3=1, \eta_p$ definition domain $0 < \eta_p < 1$ at $p > 3$, the smaller the η_p value, the larger the average spacing of adjacent terms;

4) The value of $1/p \times \eta_p \times 100\%$ is expressed as ρ_p , which means the percentage content of the infinite set of numbers containing $\geq p$ factors in odd numbers;

5) To give readers a perceptual understanding of the distribution of infinite sets of composite numbers with $\geq p$ factors on odd number axes, the K_p , of algebraic symbols is introduced, Related tie $K_p = 1/\eta_p$, The K_p definition domain is $1 \leq K_p \leq (p-1)/2$

When $p=3, K_3=1$

When $p=5, K_5=3/2, 1 \leq K_5 \leq 2$

When $p=7, K_7=15/7, 1 \leq K_7 \leq 3$

At $p \geq 5$, the perceptual significance of the K_p value reflects that the mean value of the adjacent term spacing is $2pK_p$, The larger the K_p , the larger the adjacent term spacing, the thinner the distribution.

As an example of $p=7$, the distribution of adjacent terms is irregular with 14, 28 and 42 spacing, the average spacing is $2pK_p = 2 \times 7 \times 15/7 = 30$.

When the gene segment studied extends to the $3 \leq p \leq 29$ interval, the adjacent prime terms are discontinuous due to the combination number, including the twin prime. Therefore, when analyzing in the interval of $p > 7$, we must consider how to deal with

the problem of composite number, η_p the general formula should be adjusted.

Taking the η_7 formula as an example, the $\eta_7 = 1 - 1/3 - 1/5 = 7/15$, in which one third of the terms contain the sum items formed by the prime factor 5, such as 15,45, etc., while one fifth of the terms also contain the sum items formed by the prime factor 3, such as 15,45, etc. Thus a repeat screening of $1 / 15$ occurs in $(1 - 1 / 3 - 1 / 5)$, this is correct for the η_7 results, but for the η_{17} formula, the corresponding $1/15$ compensation for the sum 15 of the previous η_7 excess sifted must be compensated to the formula:

$$\eta_{17} = 1 - 1/3 - 1/5 - 1/7 - 1/11 - 1/13 + 1/15$$

If there is a composite number between adjacent prime numbers, this η_p value calculation formula exists. The philosophy of reasonable adjustment of

compensation is: When two different prime factors multiply to form a composite number, the prime factor also reserves part of the space for its future existence when gives the composite number. In this way, we can ensure the dynamic natural balance between prime number and composite number. On the odd axis, the dense distribution of prime and compound cycles with irregular fluctuations tends to be infinite.

From the above analysis, we can see that the sum p^n , such as 9,27,25,27,25 and 49, are negligible, and the sum of more than three prime factors, such as 45,75 and 105, are also negligible.

A total of 25 composite numbers are selected in the $3 \leq p < 100$ range, of which 16 belong to the adjusted compensation object. The regularity is listed in Table 1 below:

Table 1

$p < \sqrt{100}$	$q = p \times p_n \ (p < p_n < 100/p)$								
3	15	21	33	39	51	57	69	87	93
5	35	55	65	85	95				
7	77	91							

Correctly judging the combination of control and adjusting compensation object is the key link to obtain accurate prime composite ratio.

Based on the previous logical analysis, a simplified formula for calculating the η_p can be deduced: Any two adjacent prime terms $p_a < p_b$ in an odd number sequence

There are $p_a = a, p_b = b$, In the (a, b) interval there exists composed of two different prime factors of the joint with n , expressed as $q_1, q_2, q_3 \dots q_n$, general simplified formula for η_p is

$$\eta_{pb} = \eta_{pa} - 1/a + 1/q_1 + 1/q_2 + 1/q_3 + \dots + 1/q_n$$

Using the above simplified formula, the η_p and $\rho_p\%$ data of the corresponding prime factor p can be given continuously (Table 2).

Table 2 below provides data on nine prime numbers in the $3 \leq p \leq 29$ range:

p	3	5	7	11	13	17	19	23	29
η_p	1	2/3	7/15	0.324	0.233	0.223	0.164	0.159	0.115
$\rho_p\%$	33.33	13.33	6.67	2.94	1.79	1.31	0.86	0.69	0.4

Add the $\rho_p\%$ in Table 2 to $\sum \rho_p\% = 61.32\%$.

The $S_p\%$ represents the sum of the percentage content in odd numbers, and if the ratio of $\sum \rho_p\% / S_p\%$ can be determined, the comparative accurate value of S_p can be obtained.

In this paper, two methods are given to determine $\sum \rho_p\% / S_p\%$:

First, a reference series sum with high similarity to ρ_p series is introduced, which is determined by comparison method. This reference series sum is as follows:

$$S_n = \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots + \frac{1}{1+2+3+4+\dots+n}$$

To solve the above series $S_n = \frac{n-1}{n+1}$ (Process outline)

First item 1 / 3 of the reference series and is the same as the pp first item ,when $n \rightarrow \infty$ $S = 1$,Take $n = 19$ Final

item $\frac{1}{1+2+3+4+\dots+19} = \frac{1}{190}$ is equivalent to as required ρ_{29} .

$$S_{19} = \frac{18}{20} = 0.9 \quad 0.9 \times 100\% = 90\%.$$

A second method is called η_p residual value determination.

The principle is that when $p \rightarrow \infty$, the η_p residual value is close to 0, $\sum \rho_p \% \rightarrow 100\%$. Analysis $3 \leq p \leq 29$ sections, $\eta_{29} = 0.115$, adjacent $\eta_{31} = 0.081$, the two average margin is near 0.1, we can infer that the sum of $\sum \rho_p \% \geq 61.32\%$ occupied η_p should be $1 - 0.1 = 0.9 \times 100\% = 90\%$.

The above two methods get the same ratio , and thus determine : $S_p = (61.32/0.9)\% = 68.13\%$.

The corresponding percentage content of the prime number is $(100 - 68.13)\% = 31.87\%$.

For the reliability of the above results, it is necessary to extend the study gene segment to the interval of $3 \leq p \leq 97$.

Data on 15 prime numbers in the $31 \leq p \leq 97$ intervals are given in Table 3 below:

Table 3

p	31	37	41	43	47	53	59	61
η_p	0.081	0.108	0.107	0.082	0.059	0.057	0.074	0.057
$\rho_p \%$	0.26	0.291	0.26	0.19	0.125	0.107	0.125	0.093
p	67	71	73	79	83	89	97	
η_p	0.056	0.055	0.041	0.041	0.038	0.039	0.06	
$\rho_p \%$	0.083	0.078	0.057	0.051	0.034	0.044	0.062	

The sum of the $\rho_p \%$ in Table 3 is $\sum \rho_p \% = 1.809\%$.

Between $3 \leq p \leq 97$:

$$\sum \rho_p \% = (61.32 + 1.809)\% = 63.129\%$$

The corresponding percentage increment of the prime content is :

$$[(90 - 61.32) / 61.32] \times 1.809\% = 0.89\%.$$

$3 \leq p \leq 97$ and $3 \leq p \leq 29$ total % increased to $(90 + 1.809 + 0.89)\% = 92.7\%$

It follows that the percentage content in odd numbers is:

$$63.129 / 92.7 \times 100\% = 68.1\%$$

The percentage content of the prime number is $(100 - 68.1)\% = 31.9\%$.

According to the above verification results, considering the reasonable error of approximate calculation, the prime composite ratio in odd number is determined to be 31.9:68.1.

3. An analysis of the infinite existence of twin prime numbers

The combination of prime factor 3 forms a constant equal difference series, which provides qualitative and quantitative analysis basis for the inevitability of twin prime number.

If m is used to represent a positive integer variable (domain $m \geq 1$), then $6m$ is the general term of the even arithmetic sequence with tolerance of 6 in the infinite sequence of positive integers. This even arithmetic sequence is of special significance to the analysis and demonstration of the infinite existence of twin prime numbers.

In this paper, $6m$ ($m \geq 1$) is defined as a prime dependent point.

The sum arithmetic (tolerance 6) infinite sequence formed by Prime gene 3 contains 1 / 3 of odd number, and all the sum terms are evenly distributed in the center of adjacent prime dependent

points $6m$ and $6(m+1)$. This characteristic determines that the remaining $2/3$ odd numbers (including 31.9% prime numbers and 34.77% composite numbers formed by prime factors > 3) must be distributed on the left and right adjacent positions of all prime dependent points, namely $6m-1$ and $6m+1$. This is the inevitability of the infinity of twin primes. When both $6m-1$ and $6m+1$ are prime numbers, this paper defines the $6m$ point as a twin prime point. For example, $m=1$, $m=2$ and $m=3$ are twin prime points. Therefore, twin prime points are part of all prime dependent points.

In the interval of $[6m, 6m+n]$, when $n=5$, there are five prime number dependent points, whose single digits are 0, 2, 4, 6 and 8 respectively, forming a cyclical change. In this paper, the interval is defined as the basic cyclic interval of twin prime numbers, and it is taken as the object of qualitative and quantitative logic analysis. The analysis is as follows:

The infinite set of the sum with prime factor 5 is characterized by that, except for prime 5, all digits with 5 must be the sum. This feature determines that the number of digits in the base cycle interval is 4 and $6(m > 1)$, and the two prime dependent points can only be excluded from twin prime points, because the number of digits is 4 and 6, and only two possibilities are found when $m > 1$: a) The pair is composed of prime number and sum number 1:1; b) Two combined array pairs exist. The above quantitative analysis shows that twin prime points only exist in three prime dependent points with 0, 2 and 8 digits;

2) When the range of twin prime points shrinks to 0, 2 and 8, the corresponding percentage content of composite number also changes to $(34.77-13.33)\% = 21.44\%$. Therefore, in the possible 0, 2 and 8 points of twin prime number, the number of composite prime number has an advantage in quantity, and the ratio of residual prime number to composite prime number becomes 31.9:21.44, as the average probability, the calculation is close to the reality;

3) According to the above analysis, the adjacent positions of the three points with the single digit of 0, 2 and 8 in the basic cycle interval can only be shared by the prime number with the prime composite ratio of 31.9:21.44 and the composite number with the prime factor of ≥ 7 . There are only three possible ways of distribution: a) prime and composite 1:1 pairs; b) Twin prime array pairs; c) Binary array pairs. Due to the uneven distribution and density fluctuation of the composite number, the average probability of a species can only be $1/3$, and the probability of b in the remaining two points is greater than c. according to the prime to composite ratio of 31.9:21.44, the twin prime point $b = (31.9 / 31.9 + 21.44) \times 2 = 1.2$. Therefore, the average probability of the occurrence of

twin prime points in a basic cycle interval is greater than 1.

In this paper, we study the infinity of twin prime numbers in the infinite sequence of positive integers. We define $6m-1$ and $6m+1$ as the center of prime numbers, and define $6m$ as the twin prime point. It is in line with the essential meaning of the concept of twin prime numbers to recognize even point 6 when $m=1$ as the first twin prime point.

In order to make the conclusion that the twin prime number has infinity, it is supported not only by the quantitative analysis of the average density of the twin prime number points, but also by the analysis of the special points that the distance between the twin prime number points is relatively large and the distribution is particularly sparse, that is, the special points that the continuous composite number is densely distributed.

1) When the first twin prime point $6m=6$, then the first composite number $6+3=9$ (the composite numbers in this paper are all odd composite numbers); When $m=4$, $6m=24$, two consecutive composite numbers $24+1=25$ and $24+3=27$ appear for the first time; When $m=20$, $6m=120$, there are six composite numbers for the first time $120-5=115$ 、 $120-3=117$ 、 $120-1=119$ 、 $120+1=121$ 、 $120+3=123$ 、 $120+5=125$. If k is a positive integer, the above three $6m$ points can be expressed as $6 \times 1=3!$ 、 $6 \times 4=4!$ 、 $6 \times 20=5!$.

It can be inferred that when $6m=k!$ When the prime number is $6m$, there will be a dense distribution of continuous composite numbers in the adjacent area, so that $6m=k!$ The special situation of sparse distribution of twin prime points near the point makes the distance between adjacent twin prime points $n > 5$, and increases with the increase of k .

2) When $m=1$ in $6m$, $n=1$, $n/m=1$ is the largest relative ratio; When $m > 1$, there is $0 < n/m < 1$. with the increase of m , n/m fluctuates, but the trend of continuous convergence to 0 remains unchanged. For any given twin prime point $6m$, no matter how big m is, it is a given number, and the value of n in its adjacent twin prime point $6(m+n)$ is finite.

According to the above quantitative analysis, the finite interval expression between the adjacent twin prime points is expressed in the conclusion of the analysis and demonstration in this paper, and the relative value of n/m is used to determine.

4. Conclusion

In the infinite sequence of positive integers, for any twin prime point $6m$ ($m \geq 1$, $6m-1$ and $6m+1$ are prime numbers), there must be an adjacent twin prime point $6(m+n)$ in the interval $0 < n/m \leq 1$,

so that $6(m + n) - 1$ and $6(m + n) + 1$ are prime numbers. Therefore, the existence of twin prime points must be infinite, and the average distance $6N$ of all twin prime points on the positive integer number axis is $n < 5$.

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