Applying Interval VIKOR and Fuzzy AHP Methods for Supplier Selection

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Abstract: The aim of this study is applying a new integrated method for supplier selection. In this paper, the weights of each criterion are calculated using Fuzzy AHP method. After that, Interval VIKOR is utilized to rank the alternatives. Then we select the best supplier based on these results. The outcome of this research is ranking and selecting supplier with the help of Fuzzy AHP and Interval VIKOR techniques. This paper offers a new integrated method for supplier selection.

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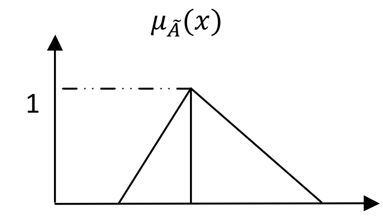
1. Introduction

Supply Chain is a network of facilities that convert raw materials to finished products and distribution of tasks among their customers [1]. A supply chain, as well as alignment the companies that offer a product or service to the market [2]. In other words, supply chains are included directly and indirectly in the completion of customer requests. Supply chain is not only for manufacturers and suppliers, but also includes transport, warehouses, retail and even customers [3]. Therefore, Supply Chain Management (SCM) includes all management activities that satisfy the needs of customers with minimal costs for all companies involved in the production and delivery of products [4]. The term of SCM was coined by consultant Oliver and Weber in 1982 [5]. Copczak refers to supply chain as group of institutions consisting of suppliers, logistic service providers, producers, distributors and sellers that are connected through flow of information, materials and products [6]. Because the price and quality of the products are directly related to the price and quality of purchased raw material, supplier evaluation and selection is very crucial in manufacturing companies' success [7]. Supplier selection decisions determine who the suppliers should be chosen as the resource to buy or how to order quantities should be allocated among the selected suppliers [8]. Choosing the best supplier is a critical decision for wide range of conclusions in a supply chain [9] .Supplier selection process requires a systematic and effective method that will help the buyer to help obtain the most effective decision. The process of supplier selection requires systematic and efficient methods that help the buyer to make best decision [10]. Considering the fact that supplier selection is a multi-criteria problem, it's recommended to apply multiple criteria for measuring the optimum solution and also multiple-criteria decision-making model. These decision-making models are categorized in two groups: multiple objective decision-making models (MODM) and multiple attribute decision-making models (MCDM). While multiple objective models are used to design, multiple attribute models are used to select the best alternative. Since this paper intends to evaluate, select and rank the suppliers, multiple attribute decision-making models (MADM) are applied. In MCDM problem, particularly in MADM, we need to have the relative importance of the criteria. This relative importance shows the importance of each criterion in respect to other criteria for decision-making. There are two different methods to obtain these weights: 1- subjective method 2- objective method. One of the most applicable techniques to obtain the relative importance is analytic hierarchy process (AHP). In traditional models all criteria values were known while in real world this cannot be true. Due to uncertainty in data, most of decision-makers are faced with imprecise data [11]. Hence the purpose of this paper is to first obtain the weights of criteria in selecting suppliers by fuzzy analytic hierarchy process (FAHP) and then select the best supplier or suppliers by interval VIKOR method. First paper on supplier selection was written by Dikson in 1966. Dikson introduced 23 criteria in supplier selection. This greatly influenced the researchers in this area [12]. Linear weighting is one of the most common methods for ranking and selecting the different suppliers regarding their performance criteria. This method was introduced in 1968 by Robinson and Wind [13]. Anthony and Buffa formulized the supplier selection problem by linear programming method to minimize the overall costs of purchase and warehouse [14]. Timmerman introduced a comprehensive weighting method to rank the suppliers [15]. Gregory (1986) applied a matrix method to rank suppliers based on scoring and weighting to factors [16].Weber and et al (1991) reviewed the literature of supplier selection and its methods. Weber and et al categorized 74 relevant articles that all of them had been published since 1966. Weber and et al categorized quantitative approaches on supplier selection into 3 groups: linear weighing models, Mathematical programming models and Statistical probabilistic approaches [17]. Ghodsypour and O'Brien introduced a model consisting of AHP and linear programming for supplier selection [18]. Liu (2000) applied DEA method based on multiple objectives to evaluate and rank the suppliers. The model introduced by him was intended to evaluate the suppliers for selecting the best one [19]. Chen (2001) presented a fuzzy MCDM model to study the supplier selection [20]. Baker and Talluri (2202) designed a new multi-step mathematical programming model for supplier selection [21]. Bayraktar and Cebi (2003) presented an integrated model based on four major criteria such as logistic, technology, business and relationships [22]. Kumar (2004) presented a supplier selection method based on goal programming [23]. In 2007, Farzipoor Saen presented a data envelopment analysis method using cardinal and ordinal variables for selecting the optimal supplier [24]. In 2009, Chen and Wang provided a fuzzy VIKOR for the application of IS/information technology (IT) outsourcing projects [25]. Sanayei, Farid, and Yazdankhah (2010) and Shemshadi, Shirazi, Toreihi, and Tarokh (2011) also integrated the VIKOR method with fuzzy concepts [26, 27]. The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory and VIKOR method. Section 3 presents the proposed method. The application of the proposed method is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. Background

2.1. Fuzzy sets and Fuzzy Numbers

Fuzzy set theory, which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework[28]. A fuzzy set $\tilde{A}$ can be defined mathematically by a membership function $\mu_{\tilde{A}}(X)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a, b, c) as illustrated in Fig 1.



F 0 : A L ngu Mfuzzy nu U r $\tilde{A}$

The membership function $\mu_{\tilde{A}}(X)$is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-c}{b-c} & b \le x \le c \\ 0 & oterwise \end{cases} \qquad (1)$$

Basic arithmetic operations on triangular fuzzy numbers $A_1 = (a_1,b_1,c_1)$, where $a_1 \le b_1 \le c_1$, and $A_2 = (a_2,b_2,c_2)$, where $a_2 \le b_2 \le c_2$,can be shown as follows:

Addition: $A_1 \oplus A_2 = (a_1 + a_2 ,b_1 + b_2,c_1 +c_2)$ (2)

Subtraction: $A_1 \ominus A_2 = (a_1 - c_2 ,b_1 - b_2,c_1 - a_2)$ (3)

Multiplication: if k is a scalar

$$k \otimes A_1 = \begin{cases} (ka_1 , kb_1, kc_1), & k > 0 \\ (kc_1 , kb_1, ka_1), & k < 0 \end{cases}$$

$A_1 \otimes A_2 \approx (a_1a_2 ,b_1b_2,c_1c_2)$, if $a_1 \ge 0$, $a_2 \ge 0$ (4)

Division: $A_1 \oslash A_2 \approx (\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2})$, if $a_1 \ge 0$, $a_2 \ge 0$ (5)

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications [29]. Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including personnel selection (e.g. rating for creativity, personality, leadership, etc.). The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation [30].A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill defined to be described in conventional quantitative terms [31].

2-2.VIKOR

The VIKOR method, introduced by Opricovic in 1998, is an effective technique in multi-criteria decision-making (MCDM) which is originated from the compromise programming method in solving problems with conflicting criteria. This method focuses on evaluating, ranking and selecting from a set of alternatives in the presence of conflicting criteria. It is particularly used when the decision-maker is not capable of uttering his preferences at the beginning of designing a system. The decision-maker needs a solution which is closest to the ideal. Assume the decision-making matrix demonstrated below (Table 1):

Table 1: Structure of the alternative performance matrix

C_j / A_i	C_1	…	C_j	…	C_n
A_1	f_{11}	…	f_{1j}	…	f_{1n}
⋮	⋮		⋮		⋮
A_i	f_{i1}	…	f_{ij}	…	f_{in}
⋮	⋮		⋮		⋮
A_m	f_{m1}	…	f_{mj}	…	f_{mn}
W_j	W_1		W_j		W_n

Where A_1, A_2, ..., Am are possible alternatives among which the decision-maker must choose. C_1, C_2, ..., C_n are criteria with which alternative performance is measured. f_{ij} is the rating of alternative Ai with respect to criterion Cj, w_j are the weights of criteria, expressing their relative importance.

Development of the VIKOR method is started with the following form of L -metric:

$$L_{pi} = \{\sum_{j=1}^{n}[(f_j^* - f_{ij})/(f_j^* - f_j^-)]^p\}^{1/p} \quad 1 \leq p \leq \infty, \quad i = 1, \dots, m. \tag{6}$$

In the VIKOR method $L_{1,i}$ (as S_i) and $L_{1,i}$ (as R_i) are used to formulate ranking measure. The solution obtained by min Si is with a minimum individual regret of the a maximum group utility (''majority'' rule), and the solution obtained by min R_i is with a minimum individual regret of the ("opponent").

(a) Determine the best f_j^* and the worst f_j^- values of all criterion functions j = 1, 2,. . ., n. If the jth function represents a benefit then:

$$f_j^* = \max_i f_{ij}^U, \qquad f_j^- = \min_i f_{ij}^L.$$

(b) Compute the values S_i andR_i; i = 1, 2, . . ., m, by following relations:

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - f_{ij})/(f_j^* - f_j^-). \tag{7}$$

$$R_i = \max_j w_j (f_j^* - f_{ij})/(f_j^* - f_j^-) \tag{8}$$

(c) Compute the valuesQ_i; i = 1, 2, . . ., m, by the relation given below:

$$Q_i = V(S_i - S^*)/(S^- - S^*) + (1 - V)(R_i - R^*)/(R^- - R^*) \tag{9}$$

where

$$S^* = \min_i S_i, \quad S^- = \max_i S_i,$$
$$R^* = \min_i R_i, \quad R^- = \max_i R_i,$$

V is introduced as weight of the strategy of ''the majority of criteria'' (or ''the maximum group utility''), here assume that, v = 0.5. It can be interpreted that the less is the amount of V, the more is accentuated the idea of the individuals and vice versa.

(d) Rank the alternatives, sorting by the values S, R and Q in decreasing order. The results include three ranking lists.

(e) Propose as a compromise solution the alternativeA', which is ranked the best by the measure Q (Minimum) if the following two conditions are satisfied:

C1. Acceptable advantage:

$$Q(A'')\text{-}Q(A') \geq DQ \tag{10}$$

where A'' is the alternative with second position in the ranking list by Q; DQ=1/(m- 1); m is the number of alternatives.

C2. Acceptable stability in decision making:

Alternative A' must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be ''voting by majority rule'' (when v > 0.5 is needed), or ''by

consensus" v≈ 0.5, or ''with veto" (v < 0.5). Here, v is the weight of the decision making strategy ''the majority of criteria" (or ''the maximum group utility").
If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives A' and A A'' if only condition C2 is not satisfied, or
- Alternatives $A', A'', \ldots, A^{(M)}$ if condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)})$) À $Q(A') < DQ$ for maximum M (the positions of these alternatives are ''in closeness").

The best alternative, ranked by Q, is the one with the minimum value of Q. The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the ''advantage rate". VIKOR is an effective tool in multi-criteria decision making, particularly in a situation where the decision maker is not able, or does not know to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum ''group utility" (represented by min S) of the ''majority", and a minimum of the ''individual regret" (represented by min R) of the ("opponent"). The compromise solutions could be the basis for negotiations, involving the decision maker's preference by criteria weights [32].

3. Proposed method

As it was discussed in the introduction, specifying the exact value of the decision matrix elements is not always possible and such values are often obtained as imprecise. The interval numbers does not indicate how probable it is to the value to be in the interval, nor does it indicate which of the many values in the interval is the most likely to occur [33]. In other way, an interval can be considered as:
(1) An extension of the concept of a real number and also as a subset of the real line.
(2) A degenerate flat fuzzy number or fuzzy interval with zero left and right spreads.
(3) An α-cut of a fuzzy number [34].
(4) The numbers obtained from the transformation of ordinal numbers to interval numbers.

Extensive researches which were conducted on interval arithmetic and its applications can be found in [35, 36 and 37]. Other studies which investigated the interval numbers and its differences with other methods such as probability and fuzzy theory can be found in [34, 38 and 39]. Therefore methods which are developed for precise data cannot be used in such conditions. As a result VIKOR method will be developed to work with interval data in which the steps followed are the same as classic VIKOR method but unlike the classic version it is intended to work with interval numbers.

Step1) Defining the decision matrix:

According to these facts that specifying determining the exact values of the decision matrix elements is not plausible in real world situation, therefore, the values are computed as intervals. The interval decision matrix is illustrated as below:

Table 2: Structure of the alternative performance matrix with interval numbers

A_i \ C_j	C_1	…	C_j	…	C_n
A_1	$[f_{11}{}^L, f_{11}{}^U]$	…	$[f_{1j}{}^L, f_{1j}{}^U]$	…	$[f_{1n}{}^L, f_{1n}{}^U]$
⋮	⋮		⋮		⋮
A_i	$[f_{i1}{}^L, f_{i1}{}^U]$	…	$[f_{ij}{}^L, f_{ij}{}^U]$	…	$[f_{in}{}^L, f_{in}{}^U]$
⋮	⋮		⋮		⋮
A_m	$[f_{m1}{}^L, f_{m1}{}^U]$	…	$[f_{mj}{}^L, f_{mj}{}^U]$	…	$[f_{mn}{}^L, f_{mn}{}^U]$
W_j	$[w_1]$		$[w_j]$		$[w_n]$

Where $[f_{ij}^L, f_{ij}^U]$ are the interval values of alternative i with respect to criterion j, and $[f_{ij}^L, f_{ij}^U]$ are lower and upper bounds respectively. $[w_j]$ are the weights of alternative i with respect to criterion j.

Step 2) Obtaining the Weights of Criteria through Fuzzy AHP

In most of MCDM problems (particularly in MADM) it is required to have the relative importance of the criteria. This relative importance shows the importance of each criterion in respect to other criteria for decision-making. There are two different methods to obtain these weights: 1-subjective method 2- objective method; one of the most applicable methods to obtain the relative importance is AHP. Despite of its wide range of applications, the conventional AHP approach may not fully reflect a style of human thinking. One reason is that decision makers usually feel more confident to give interval judgments rather than expressing their judgments in the form of single numeric values. As a result, fuzzy AHP and its extensions are developed to solve alternative selection and justification problems. Although FAHP requires tedious computations, it is capable of capturing a human's appraisal of ambiguity when complex multi-attribute decision making problems are considered. In the literature, many FAHP methods have been proposed ever since the seminal paper by Van Laarhoven and Pedrycz (1983) [40].. In his earlier work, Saaty (1980) proposed a method to give meaning to both fuzziness in perception and fuzziness in meaning. This method measures the relativity of fuzziness by structuring the functions of a system hierarchically in a multiple attribute framework [41]. Later on, Buckley (1985) extends Saaty's AHP method in which decision makers can express their preference using fuzzy ratios instead of crisp values [42]. Chang (1996) developed a fuzzy extent analysis for AHP, which has similar steps as that of Saaty's crisp AHP [43]. However, his approach is relatively easier in computation than the other fuzzy AHP approaches. In this paper, we make use of Chang's fuzzy extent analysis for AHP. Kahraman et al. (2003) applied Chang's (1996) fuzzy extent analysis in the selection of the best catering firm, facility layout and the best transportation company, respectively[44]. Let O = $\{o_1, o_2, \ldots, o_n\}$ be an object set, and U = $\{g_1, g_2, \ldots, g_m\}$ be a goal set. According to the Chang's extent analysis, each object is considered one by one, and for each object, the analysis is carried out for each of the possible goals, g_i. Therefore, m extent analysis values for each object are obtained and shown as follows:

$\tilde{M}_{g_i}^1, \tilde{M}_{g_i}^2, \ldots, \tilde{M}_{g_i}^m$, i=1, 2,…,n

Where $\tilde{M}_{g_i}^j$(j=1,2,3,…, m) are all triangular fuzzy numbers. The membership function of the triangular fuzzy number is denoted by $M_{(x)}$. The steps of the Chang's extent analysis can be summarized as follows:

Step 2.1: The value of fuzzy synthetic extent with respect to the ith object is defined as:

$$S_i = \sum_{j=1}^{m} \tilde{M}_{g_i}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1} \quad (11)$$

Where $\otimes$ denotes the extended multiplication of two fuzzy numbers. In order to obtain $\sum_{j=1}^{m} \tilde{M}_{g_i}^j$ We perform the addition of m extent analysis values for a particular matrix such that,

$$\sum_{j=1}^{m} \tilde{M}_{g_i}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad (12)$$

And to obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1}$ we perform the fuzzy addition operation of $\tilde{M}_{g_i}^j$ (j =1,2,…,m) values such that,

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right) \quad (13)$$

Then, the inverse of the vector is computed as,

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \quad (14)$$

Where $u_i, m_i, l_i > 0$

Finally, to obtain the S_j, we perform the following multiplication:

$$S_i = \sum_{j=1}^{m} \tilde{M}_{g_i}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1} = \left(\sum_{j=1}^{m} l_j \otimes \sum_{i=1}^{n} l_i, \sum_{j=1}^{m} m_j \otimes \sum_{i=1}^{n} m_i, \sum_{j=1}^{m} u_j \otimes \sum_{i=1}^{n} u_i\right) \quad (15)$$

Step 2.2: The degree of possibility of $\tilde{M}_2 = (l_2, m_2, u_2) \geq \tilde{M}_1 = (l_1, m_1, u_1)$ is defined as

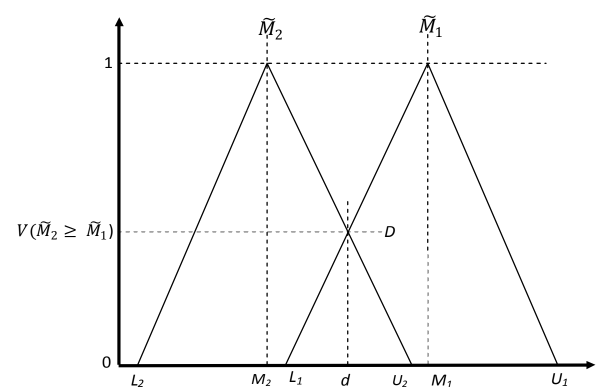


Fig 2: The degree of possibility of $\tilde{M}_1 \geq \tilde{M}_2$

$V(\widetilde{M}_2 \geq \widetilde{M}_1) = sup[\min(\widetilde{M}_1(x), \widetilde{M}_2(y))]$ (16)

This can be equivalently expressed as,

$$V(\widetilde{M}_2 \geq \widetilde{M}_1) = hgt(\widetilde{M}_1 \cap \widetilde{M}_2) = \widetilde{M}_2(d) = \begin{cases} 1 & if\ m_2 \geq m_1 \\ 0 & if\ l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2-u_2)-(m_1-l_1)}, & otherwise \end{cases} \quad (17)$$

Fig. 2 illustrates $V(\widetilde{M}_2 \geq \widetilde{M}_1)$ for the case d for the case $m_l < l_l < u_2 < m_l$, where d is the abscissa value corresponding to the highest crossover point D between $\widetilde{M}_1$ and $\widetilde{M}_2$,To compare $\widetilde{M}_1$ and $\widetilde{M}_2$, we need both of the values $V(\widetilde{M}_1 \geq \widetilde{M}_2)$ and $V(\widetilde{M}_2 \geq \widetilde{M}_1)$.

Step 2.3: The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i(i=1, 2… K) is defined as

$V(\widetilde{M} \geq \widetilde{M}_1, \widetilde{M}_2, \ldots, \widetilde{M}_k) = \min V(\widetilde{M} \geq \widetilde{M}_i)$, i =1,2,…,k

Step 2-4: Finally, $W=(\min V(s_1 \geq s_k)\ \min V(s_2 \geq s_k), \ldots, \min V(s_n \geq s_k))^T$, is the weight vector for k = 1,…,n.

In order to perform a pairwise comparison among the parameters, a linguistic scale has been developed. Our scale is depicted in Fig.3 and the corresponding explanations are provided in Table 1. Similar to the importance scale defined in Saaty's classical AHP [41], we have used five main linguistic terms to compare the criteria: ''equal importance'', ''moderate importance'', ''strong importance'', ''very strong importance'' and ''demonstrated importance''. We have also considered their reciprocals: ''equal unimportance'', ''moderate unimportance'', ''strong unimportance'', ''very strong unimportance'' and ''demonstrated unimportance''. For instance, if criterion A is evaluated ''strongly important'' than criterion B, then this answer means that criterion B is ''strongly unimportant'' than criterion A.

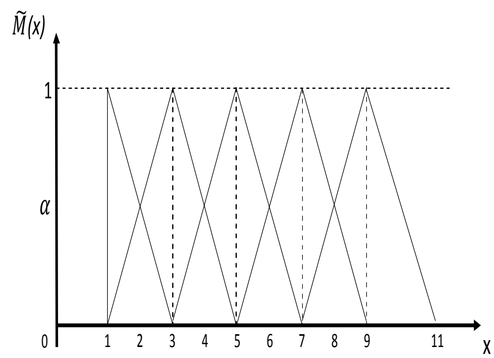


Fig 3: Membership functions of triangular fuzzy numbers corresponding to the linguistic scale

Table 3: The linguistic scale and corresponding triangular fuzzy numbers

Linguistic scale	Explanation	triangular fuzzy numbers	The inverse of triangular fuzzy numbers
Equal Importance	Two activities contribute equally to the objective	(1, 1, 1)	(1, 1, 1)
Moderate Importance	Experience and judgment slightly favor one activity over another	(1, 3, 5)	(1/5, 1/3, 1)
Strong importance	Experience and judgment strongly favor one activity over another	(3, 5, 7)	(1/7, 1/5, 1/3)
Very strong importance	An activity is favored very strongly over another; its dominance demonstrated in practice	(5, 7, 9)	(1/9, 1/7, 1/5)
Demonstrated importance	The evidence favoring one activity over another is highest possible order of affirmation	(7, 9, 11)	(1/11, 1/9, 1/7)

Step 3) Extended VIKOR Method for Interval Numbers

The extended VIKOR method consists of the following steps:

3.1) Normalize the values $[f_{ij}^L, f_{ij}^U]$ $i = 1,2,\ldots,m$ and $j = 1,\ldots,n$ by these relations:

If: $J \in J^+$ $$\left[\frac{f_{ij}^L}{\max_i f_{ij}^U}, \frac{f_{ij}^U}{\max_i f_{ij}^U}\right], \forall_{ij}. \quad (18)$$

If: $J \in J^-$ $$\left[\frac{\min_i f_{ij}^L}{f_{ij}^U}, \frac{\min_i f_{ij}^L}{f_{ij}^L}\right], \forall_{ij}. \quad (19)$$

Where J^+ is associated with benefit criteria and J^- is associated with cost criteria.
As in classic VIKOR, normalization is used to eliminate the units of criterion functions, so that all the criteria are dimensionless [32]. here in the extended VIKOR, linear normalization is used to transform all criteria into positive criteria so that the proposed method is in accordance with the classic VIKOR method introduced by Opricovic (1998) while covers its shortcomings.
3.2) Determine the best f_j^* and the worst f_j^- values of all criterion functions j = 1, 2,. . ., n. If the jth function represents a benefit then:

$$S_i^L = \sum_{j=1}^{n} w_j^L \left(\frac{f_j^* - f_{ij}^U}{f_j^* - f_j^-}\right) \quad \text{and} \quad S_i^U = \sum_{j=1}^{n} w_j^U \left(\frac{f_j^* - f_{ij}^L}{f_j^* - f_j^-}\right), \forall_{ij}. \qquad (20)$$

$$R_i^L = \max_i w_j^L \left(\frac{f_j^* - f_{ij}^U}{f_j^* - f_j^-}\right) \quad \text{and} \quad R_i^U = \max_i w_j^U \left(\frac{f_j^* - f_{ij}^L}{f_j^* - f_j^-}\right), \forall_{ij}. \qquad (21)$$

$$f_j^* = \max_i f_{ij}^U \quad \text{and} \quad f_j^- = \min_i f_{ij}^L$$

3-3) Compute the interval$Q_i = [Q_i{}^L, Q_i{}^U]$; i = 1, 2, . . . , m, by these relations:

$$Q_i^L = V\left(\frac{S_i^L - S^*}{S^- - S^*}\right) + (1 - V)\left(\frac{R_i^L - R^*}{R^- - R^*}\right) \text{ and } Q_i^U = V\left(\frac{S_i^U - S^*}{S^- - S^*}\right) + (1 - V)\left(\frac{R_i^U - R^*}{R^- - R^*}\right), \forall_i. \qquad (22)$$

Where $S^* = \min_i S_i^L$, $S^- = \max_i S_i^U$, $R^* = \min_i R_i^L$, $R^- = \max_i R_i^U$
V is introduced as weight of the strategy of ''the majority of criteria'' (or ''the maximum group utility''), here suppose that, v = 0.5.
3.4) Rank the $S_i, R_i$and Q_i, i = 1,…,m , Based on the VIKOR method, the alternative that has minimum $S_i, R_i$and Q_i is the best alternative and it is chosen as compromise solution. But here the$S_i, R_i$and Q_i, are interval numbers. So, we introduce a new method for chosen the minimum interval number and Comparison interval numbers whit each other as follows:
First, calculate $S^* = [S^{L*}, S^{U*}]$, $R^* = [R^{L*}, R^{U*}]$and $Q^* = [Q^{L*}, Q^{U*}]$ by these relations:

$$S^{L*} = \min_i\{S_i^L\} \quad , \quad S^{U*} = \min_i\{S_i^U\}, \forall_i.$$
$$R^{L*} = \min_i\{R_i^L\} \quad , \quad R^{U*} = \min_i\{R_i^U\}, \forall_i.$$
$$Q^{L*} = \min_i\{Q_i^L\} \quad , \quad Q^{U*} = \min_i\{Q_i^U\}, \forall_i.$$

Definition: Distance between $[a^L, a^U]$ and $[b^L, b^U]$ is:$D(a, b) = \frac{\sqrt{2}}{2}\sqrt{(a^L - b^L)^2 + (a^U - b^U)^2}$ [43].
Then, calculate $D_i(S_i, S^*)$, $D_i(R_i, R^*)$ and $D_i(Q_i, Q^*)$ by these relations:

$$D_i(S_i, S^*) = \frac{\sqrt{2}}{2}\sqrt{(S_i^L - S^{L*})^2 + (S_i^U - S^{U*})^2}, \forall_i. \qquad (23)$$

$$D_i(R_i, R^*) = \frac{\sqrt{2}}{2}\sqrt{(R_i^L - R^{L*})^2 + (R_i^U - R^{U*})^2}, \forall_i. \qquad (24)$$

$$D_i(Q_i, Q^*) = \frac{\sqrt{2}}{2}\sqrt{(Q_i^L - Q^{L*})^2 + (Q_i^U - Q^{U*})^2}, \forall_i. \qquad (25)$$

Finally, Rank the alternatives, sorting by S, R and Q based on the values $D_i(S_i, S^*)$, $D_i(R_i, R^*)$ and $D_i(Q_i, Q^*)$in decreasing order. The results are three ranking lists.

3.5) propose as a compromise solution the alternative A′, which is ranked the best by the measure Q (Minimum) if the following two conditions are satisfied:

C1. Acceptable advantage:
$Q(A'') - Q(A') \geq DQ$
where A′′ is the alternative with second position in the ranking list by Q; DQ=1/(m- 1); m is the number of alternatives.

C2. Acceptable stability in decision making:
Alternative A′ must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be ''voting by majority rule'' (when v > 0.5 is needed), or ''by consensus'' v ≈ 0.5, or ''with veto'' (v < 0.5). Here, v is the weight of the decision making strategy ''the majority of criteria'' (or ''the maximum group utility'').
If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives A′ and AA′′ if only condition C2 is not satisfied, or
- Alternatives A′ , A′′ , . . ., $A^{(M)}$ if condition C1 is not satisfied; $A^{(M)}$ is determined by the relation Q($A^{(M)}$)) À Q(A′) < DQ for maximum M (the positions of these alternatives are ''in closeness'').

The best alternative, ranked by Q, is the one with the minimum value of Q. The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the ''advantage rate''. VIKOR is an effective tool in multi-criteria decision making, particularly in a situation where the decision maker is not able, or does not know to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum ''group utility'' (represented by min S) of the ‘‘majority”, and a minimum of the ‘‘individual regret” (represented by min R) of the (“opponent”).The compromise solutions could be the basis for negotiations, involving the decision maker’s preference by criteria weights [32].

4. Numerical example

In this section, we present a numerical example to illustrate how the proposed method can be used. In order to demonstrate the applicability and evaluate the significance and validity of the proposed method in supplier selection problem, following data and criteria are adopted from the Talluri and Banker (2002) [45]. Assume an MADM problem for Supplier selection with eighteen alternatives (supplier) and seven criteria that contains interval and crisp data. Data and criteria are presented in Table 4, 5 respectively.

Table 4: The criteria for supplier selection

Beneficial:	Non-beneficial:
C_1= Supply Variety (SV) C_2= Number of Shipments to arrive on time (NB) C_3= Number of bills received from supplier without errors (NOT)	C_4= Price (P) C_5= Number Shipments Per months(NS) C_6= Total Cost of shipments (TC) (1000$) C_7= Distance (D) (KM)

Table 5: Related attributes for 18 suppliers

supplier	SV c_1	NB c_2	NOT c_3	Price c_4	NS c_5	TC(1000$) c_6	D(KM) c_7
1	2	[50,65]	187	[950,2000]	197	253	249
2	13	[60,70]	194	[800,1800]	198	268	643
3	3	[40,50]	220	[1000,2100]	229	259	714
4	3	[100,160]	160	[820,2150]	169	180	1809
5	24	[45,55]	204	[735,1900]	212	257	238
6	28	[85,115]	192	[650,2500]	197	248	241
7	1	[70,95]	194	[450,2200]	209	272	1404
8	24	[100,180]	195	[400,1900]	203	330	984
9	11	[90,120]	200	[607,2040]	208	327	641
10	53	[50,80]	171	[455,1890]	203	330	588
11	10	[250,300]	174	[830,2000]	207	321	241
12	7	[100,150]	209	[650,1950]	234	329	567
13	19	[80,120]	165	[960,2350]	173	281	567
14	12	[200,350]	199	[1200,2300]	203	309	967
15	33	[40,55]	188	[880,2000]	193	291	635
16	2	[75,85]	168	[655,2010]	177	334	795
17	34	[90,180]	177	[800,1990]	185	249	689
18	9	[90,150]	167	[645,2153]	176	216	913

As we can see, the second and fourth criterions are in the interval form whereas other criteria are crisps. We compare each criterion with respect to other criteria. You can see the pair-wise comparison matrix for Supplier selection criteria in Table 6.

Table 6: Inter-criteria comparison matrix

	C1			C2			C3			…	C6			C7		
C1	1.00	1.00	1.00	0.25	0.33	0.50	0.33	0.50	1.00	…	0.33	0.50	1.00	1.00	2.00	3.00
C2	2.00	3.00	4.00	1.00	1.00	1.00	3.00	4.00	5.00	…	1.00	2.00	3.00	4.00	5.00	6.00
C3	1.00	2.00	3.00	0.20	0.25	0.33	1.00	1.00	1.00	…	1.00	2.00	3.00	3.00	4.00	5.00
C4	0.25	0.33	0.50	0.33	0.50	1.00	0.20	0.25	0.33	…	0.25	0.33	0.50	1.00	2.00	3.00
C5	0.333	0.5	1	0.20	0.25	0.33	0.20	0.25	0.33	…	1.00	1.00	2.00	0.33	0.50	1.00
C6	1.00	2.00	3.00	0.33	0.50	1.00	0.33	0.50	1.00	…	1.00	1.00	1.00	3.00	4.00	5.00
C7	0.33	0.50	1.00	0.17	0.20	0.25	0.20	0.25	0.33	….	0.20	0.25	0.33	1.00	1.00	1.00

After that the weight vector is calculated as follow:

W^t = (0.1186, 0.2555, 0.1595, 0.1261, 0.0797, 0.1376, 0.1226)

Now, the decision maker wants to choose an alternative (Supplier) that has minimum C4, C5, C6, and C7 and maximum C1, C2 and C3. The values of decision matrix are not precise and interval numbers are used to describe and treat the uncertainty of the decision problem. The interval decision matrix is shown in Table 5. Now let's suppose that, v = 0.5.To solves this example using the extended Interval VIKOR method we go through the following steps.

3-1) the normalized the values $[f_{ij}^L, f_{ij}^U]$ $i = 1, 2, \ldots, 18$ and $j = 1, \ldots, 8$ are computed by (18) and (19) shown in Table 7. The normalized data which are obtained through step 3-1 are presented In Table 7.

Table 7: The normalized rates

	SV(C1)	NB(C2)	NOT(C3)	Price(C4)	NS(C5)	TC(C6)	D(C7)
1	[0.038,0.038]	[0.143,0.186]	[0.850,0.850]	[0.200,0.421]	[0.858,0.858]	[0.711,0.711]	[0.956,0.956]
2	[0.245,0.245]	[0.171,0.200]	[0.882,0.882]	[0.222,0.500]	[0.854,0.854]	[0.672,0.672]	[0.370,0.370]
3	[0.057,0.057]	[0.114,0.143]	[1.000,1.000]	[0.190,0.400]	[0.738,0.738]	[0.695,0.695]	[0.333,0.333]
4	[0.057,0.057]	[0.286,0.457]	[0.727,0.727]	[0.186,0.488]	[1.000,1.000]	[1.000,1.000]	[0.132,0.132]
5	[0.453,0.453]	[0.129,0.157]	[0.927,0.927]	[0.211,0.544]	[0.797,0.797]	[0.700,0.700]	[1.000,1.000]
6	[0.528,0.528]	[0.243,0.329]	[0.873,0.873]	[0.160,0.615]	[0.858,0.858]	[0.726,0.726]	[0.988,0.988]
7	[0.019,0.019]	[0.200,0.271]	[0.882,0.882]	[0.182,0.889]	[0.809,0.809]	[0.662,0.662]	[0.170,0.170]
8	[0.453,0.453]	[0.286,0.514]	[0.886,0.886]	[0.211,1.000]	[0.833,0.833]	[0.545,0.545]	[0.242,0.242]
9	[0.208,0.208]	[0.257,0.343]	[0.909,0.909]	[0.196,0.659]	[0.813,0.813]	[0.550,0.550]	[0.371,0.371]
10	[1.000,1.000]	[0.143,0.229]	[0.777,0.777]	[0.212,0.879]	[0.833,0.833]	[0.545,0.545]	[0.405,0.405]
11	[0.189,0.189]	[0.714,0.857]	[0.791,0.791]	[0.200,0.482]	[0.816,0.816]	[0.561,0.561]	[0.988,0.988]
12	[0.132,0.132]	[0.286,0.429]	[0.950,0.950]	[0.205,0.615]	[0.722,0.722]	[0.547,0.547]	[0.420,0.420]
13	[0.358,0.358]	[0.229,0.343]	[0.750,0.750]	[0.170,0.417]	[0.977,0.977]	[0.641,0.641]	[0.420,0.420]
14	[0.226,0.226]	[0.571,1.000]	[0.905,0.905]	[0.174,0.333]	[0.833,0.833]	[0.583,0.583]	[0.246,0.246]
15	[0.623,0.623]	[0.114,0.157]	[0.855,0.855]	[0.200,0.455]	[0.876,0.876]	[0.619,0.619]	[0.375,0.375]
16	[0.038,0.038]	[0.214,0.243]	[0.764,0.764]	[0.199,0.611]	[0.955,0.955]	[0.539,0.539]	[0.299,0.299]
17	[0.642,0.642]	[0.257,0.514]	[0.805,0.805]	[0.201,0.500]	[0.914,0.914]	[0.723,0.723]	[0.345,0.345]
18	[0.170,0.170]	[0.257,0.429]	[0.759,0.759]	[0.186,0.620]	[0.960,0.960]	[0.833,0.833]	[0.261,0.261]

3-2) in this step, we compute $[S_i^L, S_i^U]$ and $[R_i^L, R_i^U]$; i = 1, 2. . . 18, using (20) and (21).The result is presented in Table 8.

3-3) We compute the interval $[Q_i^L, Q_i^U]$; i = 1, 2, . . ., 18, by using (22).The results are shown in third column of Table 8.

Table 8: S, R and Q interval numbers

Suppliers	Si	Ri	Qi
1	[0.659,0.705]	[0.235,0.247]	[0.642,0.756]
2	[0.695,0.745]	[0.231,0.239]	[0.679,0.783]
3	[0.712,0.752]	[0.247,0.256]	[0.766,0.855]
4	[0.630,0.725]	[0.160,0.206]	[0.311,0.627]
5	[0.568,0.626]	[0.243,0.251]	[0.541,0.657]
6	[0.507,0.601]	[0.194,0.218]	[0.264,0.494]
7	[0.688,0.815]	[0.210,0.231]	[0.589,0.852]
8	[0.564,0.748]	[0.140,0.206]	[0.141,0.661]
9	[0.667,0.761]	[0.190,0.214]	[0.480,0.711]
10	[0.639,0.764]	[0.223,0.247]	[0.565,0.842]
11	[0.525,0.609]	[0.131,0.131]	[0.050,0.172]
12	[0.654,0.757]	[0.165,0.206]	[0.367,0.673]
13	[0.697,0.767]	[0.190,0.223]	[0.524,0.752]
14	[0.529,0.676]	[0.125,0.125]	[0.031,0.246]
15	[0.694,0.744]	[0.243,0.256]	[0.724,0.845]
16	[0.781,0.851]	[0.218,0.227]	[0.756,0.890]

17	[0.573,0.692]	[0.140,0.214]	[0.155,0.611]
18	[0.629,0.744]	[0.165,0.214]	[0.330,0.686]

3-4) in this step, we compute $D_i(S_i,S^*)$, $D_i(R_i,R^*)$ and $D_i(Q_i,Q^*)$ i = 1, 2. . . 18, using (23), (24) and (25). The result is presented in Table 9. Using $D_i(S_i,S^*)$, $D_i(R_i,R^*)$ and $D_i(Q_i,Q^*)$ i = 1, 2. . . 18, final ranking is obtained as follows:

Table 9: S, R and Q crisp numbers and ranks

DMUs	Si	Rank	Ri	Rank	Qi	Rank
0.130189	8	0.116702	15	0.597531	12	0.130189
0.16773	14	0.110431	13	0.629794	15	0.16773
0.179782	16	0.126911	18	0.709607	17	0.179782
0.123221	7	0.062707	4	0.37801	6	0.123221
0.046607	3	0.122791	16	0.497863	10	0.046607
0	1	0.08243	9	0.280954	3	0
0.198006	17	0.096477	11	0.622453	14	0.198006
0.111678	6	0.058665	3	0.354624	5	0.111678
0.159784	12	0.078356	8	0.496145	9	0.159784
0.148275	10	0.111045	14	0.605783	13	0.148275
0.013705	2	0.006501	2	0.013848	1	0.013705
0.151235	11	0.064279	5	0.426716	8	0.151235
0.178313	15	0.083148	10	0.53797	11	0.178313
0.055602	4	0	1	0.052025	2	0.055602
0.166388	13	0.124936	17	0.682775	16	0.166388
0.262429	18	0.098073	12	0.72171	18	0.262429
0.079562	5	0.064401	6	0.322596	4	0.079562
0.132626	9	0.069553	7	0.42064	7	0.132626

3-5) As we can see in third, fifth and seventh column, the compromise solution of Interval VIKOR method are supplier 11 and supplier 14, because they have not the best rank in S_i, R_i .Therefore, the second condition is not satisfied. On the other hand the supplier which has the second ranks in column Q_i or $Q(a'')$ is the fourteenth supplier. Therefore $Q(a'') - Q(a')$ equals to $0.052 - 0.014 = 0.038$ and lesser than $D(Q) = \frac{1}{18-1}$=0.059. Hence the first condition is not satisfied.

5. Conclusion

Supplier selection is a broad comparison of suppliers using a common set of criteria and measures to identify suppliers with the highest potential for meeting a firm's needs consistently and at an acceptable cost. Selecting the right suppliers significantly reduces the purchasing costs and improves corporate competitiveness therefore supplier selection one of the most important decision making problems. In this paper, a two-step Fuzzy AHP and Interval VIKOR methodology is structured here that Interval VIKOR uses Fuzzy AHP result weights as input weights.

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