Optimal Facility Location on Spherical Surfaces: Algorithm and Application

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Abstract: Fundamental to optimal location of facility is some measure of point-to-point distances. Distance measurements used in facility location are predominantly based on Rectilinear and Euclidean distances. This paper presents "great circle distance" which represents the shortest path for distance modeling and optimal facility location on spherical surface. Great circle distances takes into consideration the geometrical reality of the spherical Earth and offers an alternative to widely held notion that travel over water can be exactly modelled by Euclidean distances. The need for geometrical presentation of the spherical earth becomes very relevant when we take into consideration an ever increasing facility location at sea where great circle travel can be practised. Facilities being located at sea include oil rigs, mobile drilling units and dynamically positioned units. The use of "Great circle distances" opens up another avenue for convergence of Navigation and Spherical Trigonometry into advancement of logistics and facility location. In this paper an evaluation of single facility location using great circle distances is used to demonstrate the application of the concept.

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1. Introduction

It is recognized that the location of a facility determines and has great influence on the distribution system parameters including time, costs and efficiency of the system (Sule, 2001). As such, optimal location of the facility is essential for attaining improved flow of goods and services to customers served by the facility. In choosing the location of a facility both qualitative and quantitative factors are taken into account including availability of land, proximity to raw materials or market, availability of utilities and transport facilities as well as social, economic and political factors (Zarimbal, 2009; Melo et al, 2005). Distance or proximity is one of the important metric which many decision makers seeks to optimize through minimization of the mean (or total) distance as in the median concept or minimization of the maximum distance as in the centre concept (Schilling et al, 1993).

Though distance is a well known parameter, its determination in certain settings could be challenging like finding out distance between positions defined by latitudes and longitudes on Earth. Considering, the spherical nature of the Earth it is evident that distance modeling in facility location that takes into account this fact will be an improvement on the current practice dominated by Euclidean and Rectilinear models which are best suited to planar surfaces. This paper seeks to present an alternative distance measurement based on "great circle distance" which represents the shortest path on spherical surface. The need for geometrical modeling of distance of the spherical earth becomes very relevant when we take into consideration an ever increasing facility location at sea for harnessing natural resources including oil rigs, mobile drilling units and dynamically positioned units. Unlike travel on land where physical barriers have to be avoided, it is practical to travel along the great circle path during open sea navigation.

Logistics has borrowed theories from many other disciplines of study like marketing, mathematics and psychology (Stock, 1997; Sachan and Datta, 2005; Gammelgaard, 2004). The use of "Great circle distances" opens up another avenue for borrowing from navigation and spherical trigonometry into advancement of logistics and facility location. In this paper single facility location based on great circle distances is evaluated in the process of demonstrating and applying the concept.

2. Distance Functions in Location Problem

Zarimbal (2009) clearly affirm that that the distance functions play an important role in facility location problems. He identifies different distance functions used in location problem with each having its own domain, advantages, and disadvantages. He defines distance as a numerical description of how far apart objects are at any given moment in time and may refer to a physical length or a period of time. While making location decisions, network design and optimization; the distribution of travel distances among the service recipients (clients) remains an important issue.

Based on the work Zarinbal (2009) we note that Euclidean and Rectilinear distance accounts for more than 63 percent of distance functions used in location problems. Euclidean distance assumes that one can travel almost directly from one station to another following a straight line as shown in figure 1 (Montreuil, 2008; Melachrinoudis and Xanthopulos, 2003).

Rectilinear distances are applicable when travel is allowed only on two perpendicular directions such as North–South and East–West arteries as shown by the dotted line in figure 2. This distance is also popular among researchers because the analysis is usually simpler than employing other metrics (Drezner and Wesolowsky, 2001). The rectilinear distance is also called Manhattan or Taxicab Norm distances; because it is the distance a car would drive in a city lay-out in square blocks. Apart from these two dominant distance functions, other distance used in location problem includes aisle distance, distance matrix, minimum lengths path, Hilbert Curve, Mahalanobis distance, Hamming Distance and Chebyshev Distances (Klamroth, 2002; ReVelle and Eiselt, 2005). Klamroth (2002) groups these distances into multi-parameter round norms, block norms and polyhedral distances.

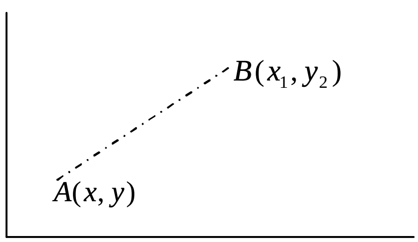

Figure 1: Euclidean Distance

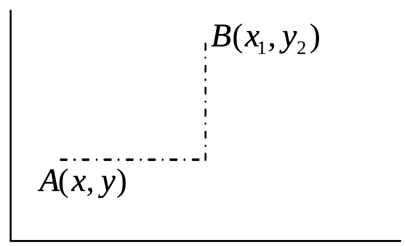

Figure 2: Rectilinear Distance

Thus Euclidian distance between two points A and B with coordinates A (x, y) and B (x_i, y_i) is expressed mathematically as;

$$d(s) = \left[(x - x_i)^2 + (y - y_i)^2\right]^{\frac{1}{2}} \quad \text{......... (1)}$$

And a Rectilinear distance $d(s)$ between A (x, y) and B (x_i, y_i) co-ordinates is expressed as:

$$d(s) = |x_i - x| + |y_i - y| \quad \text{.................... (2)}$$

2.1. Deficiency of Current Distance Modeling in Location Problem

In a realistic environment the choice of a suitable distance function plays a crucial role for a good estimation of travel distances (Klamroth, 2002 and Sminchi-Levi, 1997). In reality we are located on spherical Earth with our addresses defined by the intersections of latitudes and longitudes. Precise geographic locations can be achieved by using a geographic information system (GIS) and other satellite based systems like the Global Positioning Systems (GPS) and Glonass as well as navigation charts (Manley, 2008). Grid systems can also be used to model location and travel distance but suffer from having limited use as most of them are established based on national grid reference system hence inappropriate to evaluate facility location and networks that spans across the borders of countries with different grid reference system.

Bramel and Sminchi-Levi (1997), Klamroth (2002), Drezner and Wesolowsky (2001), and Zarinbal (2009) assert that air travel and travel over water can be exactly modeled by Euclidean distance. However, this suggestion disregard the fact that air travel and sailing at sea is made over a spherical surface whereas Euclidean modeling simply measures the distance that would be obtained if the distance between two points were measured with a ruler (Zarinbal, 2009). Using the Pythagorean Theorem (as in Euclidian Distance) and Spherical Trigonometry principles reveals disagreement between the measurements and the calculations of the sides and angles. In fact, the sum of the angles in spherical triangle is greater than the 180 degrees which is always measured in planar triangles (Ross, 2002). The discrepancy between the distances measured based on Euclidean and those based on spherical trigonometry becomes greater, the further apart the locations are from each other (Ross, 2002). Modeling distance of air travel or ocean navigation using Euclidean distances is in principle asserting that such travel is made through the interior of the sphere which is not the case. This anomaly can be corrected by use of spherical trigonometry as proposed in this paper.

2.2. Great Circles in Distance Modeling

Based on the work of Ross (2002), Frost (1988) and Earl et al (1999) we note that Trigonometry and spherical trigonometry were primarily developed for and used in astronomy, geography, and navigation. Spherical trigonometry was developed to describe and understand applications involving triangles on spheres and spherical surfaces. Spherical trigonometry offers a realistic representation of the Earth surface which is spherical in nature and is widely used in other discipline of studies but its potential particularly the use of great circle distances remains untapped in logistics. The potential areas for application of spherical trigonometry concepts include but not limited to hub-and-spoke network design and facility location at sea like oil and gas rigs.

Measurement of distances in spherical trigonometry is based on solving spherical triangles whose sides form arcs of great circles (Das *et al*, 2001). As in figure 1, great-circle arcs form the sides of a spherical triangle, and where two arcs intersect, a spherical angle is formed. In other words, the arc lengths are a measure of the angle they subtend at the center of the sphere, and the spherical angles between the arcs are a measure of the angle at which the planes that form the arcs intersect. On the Earth, the equator and circles of longitude are natural great circles. Likewise, any circular path around the Earth that cuts it into two equal hemispheres is a great circle. Spherical trigonometry involves relationships between the arc lengths (sides) and the spherical angles between the arcs.

Studies have shown that the shortest distance between any two positions on the earth's surface lies along the arc of the great circle joining these two positions. Thus on a spherical surface, a great circle path, often called a geodesic, is always the shortest path between two points (Ross, 2002). As expressed by Wikipedia between any two points on a sphere which are not directly opposite to each other there is a unique great circle.

In recognition of the fundamental difference between spherical geometry and Euclidean Geometry it is apparent that the equations for distance take different forms in these two domains of knowledge. Fundamentally, the distance between two points in Euclidean space is the length of a straight line from one point to the other while in spherical geometry straight lines are replaced with geodesics or great circle paths.

While positions of the geographical places can relatively be easily determined based on existing maps or global positioning systems like GPS and Glonass the calculation of the great circle distance and thus the shortest distance between places needs a formula. By using a system of co-ordinates of longitude and latitudes the distance along the great circle can be determined by solving the quantities of the resultant spherical triangle formed by the intersection of three great circles (Frost, 1988) namely:

a) The great circle arc joining the two positions (arc c in figure 1)
b) The meridian (longitude) through position 1 (meridian joining C and A in figure 1)
c) The Meridian (longitude) through position 2 (meridian joining C and B in figure 1)

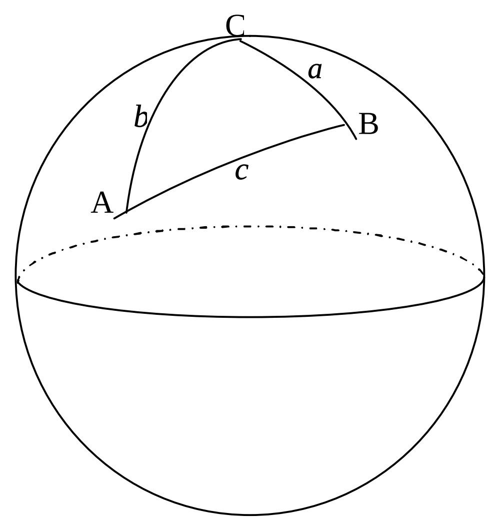

Figure 1: Spherical Triangle

Such spherical triangles and shortest distance between geographical points are solved by using the haversine formula (Bell *et al*, 2010) as shown in equation (3), (4) and (5). Thus in spherical triangle ABC in figure 1 above, given CA or b, CB or a and angle C, the haversine formula to solve arc length AB or c is expressed as:

$$hav(dist) = hav(dlong)\cos lat(A)\cos(latB) + hav(dlat) \quad \text{(3)}$$

Or

$$havAB = hav(C)\sin(a)\sin(b) + hav(a-b) \quad \text{(4)}$$

Where:

hav.dist – Haversine of distance between position A and B

hav.dlong – Haversine of the difference between the longitudes through position A and B respectively

$\cos lat.A$ – Cosine of the latitude through position A

$\cos lat.B$ – Cosine of the latitude through position B

hav.dlat – Haversine of the difference between latitudes through position A and B respectively

Alternatively, great circle distance can be calculated by finding the interior spherical angle between the two points and then multiplying that angle by the radius of the earth. Thus the length of the side of the spherical triangle (distance S) in **figure 2** is given by:

$$S = \alpha r \quad \text{………………………………………….} (5)$$

Where:

$S =$ Arc length (great circle distance on the sphere)

$r =$ Radius, in this case the radius of the earth which is 6,371.009 km or 3,958.761 miles or 3,440.069 nautical miles and

$\alpha =$ Central angle measure

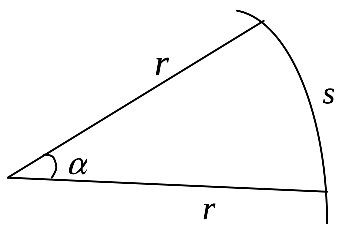

Figure 2: Arc length (S) and Central angle

Based on the haversine formula the central angle in radians is expressed as

$$\alpha = 2 \arcsin\left(\sqrt{\sin^2\frac{\Delta\theta}{2} + \cos\theta_1 \cos\theta_2 \sin^2\frac{\Delta\phi}{2}}\right) \quad (6)$$

Where:

$\alpha =$ Interior Spherical angle

$\Delta\theta = \theta_1 - \theta_2 =$ Difference in Latitude (*dlat*)

$\theta_1 =$ Latitude at position 1

$\theta_2 =$ Latitude at position 2

$\Delta\phi =$ Difference in Longitude (*dlong*)

In general the different forms of the haversine formula can be deduced from the law of cosine for spherical triangle. For spherical triangle ABC in figure 1 the cosine rule is stated as:

$$\cos(c) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(C)$$
$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)$$
$$\cos(b) = \cos(a)\cos(c) + \sin(a)\sin(c)\cos(B)$$

In order to deduce the distance from the haversine formula the haversine tables are used. Value in (4) can be computed directly using calculators or some software and programs which have been developed that solves the distances directly utilizing the excel capabilities.

The haversine formula we will assume that the Earth is a perfect sphere, even though it really isn't but somewhat ellipsoidal at the poles. To correct this anomaly a more complicated formula known as Vincenty's formula (equation 7) was developed (Jenness, 2008). Except for the antipodal point (points on the sphere directly opposite to each other), the haversine formula gives accurate distance. For demonstration in this paper the haversine formula has been used.

$$\alpha = \arctan\left(\frac{\sqrt{(\cos\theta_2 \sin\Delta\phi)^2 + (\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2 \cos\Delta\phi)^2}}{\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 \cos\Delta\phi}\right) \quad \text{……………………………….} (7)$$

3.0. Application of Spherical Trigonometry in Location Problem

The starting point in determining the optimum location is to find the centroid/centre of gravity of the spherical polygon under consideration (Jennes, 2008) as the initial coordinate of the new facility. Calculating a centroid for spherical surface is complex and still being studied, however, it is similar in concept as calculating of planner surface. The main difference is that longitudes and latitudes are not so much of coordinate but rather directions from the centre of the sphere. Since longitude and latitudes cannot be simply added and divided as the Cartesian coordinates can (Jennes, 2008), we first convert them into radians for calculating the centre of gravity. In order to facilitate calculations by excel, the positions given in degree, minutes and seconds are converted into decimal places and radians (Pearson, 2009). Likewise, in order to take in to account the hemisphere in which the position lies we introduce negative values for South Latitudes and West longitudes.

3.1. The Algorithm for Applying Great Circle Distances in Facility Location

i. Express latitudes and longitudes given in degrees, minutes and seconds as decimal values
ii. Express West Longitudes and South Latitudes as negative values otherwise positive
iii. Express location coordinates as radians by first converting degrees, minutes and second into decimal then apply

$$Radians = \frac{\deg}{180} * \pi ....(8)$$

iv. Determine the initial co-ordinate of the new facility (x^*, y^*) defined by centre of gravity formula

$$x^* = \sum w_i x_i \Big/ \sum w_i \quad \text{and}$$
$$y^* = \sum w_i y_i \Big/ \sum w_i ....(9)$$

v. Calculate the total distance (cost) from (x_i, y_i) to the optimal location using the haversine formula
vi. Make an iterative search of minimal total distance/cost based on initial position until no improvement is found

3.1 Application of the Concept and Results

ABC Company Ltd has 8 offshore rigs located at sea with coordinates as shown in table 1 (locations for demonstration purpose only). On reviewing its policy on distribution of supplies to the rigs, ABC plans to send supplies once in month to a central warehouse by using a ship. From the central warehouse the supplies are distributed to other rigs by smaller boats making 2 trips to rig no.1 and 6 and only one trip to all other rigs per week. The task is to determine the location of the central warehouse that minimizes the travel distance and therefore the distribution costs.

Table 1: Location of Oil and Gas Rigs

RIG	LOCATION (DEGREE)				LOCATION (DECIMAL)	
	LATITUDE		LONGITUDE		LATITUDE	LONGITUDE
1	01º 36' 15"	N	07º 37' 17"	E	1.6041667	7.6213889
2	02º 00' 27"	S	06º 25' 50"	E	-2.0075	6.4305556
3	00º 40' 38"	N	09º 05' 26"	E	0.6772222	9.0905556
4	01º 04' 23"	S	05º 19' 45"	E	-1.0730556	5.3291667
5	00º 03' 06"	S	08º 07' 26"	E	-0.0516667	8.1238889
6	02º 53' 52"	N	08º 59' 18"	E	2.8977778	8.9883333
7	00º 28' 46"	N	06º 57' 30"	E	0.4794444	6.9583333
8	01º 05' 36"	S	08º 27' 14"	E	-1.0933333	8.4538889

Solution:

We have noted that Great circle distance provides the shortest distance between two positions on the surface of the earth; hence great circle distances are used in solving this problem using the algorithm stated above.

The objective function: Minimize

$$S = \sum_{i=1}^{n} w \alpha_i r \text{ -----from (5)}$$

Where $w = Trips$

Step 1: Express location co-ordinates in decimal format (see Table1)
Step 2: Express West Longitudes and south latitudes as negative values otherwise positive (see Table 1 and figure 3)
Step 3: Express Location coordinates as radians (see Table 2)
Step 4: Determine the initial latitude (x^*) and initial longitude (y^*) as shown in table 2
Step 5: Calculate the total distance (cost) from (x_i, y_i) to the optimal location using the haversine formula:

Table 2: Calculation for Initial Location

Rig	Trips (w_i)	Lat in Radians x_i	Long in Radians y_i	$w_i x_i$	$w_i y_i$	$x^* = \sum w_i x_i / \sum w_i$
1	2	0.027998	0.1330183	0.055996	0.2660367	= 0.0103585 radians
2	1	-0.0350375	0.1122344	-0.0350375	0.1122344	= 00º 35' 37"
3	1	0.0118198	0.1586601	0.0118198	0.1586601	
4	1	-0.0187284	0.0930115	-0.0187284	0.0930115	
5	1	-0.0009018	0.1417886	-0.0009018	0.1417886	$y^* = \sum w_i y_i / \sum w_i$
6	2	0.0505758	0.156876	0.1011515	0.313752	= 0.1354477 Radians
7	1	0.0083679	0.1214458	0.0083679	0.1214458	= 07º 45' 38" E
8	1	-0.0190823	0.1475482	-0.0190823	0.1475482	
	10			**0.1035853**	**1.3544773**	

Based on the calculation performed in Table 2, the initial optimal location of the central warehouse will be at 00º 35' 37"N, 07º 45' 38"E. as depicted in figure 3

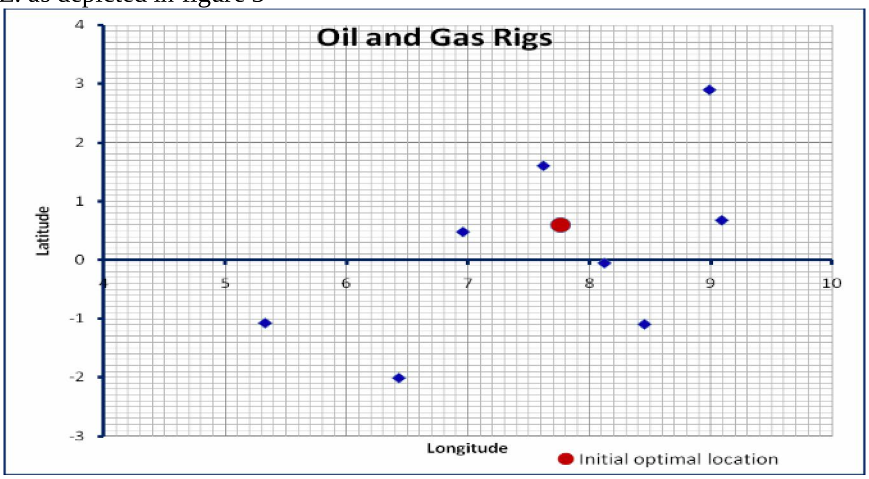

Figure 3: Location offshore rigs

Table 3: Calculation of Great Circle Distances based on Geographical Coordinates

RIG	LOCATION (DECIMAL)		Trip	$S = \alpha_i r$	$w_i \alpha_i r$ (Km)
	LATITUDE	LONGITUDE			
1	1.6041667	7.6213889	2	113.443	226.886
2	-2.0075	6.4305556	1	324.831	324.831
3	0.6772222	9.0905556	1	148.138	148.138
4	-1.0730556	5.3291667	1	327.769	327.769
5	-0.0516667	8.1238889	1	82.33	82.33
6	2.8977778	8.9883333	2	290.295	580.59
7	0.4794444	6.9583333	1	90.101	90.101
8	-1.0933333	8.4538889	1	202.794	202.794
	x^*=0.5935	y^*=7.760583		$S = \sum_{i=1}^{8} w \alpha_i r$	1983.439

3.2. Findings and Discussion

The total initial distance (cost) between the initial optimal location and all other stations is 1983.439 Kilometers. The initial position provides valuable input for subsequent iterations and decision making process taking into account both qualitative and quantitative analysis. If a new facility is to be constructed the centre of gravity (x^*, y^*) is an ideal location. Alternatively one of the existing facilities (rig) can be used for central warehouse. By inspection we note that rig 5 is closest to the centre of gravity, hence the ideal candidate for second iteration.

By locating the warehouse at rig 5 we note that the total distribution distance (cost) becomes 2161.965 Km an increase of 178.526 Km as compared to locating a new facility at the centre of gravity. The additional annual distribution cost related to the extra 178.526 Km per week needs to be compared to the annual fixed cost of establishing a new facility. This will help in establishing the trade-offs between establishing a new facility and locating a warehouse at the existing facilities. Iterations can be made for all the remaining facilities and evaluations made accordingly.

4. Conclusion

Analysis made in this paper shows that distances for facility location can be modeled more realistically by applying the great circle distances concept that takes into account the spherical nature of the Earth we live in. The contribution of this paper has been the introduction of an alternative approach to distance modeling for travel over water in place of Euclidean distance by developing an algorithm for deducing distances from geographical address defined by the grid of latitudes and longitudes and applying spherical trigonometry principles in the logistics of facility location. As such, the application of "Great circle distances" which is very much used in navigation and Spherical Trigonometry will contribute to advancement of logistics and facility location by broadening the scope of the set of knowledge from which the logistics discipline borrows.

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