**Jiang’s function  in prime distribution**

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**Abstract:** We define that prime equations

 （5）

are polynomials (with integer coefficients) irreducible over integers, where  are all prime. If Jiang’s function  then （5）has finite prime solutions. If  then there are infinitely many primes  such that  are primes. We obtain a unite prime formula in prime distribution



 （8）

Jiang’s function is accurate sieve function. Using Jiang’s function we prove about 600 prime theorems [6]. Jiang’s function provides proofs of the prime theorems which are simple enough to understand and accurate enough to be useful.

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*Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.*

Leonhard Euler

*It will be another million years, at least, before we understand the primes.*

Paul Erdös

Suppose that Euler totient function

 as ， （1）

where  is called primorial.

Suppose that, where . We have prime equations

 （2）

where .

（2）is called infinitely many prime equations (IMPE). Every equation has infinitely many prime solutions. We have

, （3）

where  denotes the number of primes  in  ,  the number of primes less than or equal to .

We replace sets of prime numbers by IMPE. (2) is the fundamental tool for proving the prime theorems in prime distribution.

Let  and . From (2) we have eight prime equations

, , , , ,

, , ,  （4）

Every equation has infinitely many prime solutions.

**THEOREM**. We define that prime equations

 （5）

are polynomials (with integer coefficients) irreducible over integers, where  are primes. If Jiang’s function  then (5) has finite prime solutions. If  then there exist infinitely many primes  such that each  is a prime.

**PROOF**. Firstly, we have Jiang’s function [1-11]

, （6）

where  is called sieve constant and denotes the number of solutions for the following congruence

, （7）

where .

 denotes the number of sets of  prime equations such that  are prime equations. If  then (5) has finite prime solutions. If  using  we sift out from (2) prime equations which can not be represented , then residual prime equations of (2) are  prime equations such that   are prime equations. Therefore we prove that there exist infinitely many primes  such that   are primes.

Secondly, we have the best asymptotic formula [2,3,4,6]



 （8）

（8）is called a unite prime formula in prime distribution. Let , . From (8) we have prime number theorem

. （9）

Number theorists believe that there are infinitely many twin primes, but they do not have rigorous proof of this old conjecture by any method. All the prime theorems are conjectures except the prime number theorem, because they do not prove that prime equations have infinitely many prime solutions. We prove the following conjectures by this theorem.

**Example 1.** Twin primes ****(300BC)**.**

From (6) and (7) we have Jiang’s function

.

Since  in (2) exist infinitely many  prime equations such that  is a prime equation. Therefore we prove that there are infinitely many primes  such that is a prime.

Let  and . From (4) we have three  prime equations

.

From (8) we have the best asymptotic formula





In 1996 we proved twin primes conjecture [1]

Remark.  denotes the number of  prime equations,  the number of solutions of primes for every  prime equation.

**Example 2.** Even Goldbach’s conjecture **.** Every even number  is the sum of two primes**.**

From (6) and (7) we have Jiang’s function

.

Since  as  in (2) exist infinitely many  prime equations such that  is a prime equation. Therefore we prove that every even number  is the sum of two primes.

From (8) we have the best asymptotic formula



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In 1996 we proved even Goldbach’s conjecture [1]

**Example 3.** Prime equations **.**

From (6) and (7) we have Jiang’s function

,

 is denotes the number of  prime equations such that  and  are prime equations. Since  in (2) exist infinitely many  prime equations such that  and  are prime equations. Therefore we prove that there are infinitely many primes  such that  and  are primes.

Let . From (4) we have two  prime equations

.

From (8) we have the best asymptotic formula



**Example 4**. Odd Goldbach’s conjecture . Every odd number  is the sum of three primes.

From (6) and (7) we have Jiang’s function

.

Since  as  in (2) exist infinitely many pairs of  and  prime equations such that  is a prime equation. Therefore we prove that every odd number  is the sum of three primes.

From (8) we have the best asymptotic formula

.

.

**Example 5**. Prime equation .

From (6) and (7) we have Jiang’s function



 denotes the number of pairs of  and  prime equations such that  is a prime equation. Since  in (2) exist infinitely many pairs of  and  prime equations such that  is a prime equation. Therefore we prove that there are infinitely many pairs of primes  and  such that  is a prime.

From (8) we have the best asymptotic formula



Note. deg.

**Example 6** [12]. Prime equation .

From (6) and (7) we have Jiang’s function

,

where  if ;  if ;  otherwise.

Since  in (2) there are infinitely many pairs of  and  prime equations such that  is a prime equation. Therefore we prove that there are infinitely many pairs of primes  and  such that  is a prime.

From (8) we have the best asymptotic formula



**Example 7** [13]. Prime equation .

From (6) and (7) we have Jiang’s function



where  if ;  if ;  otherwise.

Since  in (2) there are infinitely many pairs of  and  prime equations such that  is a prime equation. Therefore we prove that there are infinitely many pairs of primes  and  such that  is a prime.

From (8) we have the best asymptotic formula



**Example 8** [14-20]. Arithmetic progressions consisting only of primes. We define the arithmetic progressions of length .

. （10）

From (8) we have the best asymptotic formula



.

If  then (10) has finite prime solutions. If  then there are infinitely many primes  such that  are primes.

To eliminate  from (10) we have

.

From (6) and (7) we have Jiang’s function



Since  in (2) there are infinitely many pairs of  and  prime equations such that  are prime equations. Therefore we prove that there are infinitely many pairs of primes  and  such that are primes.

From (8) we have the best asymptotic formula



 .

**Example 9**. It is a well-known conjecture that one of  is always divisible by 3. To generalize above to the primes, we prove the following conjectures. Let  be a square-free even number.

1. ,

where .

From (6) and (7) we have , hence one of  is always divisible by 3.

2. ,

where 

From (6) and (7) we have , hence one of  is always divisible by 5.

3. ,

where 

From (6) and (7) we have , hence one of  is always divisible by 7.

4. ,

where 

From (6) and (7) we have , hence one of  is always divisible by 11.

5. ,

where 

From (6) and (7) we have , hence one of  is always divisible by 13.

6. ,

where 

From (6) and (7) we have , hence one of  is always divisible by 17.

7. ,

where 

From (6) and (7) we have , hence one of  is always divisible by 19.

**Example 10**. Let  be an even number.

1. ,

From (6) and (7) we have . Therefore we prove that there exist infinitely many primes  such that  are primes for any .

2. .

From (6) and (7) we have . Therefore we prove that there exist infinitely many primes  such that  are primes for any .

**Example 11**. Prime equation 

From (6) and (7) we have Jiang’s function

.

Since  in (2) there are infinitely many pairs of  and  prime equations such that  is prime equations. Therefore we prove that there are infinitely many pairs of primes  and  such that  is a prime.

From (8) we have the best asymptotic formula



In the same way we can prove  which has the same Jiang’s function.

Jiang’s function is accurate sieve function. Using it we can prove any irreducible prime equations in prime distribution. There are infinitely many twin primes but we do not have rigorous proof of this old conjecture by any method [20]. As strong as the numerical evidence may be, we still do not even know whether there are infinitely many pairs of twin primes [21]. All the prime theorems are conjectures except the prime number theorem, because they do not prove the simplest twin primes. They conjecture that the prime distribution is randomness [12-25], because they do not understand theory of prime numbers.

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**References**

1. Chun-Xuan Jiang, On the Yu-Goldbach prime theorem, Guangxi Sciences (Chinese) 3(1996), 91-2.
2. Chun-Xuan Jiang, Foundations of Santilli’s isonumber theory, Part I, Algebras Groups and Geometries, 15(1998), 351-393.
3. ChunXuan Jiang, Foundations of Santilli’s isonumber theory, Part II, Algebras Groups and Geometries, 15(1998), 509-544.
4. Chun-Xuan Jiang, Foundations Santilli’s isonumber theory, In: Fundamental open problems in sciences at the end of the millennium, T. Gill, K. Liu and E. Trell (Eds) Hadronic Press, USA, (1999), 105-139.
5. Chun-Xuan Jiang, Proof of Schinzel’s hypothesis, Algebras Groups and Geometries, 18(2001), 411-420.
6. Chun-Xuan Jiang, Foundations of Santilli’s isonmuber theory with applications to new cryptograms, Fermat’s theorem and Goldbach’s conjecture, Inter. Acad. Press, 2002, MR2004c: 11001, http://www.i-b-r.org/jiang.pdf
7. Chun-Xuan Jiang Prime theorem in Santilli’s isonumber theory, 19(2002), 475-494.
8. Chun-Xuan Jiang, Prime theorem in Santilli’s isonumber theory (II), Algebras Groups and Geometries, 20(2003), 149-170.
9. Chun-Xuan Jiang, Disproof’s of Riemann’s hypothesis, Algebras Groups and Geometries, 22(2005), 123-136. http://www.i-b-r.org/docs/Jiang Riemann.pdf
10. Chun-Xuan Jiang, Fifteen consecutive integers with exactly  prime factors, Algebras Groups and Geometries, 23(2006), 229-234.
11. Chun-Xuan Jiang, The simplest proofs of both arbitrarily long arithmetic progressions of primes, preprint, 2006.
12. D. R. Heath-Brown, Primes represented by , Acta Math., 186 (2001), 1-84.
13. J. Friedlander and H. Iwaniec, The polynomial  captures its primes, Ann. Math., 148(1998), 945-1040.
14. E. Szemerédi, On sets of integers containing no  elements in arithmetic progressions, Acta Arith., 27(1975), 299-345.
15. H. Furstenberg, Ergodic behavior of diagonal measures and a theorem of Szemerédi on arithmetic progressions, J. Analyse Math., 31(1997), 204-256.
16. W. T. Gowers, A new proof of Szemerédi’s theorem, GAFA, 11(2001), 465-588.
17. B. Kra, The Green-Tao theorem on arithmetic progressions in the primes: An ergodic point of view, Bull. Amer. Math. Soc., 43(2006), 3-23.
18. B. Green and T. Tao, The primes contain arbitrarily long arithmetic progressions, Ann. Math., 167(208), 481-547.
19. T. Tao, The dichotomy between structure and randomness, arithmetic progressions, and the primes, In: Proceedings of the international congress of mathematicians (Madrid. 2006), Europ. Math. Soc. Vol. 581-608, 2007.
20. B. Green, Long arithmetic progressions of primes, Clay Mathematics Proceedings Vol. 7, 2007,149-159.
21. H. Iwanice and E. Kowalski, Analytic number theory, Amer. Math. Soc., Providence, RI, 2004
22. R. Crandall and C. Pomerance, Prime numbers a computational perspective, Spring-Verlag, New York, 2005.
23. B. Green, Generalising the Hardy-Littlewood method for primes, In: Proceedings of the international congress of mathematicians (Madrid. 2006), Europ. Math. Soc., Vol. II, 373-399, 2007.
24. K. Soundararajan, Small gaps between prime numbers: The work of Goldston-Pintz-Yildirim, Bull. Amer. Math. Soc., 44(2007), 1-18.
25. Granville, Harald Cramér and distribution of prime numbers, Scand. Actuar. J, 1995(1) (1995), 12-28.

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