**Inequality of Nikolsky and Bernshteins’s type classification within **

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**Abstract:** In this work the analytical functions in the upper semi plane are learned. Therefore, there received inequalities for norms of Hardy space **,** which is the analogue in some ideas inequality of S.M. Nokolsiy and S.N. Bernstein.

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**Keywords:** space **,** inequality ofS.M. Nokolsiy, S.N. Bernstein.

**1. Introduction.**

Let  is a space of analytical in the upper semi plane functions  meeting the condition







Let  means a space of all measured on  functions for which

 (A)

and when 



Clearly, if , the set  is space with the norm defined by (A). If  the formula (A) does not define a norm since the triangle inequality is not satisfied. However, in this case  is a linear metric space.

For the entire functions of the degree  within the space  an inequality (see [1], p.150)

 (1)

is known as Nikolsky’s inequality, (see [1], p.137-138) also an inequality

 (2)

is known as Bernshtein’s inequality.

Let’s underline, that an analog of an inequality (2) when  is calculated by the author [2] for natural numbers  , while for the fractional number  if in [3]. Some properties of the function  possessing derivatives of a fraction order were investigated by us in works [4] and [5].

**2. Problem formulation.**

The aim of the work is to receive an analog of an inequality (1), (2) and an analog of one inequality of Hardy-Littlewoods [6] within the spaces.

**3. Subsidiary facts.**

For proving the basic result the following is urgent.

1. It is known (see [7], formulae (2.7)) that for the function  there is a representation

. (3)

1. The analytical function  in the upper semi plane has a representation (see [7])

 *y>y1*>0 (4)

1. Integral

 (5)

as a function from  does not increase (see [8])

1. The inequality (see [7])

 (6)

occurs.

1. If



 (7)

then correlations received by Hardy and Littlewoods occur (see [9]).

**4. Basic results.**

**Theorem 1.** If , then an inequality occurs:

 0<p<1,  y>y0≥0, (8)

 (9)

Let’s mark that the constant  and if then **.**

**Theorem 2.** If  and there is a derivative of the order, then an inequality:

 (10)

occurs, when  constant  doesn’t depend on.

Further  means a constant, depending on 

**Theorem 3.** If function, then when  an inequality

 (11)

where -is a module of continuity (see [1], p. 174-180) of the boundary function  in, i.e.

.

From theorem 2 and 3 the following stems:

***Corollary fact 1.*** If the condition



is fulfilled, then

  

This is an analog of one result of Hardy and Littlewoods [10], calculated for periodical functions in the class.

***Corollary fact 2.*** If the boundary function  meets the condition



then





Inequality (8) and (9) are at Nikolsky’s type classification (see (1)). Inequality (10) is of Bernstein’s type classification (see (2)).

Let’s note that inequality (11) is an analog of inequality calculated by Yu. A. Brudniy and Hopengauz for analytical functions of the unit disk at  and at by E. A. Storojenko and Ya. Valashek [11] (for poly harmonically functions in disk M. F. Timan [12]).

Inverse inequality to inequality (11) for integral functions of the degree ≤σ within the space gives us lemma 1 in [13].

**Proving theorem 1.**

In equality (3) we shall replace function  in to functions (see [14], p.101).

 (12)

We shall

 (13)

from (12)

 (14)

stems. Hence it follows

 (15)

as  and , then applying Minkovsky’s inequality in the right part of inequality (15) and considering (13) we receive:

 (16)

Now, let’s estimate integral

 (17)







Thus, integral (17) is estimated as

 (18)

Considering estimates (8) under inequality (16) we find:

,

i.e. the theorem is proved for Now, let’s proved a general case. From equality (14) we have:

,

as, then, applying generalized inequality of Minkovsky we receive:

.

Considering designation (13), from the last inequality we receive

 (19)

Inner integral in the right part of inequality (19) is estimated in the same way as (17), then, calculating in detail we receive:

 (20)

Considering estimation (20) from inequality (19) we get

 , , 

The theorem is proved for.

Let’s consider when . From (12) we got

 .

Hence it follows

,

i.e. the affirmation of the theorem when q=∞. The theorem is proved completely.

**Proving theorem 2.**

Applying inequality (4) to functions , where, is a free substantial and 

.

Hence, differentiating by, we find out that

 (21)

Supposing  at  from the last inequality we receive



. (22)

Consider integral in the right part (22)

. (23)

Applying theorem 1 when  for integral (23) we receive:

. (24)

We choose a substantial  so that  and considering that



from inequality (24) we receive

. (25)

Now, from inequalities (22), (23), (25) we find out that



. (26)

Let’s note that

. (27)

as  and is unconditioned, then supposing that, where, from inequality (26) considering (27), we receive:

 . (28)

As integral (5) doesn’t increase, then from inequality (28) we receive:



i.e. the theorem is proved for  Repeating reasoning given above  times; we get the affirmation of the theorem for any  when. When we adduce reasoning similar to, but in this case when integrating inequality (22) we apply Minkovsky’s generalized inequality.

**Proving theorem 3.**

Let’s denote that, then from inequality (3) we receive:



Here is considered that

,  .

Further, replacing variables, we receive:

 (29)

Let. Applying Minkovsky’s generalized inequality we calculate that

 . (30)

Stemming from (6) we receive

.

Under (30) we receive:

 . (31)

Let’s consider  and 

.

Under monotony of the module of continuity we receive:

.

Under continuity module we receive that:



Consequently,

, (32)

similarly

 (33)

and at last we have, that (see (31), (32) and (33))



The theorem is proved.

**5. Conclusion.**

Let’snote that theorem 1, shows the correlation between quantities  at various parameters of and , being an analog of Nikolsky’s (1) inequality. Theorem 2 shows the connection between functions  and its derivative  within the spaces, being an analog of Bernstein’s (2) inequality. Theorem 3 is an analog of Brudny and Hopengauz’s results (consequences 1 and 2 are analogs of Hardy-Littlewoods) received for analytical functions in the unit disk.

Consider, that some issues of approximation functions in spaces , by whole functions of final levels learned in this work [13].

**6. Comment.**

Theorem 1 and 2 are proved by G.Gaimnazarov and theorem 3 is proved by O.G.Gaimnazarov.

**References**

1. Nikolskiy S.M. *Approximation function many variable and theorems of the embedding.* M.: Nauka, 1969. -480 p.
2. Gaymnazarov G. *Some inequality in space*. Dokl. AN Tadzh, 1985. –V. 28. - №12. –p. 685-687.
3. Gaimnazarov G., Gaimnazarov O.G. *On* *some inequalities for functions having derivative of fractional order*. Reports of Academy of Sciences Republic of Uzbekistan, 2011, No 2, pp. 16-21.
4. Gaymnazarov G. *About module of smoothness of the fractional order function, given on the whole material axis*. Dokl. AN Tadzh., 1981. -V. 24. - 3. -p. 148-149.
5. G. Gaimnazarov, H. Narjigitov and O. G. Gaimnazarov *On some properties of function associated with derivative of fractional order in space of Lp(-∞,∞)*. Far East Journal of Mathematical Sciences (FJMS) Volume 76, Number 2, 2013, pp 319-336.
6. Hardy G.H., Littlewood J. E . *Some properties of conjugate functions.* j. reinе and аngеw. Moth 1931, v167. p. 405-423.
7. Helle E., Tamarkin J. *On the absolute integrality of Fourier transforms*. Fundam. Math, v.25, 1935, p.329-351.
8. M. F. Timan, “The imbedding of the **classes of functions”, *Izv. Vyssh. Uchebn. Zaved. Mat.*, 1974, no. 10, 61–74.
9. Hardy G.Н. and Littlewoods J.E. *Theorems concerning Сezaro means of pоwеr series.* Рrоc, London Math. Soc.1934, V36 .p 516-531 .
10. Yu. A. Brudnyi, I. E. Gopengauz, “Generalization of a theorem of Hardy and Littlewood”, *Mat. Sb. (N.S.)*, 52(94):3 (1960), 891–894.
11. Storozhenko Z.A., Valashek YA. *Generalization of one theorem Hardi-Littllwoods.* In book Constructive theory function-81.- Works to international conference on constructive theory function. -Varna, June 1-5, 1981. -Sophia: BAN, 1983.-p.164-167.
12. Krilov V.I. *About function, the regular semi planes in floor*. Mathem.sb.,1939.-V.6.-№1.-p. 95-137.
13. R. R. Akopyan, “Approximation of the Hardy–Sobolev class of functions analytic in a half-plane by entire functions of exponential type”, Trudy Inst. Mat. i Mekh. UrO RAN, 16, no. 4, 2010, 18–30.

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