



Proof of Goldbach conjecture by function method

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Abstract: Through the in-depth reading of the proposition of the Goldbach conjecture, the paper holds that the proof of the screening method is not in line with the meaning of the question, and thus puts forward the proof of the function method. In this paper, two independent linear prime functions of two variables p_1 and p_2 are constructed in the variable interval $[1, 2m]$, and then two equivalent linear even functions of two variables N and $2m$ are logically deduced. The present study came to a consistent conclusion based on the qualitative and quantitative analysis of these two equivalent even functions: in the same definitive domain (greater than 2), the equations $N = 2m$ must be established. From this, it is deduced that $2m = p_1 + p_2$ surely is set up.

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1. Introduction

Any even number greater than 2 can be represented as the sum of two primes. This is the world famous Goldbach conjecture proposition written expression.

The premise to get a correct solution to a mathematical problem is to have a reasonable interpretation of the proposition after logical analysis. The following is a logical analysis of the meaning of the proposition of Goldbach conjecture:

A) The mathematical language representation of the proposition is an algebraic equation;

B) To the left of this algebraic equation is "any even number greater than 2", which exactly means that the even variable has a domain >2 , and should not be a certain even constant. This even variable represents all the even numbers in the domain, in fact, all the even composite numbers;

C) The sum of two primes on the right side of the algebraic equation, whose exact meaning is the addition of two prime variables, limited to the value in the prime infinite series, and excluding odd composite numbers from the equation;

D) Two prime variables should be independent and interrelated prime functions with different value domains. These two prime functions can represent all primes in the whole prime infinite series.

The algebraic equation that fits the propositional question must be a function.

Since Goldbach put forward his conjecture proposition in 1742, it has attracted worldwide attention and made a long-term effort to prove the

proposition. Screening method as the mainstream method of proof has a history of nearly 100 years. In China, famous mathematicians Yuan Wang, Cheng-Dong Pan and Jing-Run Chen have become world famous leaders in screening proof. Among them, the "1 + 2" proved by Jing-Run Chen is called "the peak of the development of screening method" (1). But this is also a dead end, now recognized in the world of mathematics, "1 + 2" theory and method can not prove "1 + 1" to provide any guidance and help, but look forward to new mathematical ideas to study "1 + 1" (2). The helpless result of the proof of screening method stems from the fact that the proof method deviates from the rigorous logic principle of mathematics, and the emergence of undue errors in the interpretation of the meaning of the Goldbach conjecture.

1) The screening method replaces even continuous variables with a constant value "large even number" on the left side of algebraic equation, and the function formula is misinterpreted as solving algebraic equation. No matter what "sufficient" or "enough" attribute is added before "large even number", there is no logical proof force. At best, it is only a case analysis of "large even number" to deduce the proposition of Goldbach's conjecture.

2) The screening method, which adopts "the sum of two composite numbers" or "the sum of a prime number and a composite number" on the right side of an algebraic equation to determine the "large even number" on the left side of an equation, is a helpless reverse choice that does not fit the meaning

of the proposition. Because the screening method can not use "the sum of two prime numbers" to determine "large even number", otherwise the significance of proof will be lost. However, the establishment of such an algebraic equation as a proof platform completely blocks all possibilities of cleanly screening out the composite numbers in the algebraic equation and qualitatively changing them into the sum of two equal prime numbers. The more and less prime factors contained in a composite number are only the difference of the quantity of a representation, and do not involve the change of the essence of composite number.

On the basis of the above analysis, a clear conclusion can be drawn that the negative screening method can be used to prove this. In this paper, a functional method is proposed to prove it.

The mathematical language expression of Goldbach conjecture proposition by function method is as follows: if the continuous even variable $2m$ (domain > 2) is used to represent "any even number greater than 2", the mutually independent prime function p_1 and p_2 are used to represent two primes, respectively. Confirm that: $2m = p_1 + p_2$ is established. $2m = p_1 + p_2$ is equivalently converted to $m - p_1 = p_2 - m$. This is the platform in which the function-based proof suits the meaning of the proposition. From a logical analysis of the above equations, the following point can be understood from the logical analysis of the above equations:

A) $0 < p_1 \leq m, m \leq p_2 < 2m$, so p_1 and p_2 are in the variable interval $[1, 2m]$, the function method takes this variable interval as the object of study, with $2m$ continuous increasing, this interval represents the whole even number infinite sequence;

B) If $m - p_1 = p_2 - m$ holds, it means that p_1 and p_2 have a symmetric relation with m as the center, so the qualitative analysis conclusion should be related to the symmetric function.

C) $p_1 + p_2$ must be an even number, and the algebraic equation $N = p_1 + p_2$ is an even function, but N does not necessarily have the same domain and necessary continuity as $2m$. Therefore, the qualitative analysis proved by function method must have the logical inference of symmetry continuity in addition to confirming that N and $2m$ have the same domain > 2 .

D) In the interval $[1, 2m]$, there exists an odd pair with m as its central symmetric distribution, which is determined by: $m/2$ (m is even) or $(m+1)/2$ (m is odd). The conclusion of quantitative analysis should be $d \geq 1$ as long as it is logically proved that there is at least a pair of prime combinations $p_1 + p_2$ (the quantity is expressed by d) in the symmetrical odd combination.

E) Because the object of study of function method is even function $N = p_1 + p_2$ of variable interval $[1, 2m]$, quantitative analysis is to solve series of equations of even function N with different definitions in the range of values. Relevant data obtained, such as a, b, c, d listed in this paper, are composed of the sequence of numbers, as long as they are changed according to the trend of these sequence, whether the potential is divergent or convergent, we can analyze the conclusion.

Finding the entry point for proof that conforms to the objective essential rules of the infinite sequence of prime numbers, and constructing a reasonable functional relationship are the keys to the proof of the Goldbach conjecture. Choosing a proper dynamic symmetric coordinate system is the premise and foundation to grab these keys.

A brief description of the coordinate system selected for function method proof is as follows:

Since Goldbach conjecture only involves numbers in the range of positive integers, it is preferred that the number axis of positive integer independent variable m ($m \geq 0$) is used as the basic coordinate system, let's take this ray that goes one way to the right with 0 as the origin of the coordinate. The number axes belonging to different categories on the principal axis, such as even number, odd number, prime number, composite number and so on, also include the symmetry axis $N = 2m$ of even symmetric function, all of which are implicitly attached to the principal axis, forming the state of unity of multiple number axes.

Because the proof of the function method involves the symmetry analysis of the function, it is necessary to set up a dynamic symmetric coordinate system autonomously in a local interval on the principal axis as the basis for the rational construction of the prime function. The elements of the dynamic symmetric coordinate system include: 1) the interval on the principal axis is $[1, 2m]$; 2) the coordinate origin 0 coincides with the principal independent variable m ; 3) the auxiliary parameter independent variable n_1 is oriented to the left and n_2 is opposite to the right. 4) the coordinate value range marked on n_1 and n_2 axes is determined by n_1 and n_2 domain.

After the prime number functions p_1 and p_2 carry out algebraic sum operation, the dynamic symmetric coordinate system above will be transformed automatically: ① the interval on the principal axis becomes $(m, 3m)$; ② Coordinate origin, n_1 and n_2 change from m point on the principal axis to $2m$ point; ③ The auxiliary parameter independent variable changes from the independent n_1, n_2 to the

combination independent variable, where $(n_1 - n_2)$ number axis direction is left, $(n_2 - n_1)$ number axis direction is opposite to right; ④ The range of coordinate values marked on the $(n_1 - n_2)$ axis and on the $(n_2 - n_1)$ axis is determined by the domain.

In order to provide readers with intuitionistic and perceptual cognition of the above coordinate system, four coordinate charts are attached to the proof part. In order to display other relevant axes hidden on the spindle, the coordinate diagram marks the number axes that need to be separately marked outside the spindle, such as the prime p axis in figs. 1 and 2, and marks the name, direction and coordinate value range by moving parallel to the lower part of the spindle, and the name, direction and range of coordinate values are marked by moving the dynamically symmetrical coordinate system parallel to the upper section of the spindle. Considering that the prime functions p_1 and p_2 involved in the function method do not contain even primes 2, in order to show this difference on the p axis, the prime p axis is drawn with a dotted line from the origin point 0 to 3.

Take fig. 1 as an example, $p_1=3$ and $p_2=11$ on the prime p axis have no sense of symmetry, but in the dynamic symmetric coordinate system, the symmetry can be seen clearly: $n_1=n_2=2$, $p_1=7-4$, $p_2=7+4$. For the origin of the dynamic coordinate, the left and right symmetries are equal.

Setting the dynamic symmetric coordinate system autonomously becomes the key link of the logical proof of the function method.

Function method to prove the relevant mathematical basis:

Lemma 1: prime infinite theorem

Euclid's proof:

Euclid proved that for any set of finite primes, there is always a prime that is not in it, so the prime must be infinite.

Since all even numbers greater than 0 are composite numbers except 2, the prime of the infinite prime theorem refers to odd prime numbers. The prime numbers involved in this paper (except the special even prime number 2) all refer to the odd

prime numbers. Now the prime numbers are shown as follows:

3、5、7、11、13、17..... p 、 $p+2n$ (n is a positive integer)

Lemma 2: Bertrand Chebyshev theorem

Bertrand - chebyshev's theorem:

If the integer $n > 3$, there is at least one prime number p , which corresponds to $n < p < 2n - 2$.

It can be inferred from the theorem that when $m \geq 3$, at least one prime number exists in the variable interval $[m,2m]$.

2. Proof

In a positive integer infinite sequence, any even independent variable $2m$ greater than 2 can be expressed as the sum of two identical or adjacent odd numbers.

When m is odd, there's the equation: $2m = m + m$.

For the interval of $[1,2m]$, the variable m is taken as the origin of coordinate, first the prime number p_1 of the interval of $[1,m]$, which is the same as m (m is a prime) or adjacent (m is not a prime), is expressed as a function of the independent variable m . There is a function $p_1 = m - 2n_1$, in which n_1 is the auxiliary parameter variable with the direction of the number axis to the left. The domain is $0 \leq n_1 < m / 2$.

In $[m, 2m]$ interval, according to Lemma 2, again the prime number p_2 which is the same as m (m is prime) or adjacent (m is not prime) can also be expressed as a function of the independent variable m . There is a function $p_2 = m + 2n_2$, in which n_2 is an auxiliary parameter variable with the direction of the number axis to the right, and the domain is the same as n_1 : $0 \leq n_2 < m / 2$.

On the axis of positive integer, the dynamic symmetric coordinate system with local interval $[1,2m]$ is constructed by taking m as the origin of dynamic coordinate and the directions of the two axes of n_1 and n_2 are left and right opposite. When the m value is 7, the coordinate image of the prime function p_1 and p_2 is shown in Figure 1.

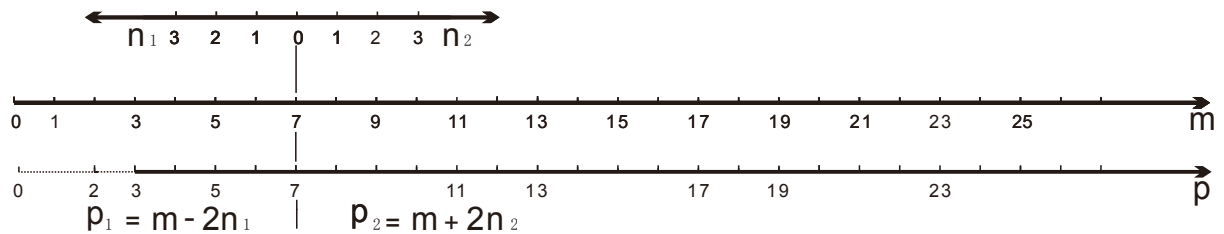


Figure 1

In the above two prime functions, when m is a prime number, $n_1 = n_2 = 0$. Then $p_1 + p_2 = m$. When different integer values are taken for n_1 and n_2 in the domain, the two functions represent all the prime numbers in the interval $[1, 2m]$. When m is continuously incremented, the two prime functions represent the entire infinite prime sequence.

The algebraic sum operation of the above two prime functions was conducted, and there is equation $p_1 + p_2 = m - 2n_1 + m + 2n_2 = 2m - 2(n_1 - n_2)$.

When m in the even independent variable $2m$ is an even number greater than 3, it can also be expressed as the sum of two adjacent odd numbers. The equation is $2m = (m-1) + (m+1)$. For the same reason, there must be at least a pair of primes p_1 and

p_2 in the interval $[1, 2m]$, which are the same or adjacent to $(m-1)$ and $(m+1)$, respectively. There are the following functions: $p_1 = (m-1) - 2n_1$ and $p_2 = (m+1) + 2n_2$, where the dynamic coordinates origin of the auxiliary parameter variables n_1 and n_2 is the m point ($n_1 = n_2 = 0$) on the axis. Since only integer values are taken for n_1 and n_2 , the parameters of $(m-1)$ and $(m+1)$ are the same as point m , namely, $n_1 = n_2 = 0$. Furthermore, n_1 and n_2 share the same domain: $0 \leq n_1 < m/2$, and $0 \leq n_2 < m/2$. When $(m-1)$ or $(m+1)$ is prime number, $n_1 = 0$ or $n_2 = 0$. When the m value is 10, the coordinate image of the prime function p_1 and p_2 is shown in Figure 2.

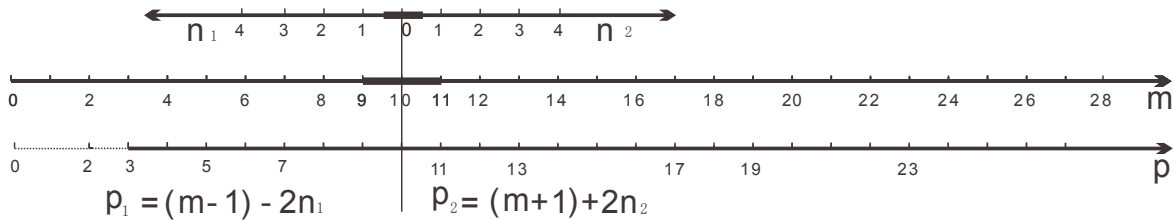


Figure 2

Similarly, the algebraic sum operation of the above two prime functions was conducted, and the equation is $p_1 + p_2 = (m-1) - 2n_1 + (m+1) + 2n_2 = 2m - 2(n_1 - n_2)$.

The above equation indicates that an even function could be obtained by adding two prime functions. By making $N = 2m - 2(n_1 - n_2)$, the equation $N = p_1 + p_2$ is obtained.

The perceptual image meaning of the even function $N = 2m - 2(n_1 - n_2)$: the even function N in the equation represents the data group or data chain obtained from the combination $(p_1 + p_2)$ of all prime numbers p_1 and p_2 in the interval $[1, 2m]$. If a represents the number of p_1 and b represents the number of p_2 , then the total data chain of N is ab . This ab data chain performs a sandpile follow-up coverage on even numbers in interval $(m, 3m)$, which is centered on $2m$. When at least one pair in the N data chain ab satisfies $n_1 - n_2 = 0$, the equation $N = 2m$ holds, indicating that the corresponding point of the function value N on the positive integer even axis coincides with the independent variable $2m$.

In order to provide a strict qualitative and quantitative logic argument for the relationship

between N and $2m$, it is first necessary to determine whether the range of the even function N is the same as the domain of $2m$.

In the function $N = 2m - 2(n_1 - n_2)$, when m is an odd number, $2m > 2$, then $m > 1$. The smallest odd value is $m = 3$, and correspondingly $2m = 6$. All prime numbers present in the interval $[1, 6]$ are $p_1 = p_2 = 3$, the corresponding parameter variables $n_1 = n_2 = 0$, $p_2 = 5$, and the corresponding parameter variable $n_2 = 1$. So $a = 1$ and $b = 2$, all prime pairs have two combinations ($1 \times 2 = 2$): $N_1 = 3 + 3 = 6$, and $N_2 = 3 + 5 = 8$. Furthermore, there are three even numbers (4, 6 and 8) in the interval $[3, 9]$, and the coverage of the N data chain is 6 (1 pair) and 8 (1 pair). Therefore, the value range of N is determined as $N \geq 6$.

When m is an even number, $2m > 2$, then $m > 1$. The smallest even number is $m = 2$, and the corresponding $2m = 4$. The prime number in the interval $[1, 4]$ is $p_2 = 3$, corresponding parameter variable $n_2 = 0$. So $a = 0$, $b = 1$, and $ab = 0$ (that is, there is no odd prime pair). Therefore, even prime pairs were used to express $2 + 2 = 4$, and $N > 2$.

Based on the above analysis of the initial interval of even number sequence, it is concluded that the value range of function N is the same as that of the independent variable $2m$: $N > 2$ and $2m > 2$.

The even function $N=2m-2(n_1-n_2)$ is transformed into $2m=N-2(n_2-n_1)$ by the equivalent transformation of terms. These two linear functions of two variables interchange the independent variables and functions. Essentially, the same algebraic equation is expressed in different functional formulas. The previous discussion has proven that N and $2m$ share the same domain. Since parameter variables n_1 and n_2 share the same domain, $0 \leq n_1 < m/2$ and $0 \leq n_2 < m/2$, the domains of parameter variables (n_1-n_2) and (n_2-n_1)

are also the same: $-m/2 < (n_1-n_2) < m/2$, and $-m/2 < (n_2-n_1) < m/2$.

The first conclusion drawn from the above qualitative logic analysis of the relationship between the two linear functions of two variables is the identity, which includes the identity of the coordinate system, the functional relationship, and the independent variable domain. When $2m = 14$, the image of two equivalent even function coordinate is shown in Figure 3.

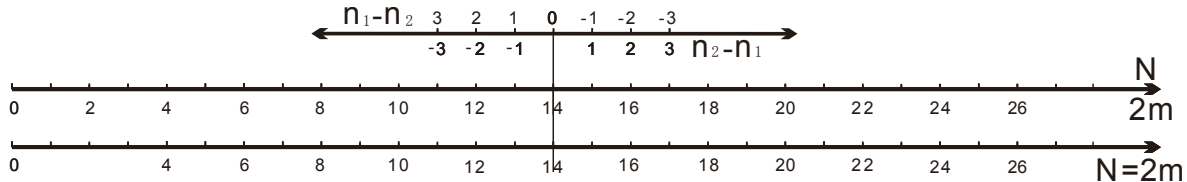


Figure 3

According to the identity of the domains of parameter variables (n_1-n_2) and (n_2-n_1) , when $n_1 - n_2 = n_2 - n_1$, the two linear functions of two variables become linear functions of one variable $N=2m$. The perceptual image meaning is: $n_1 - n_2 = n_2 - n_1$ is the function relation $N=2m$ of the dynamic coordinate origin of two linear functions of two variables $(n_1 - n_2 = n_2 - n_1 = 0)$, which is the same image as the symmetric axis $N=2m$ and the three axes are identical. Therefore, the second conclusion from the above qualitative analysis is symmetry: in the domain $2m > 2$, the points on axes N and $2m$ coincide one to one.

When $n_1 - n_2 \neq n_2 - n_1$, there exists $|n_1 - n_2| = |n_2 - n_1|$, which shows that in the $(m, 3m)$ interval centered on $2m = N$, the two linear functions of two variables $N = 2m - 2(n_1 - n_2)$ and $2m = N - 2(n_2 - n_1)$ are inverse function to each others. Hence, the two linear functions of two variables were bilaterally left and right symmetrical functions. Since $2m$ is continuous in the domain $2m > 2$, N must also be continuous in the domain, and there must be no discontinuity point. This is the third conclusion from the qualitative logic analysis.

Identity, symmetry and continuity are the three characteristics obtained from the above qualitative analysis, which can be expressed as: in the domain $2m > 2$, the equations $N = 2m$ surely is set up.

Based on the construction of the even function $N=2m-2(n_1-n_2)$, a quantitative logic analysis was conducted again:

(1) In the positive integer sequence, the total amount of all prime numbers in the interval $[1, 2m]$ was expressed as k , and the amount of prime numbers in the interval $[1, m]$ and $[m, 2m]$ was expressed as a and b , respectively. The following deduction can be made according to Lemma 1:

① With the continuous increment of $2m$, sequence k turns into a monotonically increasing divergence sequence ;

② Similarly, sequences a and b also increase with the continuous increment of $2m$ into a monotonically increasing divergence sequence.

(2) In the interval $[1, 2m]$, ab represents the total amount of data chains of the even function N obtained from the combination p_1+p_2 of all prime pairs. The data chain ab performs the sandpile follow-up coverage on the even numbers $(\leq m)$ in the interval $[m, 3m]$. When c represents the ratio of ab and the amount of even numbers covered by the N data chain ab in the interval $(m, 3m)$, and $m \geq 9$, $c \geq ab/m$ is established. When $m < 9$, $N=2m-2(n_1-n_2)$ has a solution $(d \geq 1)$, and the sequence c obtained from the continuous increment of $2m$ turns into a monotonically increasing divergence sequence.

(3) The value c reflects the average number of repetitions of even numbers in the interval $(m, 3m)$ covered by the N data chain ab . The peak value of the sandpile over-coverage may only appear in the central neighborhood of $2m$. If d is used to express the amount of combinations of prime pairs that meet $N=2m(n_1-n_2=0)$, then $d \geq c$ is established. With the continuous increment of $2m$, the sequence d turns into a divergence sequence with fluctuant increase.

Therefore, it was concluded that $d \geq c \geq 1$ surely is set up.

Attached figure 4 shows the perceptual image of the even number of the sandpile follow-up coverage

times in the interval $[10,30]$ of the data chain ($ab = 3 \times 4$) of the even number function N when $2m = 20$.

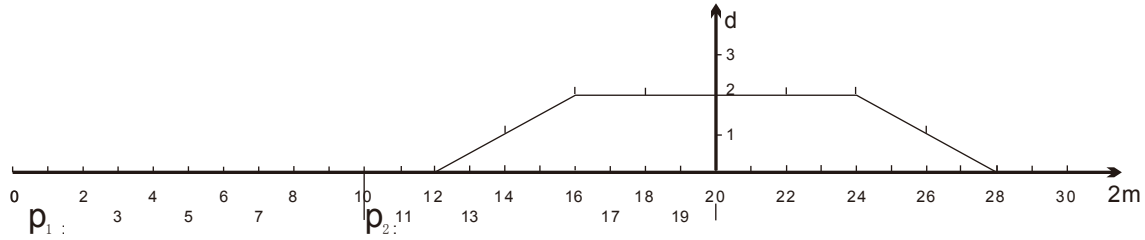


Figure 4

3. Conclusion

Based on the comprehensive qualitative and quantitative logic analysis of the interval $[1, 2m]$ above, this was finally concluded, as follows:

In the positive integer infinite sequence, for any even number $2m$ greater than 2, there must be at least one pair of prime number p_1 and p_2 in the interval $[1, 2m]$ that enables the equation $2m = p_1 + p_2$ to be established. Since $N=2m-2(n_1-n_2)$ and $2m=N-2(n_2-n_1)$ form symmetrical functions, there is $n_1 - n_2 = n_2 - n_1 = 0$, that is, $n_1 = n_2$. It can be philosophically observed that the existence of the symmetric correlation of even functions is completely rooted in the objective existence of the symmetric distribution of prime numbers on the positive integer axis. If the Goldbach conjecture is proved to be accepted in this paper, the following inference is drawn :

In a positive integer sequence, for any independent variable $m \geq 2$, there is at least one pair of prime numbers symmetrically centered about m .

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