Power of Monte Carlo Test for Different Returns to Scale in Production Function

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Abstract: The aim of this study is to find the power of Monte Carlo test involving different returns to scale in production function. In a Monte Carlo study in which the above test have conducted, sample sizes of 20, 40 and 80 are used with each experiment replicated 20 times. The null hypothesis H_0 against the alternative hypothesis H_1 for decrease returns scale constant returns to scale and increase returns scale were conducted at 5 % significant level. The result obtained from the test conducted showed that for $\lambda = 0.65$, the powers of the test for T=20 is 0.7000(70 %), for T=40 is 0.9000 (90 %) and for T=80 is 1.0000(100%) respectively. For $\lambda = 1.00$, the power of the test for T=20 is 0.7000(70 %), for T=40 is 0.9000(90 %) and for T=80 is 1.0000(100 %) respectively. It the same for $\lambda = 1.35$. It has been found that as the sample size increases, the power of the test decreases and vice-visa.

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1. Introduction

Cobb-Douglas production function plays an important role in modeling certain phenomena. In economics, it is frequently used in research works on production, cost function and demand. A production function is a quantitative link between production inputs and outputs. It summarizes the conversion of inputs into outputs.

Johnson and Samuel (2013) affirms that production function establishes the functional relationship between the quantity of a specific product that can be produced within a time and a set of inputs used, given the existing technology in a socio-cultural environment.

According to shaiara and Md (2016) production function provides quantitative link between inputs and output. Production function can be applied to a single firm, an industry, or an entire nation. The traditional theory of production function of a firm expresses output as a function of two inputs capital (K) and labour (L) in the form of Cobb-Douglas function. The study aimed at Monte Carlo power of test for Return to Scale in production function.

1.1 Cobb-Douglas Production Function

The Cobb-Douglas production function is the simplest production function widely used to represent the technological relationship between the amounts of two or more inputs, and the amount of output that can be produced by those inputs.

According to Bao (2008) the Cobb-Douglas production function was proposed by Knut Wicksell (1851-1926) and was used by Charles Cobb-Douglas and Paul Douglas in the study in which they modeled the growth of American Economy during the period (1899-1922). The Cobb-Douglas production function omitting the error term is of the form.

$$Y = f(K, L) = \Theta_0 K^{\Theta_1} L^{\Theta_2} \quad \textbf{1}$$

Where,

Y = Production output
K = Capital input
L = Labour input
Θ_0 = Constant parameter

Θ_1 and $\Theta 2$ are the positive parameter

Θ_1 and Θ_2 are the output elasticities of capital and labour respectively, and their sum $\Theta_1 + \Theta_2$ is called as a measure of returns to scale.

Let $\lambda = \Theta_1 + \Theta_2$
When $\lambda > 1$, means increase returns to scale.
When $\lambda = 1$, means constant returns to scale.
When $\lambda < 1$, means decrease returns to scale.

Case 1: $\boldsymbol{\lambda} > 1$, if the inputs (Capital and Labour) are increased by an amount say n, then output increases by an amount greater than n

Case 2: For $\boldsymbol{\lambda} = 1$, if the inputs (Capital And Labour) are increased by an amount say (n), then output also increases by an amount n.

Case 3: For $\boldsymbol{\lambda} < 1$, if the inputs (Capital and Labour) are increased by an amount say (n), then output increases by an amount less n.

In spite of the important role played by the producing sectors or industries in Nigeria and other countries, they faced with problems of estimation of parameters, measuring of returns to scale, finding power of Monte Carlo test involving different return to scale of Cobb-Douglas production function. The objectives of the study are:

i. Estimate Cobb-Douglas Production Function.

ii. Find power of Monte Carlo test involving different returns to scale.

1.2 Literature Review

Ashfag and Muhummad (2015); estimated Cobb-Douglas production function to investigate the relationship between the production of cement and inputs labour and capital. The results of the estimates showed that there is a constant return to scale in the cement industry, moreover, the empirical results also showed that the capital contributes relatively less than the labour during the production process. From this study, it was concluded that there is a strong relationship between the input and output variables.

Iyabode and Benjamin (2017); investigated the presence of Heteroscedasticity using the Cobb-Douglas and Exponential production function models. The nonlinear were transformed to an linear model by the natural logarithm. The result of the tests they carried out at 0.01 and 0.05 significance levels showed that as the simple size increases, for every level of heteroscedasticity, the power of the Glejser and park increases in detecting heteroscedasticity compare to other tests. The results obtained also revealed that the power of the tests is more powerful at every level of heteroscedasticity for the two models. Hien and Shino (2017) empirically examined the relationship between firm size, production efficiency, and returns to scale.

They applied a developed stochastic frontier approach on the data obtained from Vietnam and their analysis showed that across all the sectors they considered, production efficiency is most variable among the middle -sized firms in addition, most firms across different sized groups showed constant returns to scale technologies. The result of the analysis using spearman coefficient revealed that there is a significant difference in technological and this difference varies across size groups in all the sectors. The study also showed that the least efficient size also differs across sectors.

Hossain and Al-Amri (2010); affirmed that for most of the selected industries, the Cobb-Douglas function fits the data very well in terms of labour and capital elasticity, return to scale measurements, standard errors, economy of the industries, high value of R^2 and reasonably good Durbin- Watson statistics. From the study, the estimated results implied that the manufacturing industries of Oman generally seem to indicate the case of increasing return to scale. Seven of the nine industries exhibit increasing return to scale and only the rest two showed decreasing return to scale. They also found that no industries with constant return to scale. A recent study by Hossain and Mahunder (2015); showed that estimates of both capital and labour elasticity of Cobb-Douglas function with additive errors are more efficient than those estimates of Cobb-Douglas production function with multiplicative errors.

1.3 Materials and methods

The data for this study are generated data of crude oil production in Nigeria.

1.4 Estimation Method

This study considers Cobb-Douglas production function with Multiplication error term. The model is given by

$$Y = \Theta_0 K^{\Theta_1} L^{\Theta_2} e^u \qquad \mathbf{2}$$

When,

Y = Production output

K = Capital invested in the production

L = Labour used in the production

Θ_0 =Positive constant or Technological constant.

Θ_1 and Θ_2 are positive parameters output elasticities of capital and Labour

U = Random or stochastic error

e = Base of natural logarithm

The model in (3.1) can be transformed to linear model by taking the natural logarithm of both sides of the equation to obtain a regression model of the form.

$$Lny = Ln\,\Theta_0 + \Theta_1 Ln\,K + \Theta_2\,Ln + u \qquad \mathbf{3}$$

The ordinary least square (OLS) estimation is used for the linear model to obtain the estimate $\Theta = (\Theta_0, \Theta_1, \Theta_2)$.

The choice of model parameters $(\Theta_0, \Theta_1, \Theta_2)$ is such that $\Theta_1 + \Theta_2 < 1$

$\Theta_1 + \Theta_2 = 1$ and $\Theta_1 + \Theta_2 > 1$, while the value of Θ_0 is arbitrary and kept constant at

$Ln\,\Theta_0 = 3$, $\Theta_0 = 20.09$

We use the following three sets of parameters:

$V_1 = (20.09, 0.35, 0.30)$, $V_2 = (20.09, 0.55, 0.45)$, $V_3 = (20.09, 0.75, 0.60)$

The input matrix is made of two variables K (Capital) and L (Labour) which are randomly generated and normally distributed independently.

1.5 Simulation

The Mont Carlo Study uses Sample size of 20, 40 and 80 with each experiment replicated 20 times under the following three conditions, varied one at a time while the others are kept: the sample size T and the parameters set $\Theta = (\Theta_0, \Theta_1, \Theta_2)$ used in the data generating process.

1.6 Empirical Results and Discussion

We have estimated a total of 180 equations. θ = (20.09, 0.35, 0.30) with sample size T = 20 and 20 replications. In all the tables, N stands for the number of replication. The model (3.2) is a multiplicative error based model which is filled to the data generated. The strata software package is use to analysed the data.

1.7 Hypothesis testing

Here, we conducted tests of null hypothesis H_o against the alternative Hypothesis against the alternative hypothesis H_1 at 5% level of significance, for different returns to scale in Cobb-Douglas production function. If the probability value (P-value) is greater than the significance level (α= 0.05).

We accept Ho and reject H_1 and if the probability value (p-value is less than the significance level (α =0.05), then we reject H_o. The results of the power of Monte Carlo test involving different returns to scale (decrease return to scale, constant return to scale and increase return to scale are summarized and presented in tables 4.1 to 4. 10 below. Table 1 shows the Power of Monte Carlo Test for Returns to scale 20 Samples.

Ho: $\theta_1+\theta_2 = 0.50$ vs H_1: $\theta_1 + \theta_2 = 0.65$

Power of T = 20, N = 20

$Y = \text{In } \theta_0 + \theta_1 \text{ lnk} + \theta_2 \ln L + v - u.$

Table 1: Power of Monte Carlo Test for Returns (20 Samples)

Replication	P-value	Decision α - 0.05
1	0.2498	Accept H_o
2	0.3899	Accept H_o
3	0.4848	Accept H_o
4	0.0001	Reject H_o
5	0.0000	Reject H_o
6	0.0000	Reject H_o
7	0.0000	Reject H_o
8	0.0003	Reject H_o
9	0.0000	Reject H_o
10	0.0000	Reject H_o
11	0.0947	Accept H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0000	Reject H_o
15	0.0109	Reject H_o
16	0.0733	Accept H_o
17	0.0000	Reject H_o
18	0.0000	Reject H_o
19	0.2292	Accept H_o
20	0.0000	Reject H_o
		R = 14/20= 0.700

Table 2 shows the Power of Monte Carlo Test for Return to Scale for 40 samples.

H_o: $\theta_1+ \theta_2 = 0.50$ vs H_1: $\theta_1+\theta_2 = 0.65$

T = 40, N = 20, θ = (20.09, 0.35, 0.30)

$Y = \text{In } \theta_0 + \theta_1 \text{ lnk} + \theta_2 \text{ lnL} + v - u.$

Table 3 Power of Monte Carlo Test for Returns to Scale for 80 samples.

H_o: $\theta_1+ \theta_2 = 0.50$ vs H_1: $\theta_1+\theta_2 = 0.65$

T = 80, N = 20, θ= (20.09, 0.35, 0.30)

$Y = \text{In } \theta_0 + \theta_1 \text{ lnk} + \theta_2 \text{ lnL} + v - u.$

Table 2: Power of Monte Carlo Test for Return to Scale (40 Samples)

Replication	Test for Returns to Scale	
	P-value	Decision α - 0.05
1	0.0555	Accept H_o
2	0.0000	Reject H_o
3	0.0000	Reject H_o
4	0.0173	Reject H_o
5	0.0001	Reject H_o
6	0.0012	Reject H_o
7	0.0000	Reject H_o
8	0.0030	Reject H_o
9	0.0000	Reject H_o
10	0.0349	Reject H_o
11	0.0000	Reject H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0001	Reject H_o
15	0.0000	Reject H_o
16	0.2526	Accept H_o
17	0.0000	Reject H_o
18	0.0000	Reject H_o
19	0.0000	Reject H_o
20	0.0000	Reject H_o
		18/20 = 0.900

Table 3: Power of Monte Carlo Test for Returns to Scale (80 Samples)

Replication	Test for Returns to Scale	
	P-value	Decision α - 0.05
1	0.0035	Reject H_o
2	0.0000	Reject H_o
3	0.0000	Reject H_o
4	0.0000	Reject H_o
5	0.0003	Reject H_o
6	0.0000	Reject H_o
7	0.0010	Reject H_o
8	0.0052	Reject H_o
9	0.0000	Reject H_o
10	0.0010	Reject H_o
11	0.0000	Reject H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0041	Reject H_o
15	0.0000	Reject H_o
16	0.0000	Reject H_o
17	0.0212	Reject H_o
18	0.0000	Reject H_o
19	0.0002	Reject H_o
20	0.0000	Reject H_o
		R = 20/20 = 1.000

Table 4 shows the Power of Monte Carlo Test for scale. for 20 samples.

H_o: $\theta_1 + \theta_2 = 0.85$ vs H_1: $\theta_1 + \theta_2 = 1.00$

T = 20, N = 20, $\theta = (20.09, 0.55, 0.45)$

$Y = \text{In } \theta_0 + \theta_1 \text{ lnk} + \theta_2 \text{ lnL} + v\text{-}u$.

Table 4: Power of Monte Carlo Test for scale (20 Samples)

Replication	Test for Returns to Scale	
	P-value	Decision α - 0.05
1	0.2498	Accept H_o
2	0.3899	Accept H_o
3	0.4848	Accept H_o
4	0.0001	Reject H_o
5	0.0000	Reject H_o
6	0.0000	Reject H_o
7	0.0000	Reject H_o
8	0.0003	Reject H_o
9	0.0000	Reject H_o
10	0.0000	Reject H_o
11	0.0947	Accept H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0000	Reject H_o
15	0.0109	Reject H_o
16	0.0733	Accept H_o
17	0.0000	Reject H_o
18	0.0000	Reject H_o
19	0.2292	Accept H_o
20	0.0000	Reject H_o
		R = 14/20 = 0.7000

Table 5 Power of Monte Carlo Test for Returns to scale for 40 samples.

H_o: $\theta_1 + \theta_2 = 0.85$ vs H_1: $\theta_1 + \theta_2 = 1.00$

T = 40, N = 20, $\theta = (20.09, 0.55, 0.45)$

$Y = \text{In } \theta_0 + \theta_1 \text{ lnk} + \theta_2 \text{ lnL} + v\text{-}u$.

Table 5: Power of Monte Carlo Test for scale (40 Samples)

Replication	Test for Returns to Scale	
	P-value	Decision α - 0.05
1	0.0555	Accept H_o
2	0.0000	Accept H_o
3	0.0000	Reject H_o
4	0.0173	Reject H_o
5	0.0001	Reject H_o
6	0.0012	Reject H_o
7	0.0000	Accept H_o
8	0.0030	Reject H_o
9	0.0000	Reject H_o
10	0.0349	Reject H_o
11	0.0000	Reject H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0001	Reject H_o
15	0.0000	Reject H_o
16	0.2526	Accept H_o
17	0.0000	Reject H_o
18	0.0000	Reject H_o
19	0.0000	Reject H_o
20	0.0000	Reject H_o
		R = 18/20 = 0.9000

Table 6 Power of Monte Carlo Test for to scale for b80 samples.

H_o: $\theta_1 + \theta_2 = 0.85$ vs H_1: $\theta_1 + \theta_2 = 1.00$

T = 80, N = 20, θ = (20.09, 0.55, 0.45)

$Y = \text{In } \theta_0 + \theta_1 \text{ lnk} + \theta_2 \text{ lnL} + v - u.$

Table 6: Power of Monte Carlo Test for scale (80 Samples)

Replication	Test for Returns to Scale	
	P-value	Decision α - 0.05
1	0.0035	Reject H_o
2	0.0000	Reject H_o
3	0.0000	Reject H_o
4	0.0000	Reject H_o
5	0.0003	Reject H_o
6	0.0000	Reject H_o
7	0.0010	Accept H_o
8	0.0052	Reject H_o
9	0.0000	Reject H_o
10	0.0010	Reject H_o
11	0.0000	Reject H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0041	Reject H_o
15	0.0000	Reject H_o
16	0.0000	Accept H_o
17	0.0212	Reject H_o
18	0.0000	Reject H_o
19	0.0002	Reject H_o
20	0.0000	Reject H_o
		R = 20/20 = 1.000

Table 7 Power of Monte Carlo Test for Returns to scale for 20 samples.

H_o: $\theta_1 + \theta_2 = 1.20$ vs H_1: $\theta_1 + \theta_2 = 1.35$

T = 20, N = 20, θ = (20.09, 0.75, 0.60)

$Y = \text{In } \theta_0 + \theta_1 \text{ lnk} + \theta_2 \text{ lnL} + v - u.$

Table 7: Power of Monte Carlo Test for scale (20 Samples)

Replication	Test for Returns to Scale	
	P-value	Decision α - 0.05
1	0.2498	Accept H_o
2	0.3899	Accept H_o
3	0.4848	Accept H_o
4	0.0001	Reject H_o
5	0.0000	Reject H_o
6	0.0000	Reject H_o
7	0.0000	Reject H_o
8	0.0003	Reject H_o
9	0.0000	Reject H_o
10	0.0000	Reject H_o
11	0.0947	Accept H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0000	Reject H_o
15	0.0109	Reject H_o
16	0.0733	Accept H_o
17	0.0000	Reject H_o
18	0.0000	Reject H_o
19	0.2292	Accept H_o
20	0.0000	Reject H_o
		R = 14/20 = 0.700

Table 8 Power of Monte Carlo Test for Returns to scale for 40 samples.

H_o: $\theta_1 + \theta_2 = 1.20$ vs H_1: $\theta_1 + \theta_2 = 1.35$

T = 40, N = 20, θ = (20.09, 0.75, 0.60)

$Y = \text{In } \theta_0 + \theta_1 \text{ lnk} + \theta_2 \text{ lnL} + v - u.$

Table 8: Power of Monte Carlo Test for scale (40 Samples)

Replication	Test for Returns to Scale	
	P-value	Decision

		α - 0.05
1	0.0555	Accept H_o
2	0.0000	Reject H_o
3	0.0000	Reject H_o
4	0.0173	Reject H_o
5	0.0001	Reject H_o
6	0.0012	Reject H_o
7	0.0000	Reject H_o
8	0.0030	Reject H_o
9	0.0000	Reject H_o
10	0.0349	Reject H_o
11	0.0000	Reject H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0001	Reject H_o
15	0.0000	Reject H_o
16	0.2526	Accept H_o
17	0.0000	Reject H_o
18	0.0000	Reject H_o
19	0.0000	Reject H_o
20	0.0000	Reject H_o
		R= 18/20 = 0.9000

Table 9 shows the Power of Monte Carlo Test for Returns to Scale for 80 samples.

H_o: θ_1+ θ_2 = 1.20 vs H_1: θ_1+θ_2 = 1.35

T = 80, N = 20, θ = (20.09, 0.75, 0.60)

Y = In θ_0 + θ_1 lnk + θ_2 lnL + v-u.

Table 9: Power of Monte Carlo Test for scale (80 Samples)

Replication	Test for Returns to Scale	
	P-value	Decision α - 0.05
1	0.0035	Reject H_o
2	0.0000	Reject H_o
3	0.0000	Reject H_o
4	0.0000	Reject H_o
5	0.0003	Reject H_o
6	0.0000	Reject H_o
7	0.0010	Reject H_o
8	0.0052	Reject H_o
9	0.0000	Reject H_o
10	0.0010	Reject H_o
11	0.0000	Reject H_o
12	0.0000	Reject H_o
13	0.0000	Reject H_o
14	0.0041	Reject H_o
15	0.0000	Accept H_o
16	0.0000	Reject H_o
17	0.0212	Reject H_o
18	0.0000	Reject H_o
19	0.0002	Reject H_o
20	0.0000	Reject H_o
		R = 20/20 = 1.000

Table 10 shows the Power of Monte Carlo test for Returns to scale for samples 20, 40 and 80 respectively.

Lny = lin θ_o + θ_1 lnK + θ2 lnl + v - u.

Table 10: Power of Monte Carlo Test for scale for samples 20, 40 and 80

T	K λ = 0.65	K λ = 1.00	K λ = 1. 35
20	0.7000	0.7000	0.7000
40	0.9000	0.9000	0.9000
80	1.0000	1.0000	1.0000

1.8 Conclusion

In economics, the sum of power of the input variables K and L is interpreted as a measure of returns to scale. The results have shown that for different returns to scale for various sample sizes T, the power of Monte Carlo test are same. The results also showed that as the sample size T increases, the power of the test increases.

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