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Derivation Of A Class Of Continuous Generalized Adams Methods For The Solution Of Ordinary Differential Equations

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Abstract: In this paper, we derived the continuous form of a class of block Generalized Adams Methods for step numbers k=4 and 8 with continuous coefficients based on multistep collocation using the matrix inverse collocation approach of Sirisena (1997). The convergence and order of the derived schemes were analysed and the stability regions of the block method were plotted. The schemes were A-stable and were of uniform order. The new block method derived were applied on IVPs and the solutions were compared with the conventional Adams method and we found that they perform relatively better for k=4 and 8.
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1. Introduction

Most Linear Multistep Method with step number k ≥ 2 suffers the problem of starting values. These methods need additional one-step method like the trapezoidal method or Runge Kutta Method to solve the problem of starting values. The Generalized Adams Methods (GAMs) were introduced by Brugnano and Trigiante (1998) for both the even and odd step numbers called extended Trapezoidal Rules. These methods were introduced because they have better stability properties than the standard Adams Moulton method and reach the highest possible order than the reverse Adams method, though it has better stability properties than the standard Adams Moulton Methods. These discrete forms of GAMs derived by Brugnano and Trigiante are not self-starting, so there is need for additional equation to solve the problem of non-self-starting.

To solve differential equations using the discrete generalized Adams Method constructed by Brugnano and Trigiante (1998) needs additional equation to meet the need of starting values. The discrete schemes obtained from the continuous formulation of a class of Generalized Adams Methods can be used in block form to obtain the block solution. The use of this block schemes has solved the problem of starting values.

In this paper, the reformulation of the block generalized Adams methods is considered for the solution of initial value problems of the form

$$y' = f(x, y), \qquad x \; \varepsilon \; [x_0, b]$$
$$y(x_0) = y_0$$

On a given mesh

$$a = x_0 < x_1 < \ldots < x_n < x_{n+1} < \ldots < x_{n+k} < \ldots < x_k = k$$

With a mesh size h a parameter taken as a constraint is defined as

$$h = x_{n+1} - x_n$$

The New Method

The general form of the k-step Generalized Adams Method is given by:

$$y_{n+v} - y_{n+v-1} = h\sum_{j=0}^{k} \beta_j f_{n+j}$$

Where,

$$v = \begin{cases} \dfrac{k}{2} & \text{if k is even} \\ \dfrac{k+1}{2} & \text{if k is odd} \end{cases}$$

These methods are often referred to as the conventional Adams Method. They are zero-stable and have the first characteristic polynomial located at the origin. The coefficient { β_j } are determined by imposing the formula to reach the highest possible order i.e. k+1.

In this paper, we shall use the multistep collocation method to recover the discrete schemes derived by Brugnano and Trigiante. These block methods were shown to have good stability properties making them useful in the solution of stiff ODEs. The block generalized Adams methods for step numbers k = 4, 8 are as follows;

Block generalized Adams method for step number 4 with uniform order 5 respectively.

$$y_n - y_{n+1} = -\frac{1}{720} h\left(251f_n + 646f_{n+1} - 264f_{n+2} + 106f_{n+3} - 19f_{n+4}\right)$$
$$y_{n+2} - y_{n+1} = -\frac{1}{720}\left(19f_n - 346f_{n+1} - 456f_{n+2} + 74f_{n+3} - 11f_{n+4}\right)h$$
$$y_{n+3} - y_{n+1} = -\frac{1}{90} h\left(f_n - 34f_{n+1} - 114f_{n+2} - 34f_{n+3} + f_{n+4}\right)$$
$$y_{n+4} - y_{n+1} = -\frac{3}{80} h\left(f_n - 14f_{n+1} - 24f_{n+2} - 34f_{n+3} - 9f_{n+4}\right)$$

Block generalized Adams method for step number 8 with uniform order 9

$$y_n - y_{n+3} = -\frac{1}{44800}\left(12881f_n + 70902f_{n+1} + 3438f_{n+2} + 79934f_{n+3} - 56160f_{n+4} + 34434f_{n+5} - 14062f_{n+6} + 3402f_{n+7} - 369f_{n+8}\right)h$$

$$y_{n+1} - y_{n+3} = \frac{1}{113400}\left(833f_n - 39874f_{n+1} - 152596f_{n+2} - 27478f_{n+3} - 15130f_{n+4} + 11162f_{n+5} - 4756f_{n+6} + 1166f_{n+7} - 127f_{n+8}\right)h$$

$$y_{n+2} - y_{n+3} = -\frac{1}{3628800} h\left(7297f_n - 99626f_{n+1} + 1638286f_{n+2} + 2631838f_{n+3} - 833120f_{n+4} + 397858f_{n+5} - 142094f_{n+6} + 31594f_{n+7} - 3233f_{n+8}\right)$$

$$y_{n+4} - y_{n+3} = -\frac{1}{3628800}\left(3233f_n - 36394f_{n+1} + 216014f_{n+2} - 1909858f_{n+3} - 2224480f_{n+4} + 425762f_{n+5} - 126286f_{n+6} + 25706f_{n+7} - 2497f_{n+8}\right)h$$

$$y_{n+5} - y_{n+3} = -\frac{1}{113400} h\left(23f_n - 334f_{n+1} + 2804f_{n+2} - 46378f_{n+3} - 139030f_{n+4} - 46378f_{n+5} + 2804f_{n+6} - 334f_{n+7} + 23f_{n+8}\right)$$

$$y_{n+6} - y_{n+3} = -\frac{1}{44800} h\left(49f_n - 522f_{n+1} + 2862f_{n+2} - 23234f_{n+3} - 44640f_{n+4} - 50814f_{n+5} - 19118f_{n+6} + 1098f_{n+7} - 81f_{n+8}\right)$$

$$y_{n+7} - y_{n+3} = \frac{1}{14175} h\left(13f_n - 104f_{n+1} + 244f_{n+2} + 4402f_{n+3} + 19270f_{n+4} + 9232f_{n+5} + 18724f_{n+6} + 5026f_{n+7} - 107f_{n+8}\right)$$

$$y_{n+8} - y_{n+3} = -\frac{5}{145152} h\left(245f_n - 2290f_{n+1} + 9830f_{n+2} - 34186f_{n+3} + 800f_{n+4} - 63670f_{n+5} - 1510f_{n+6} - 46030f_{n+7} - 8341f_{n+8}\right)$$

2. Construction of the New Method Through Multistep Collocation

The method employed by Sirisena (1997) will be used in this construction where a k-step multistep collocation method with m collocation points is expressed as:

$$\bar{y}(x) = \sum_{j=0}^{t-1} \alpha_j(x)\, y(x_{n+j}) + h\sum_{j=0}^{m-1} \beta_j(x)\, f(x, \bar{y}(x))$$

Where t denotes the number of interpolation points and m denotes the number of distinct collocation points and $\alpha_j(x)$ and $\beta_j(x)$ are the continuous coefficients defined by:

$$\alpha_j(x) = \sum_{i=0}^{t+m-1} \alpha_{j,i+1} x^i, \quad j \in (0,1,\dots,t-1)$$
$$h\beta_j(x) = \sum_{i=0}^{t+m-1} \beta_{j,i+1} x^i, \quad j \in (0,1,\dots,m-1)$$

With constant coefficients $\alpha_{j,i+1}(x)$, $h\beta_{j,i+1}(x)$ to be determined using the interpolation and collocations conditions. (Onumanyi et.al 1994).

From the interpolation conditions and their expression for $\bar{y}(x)$, the following conditions are imposed on $\alpha_j(x)$ and $\beta_j(x)$. (Sirisena 1997).

$$\alpha_j x_{n+i} = \delta_{ij} \qquad j = 0,\dots t-1,\ i = 0,\dots t-1$$

$$\beta_j x_{n+i} = 0 \qquad j = 0,\dots \mathrm{m}-1,\ i = 0,\dots t-1$$

$$\alpha_j(x_i) = 0 \qquad j = 0,\dots t-1,\ i = 0,\dots \mathrm{m}-1$$

$$h\beta_j(\bar{x}_i) = \delta_{ij} \qquad j = 0,\dots \mathrm{m}-1,\ v = 0,\dots \mathrm{m}-1$$

We obtain matrices D and C where DC = I. Where I is the identity matrix.

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & \cdots & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2 & \cdots & x_{n+1}^{t+m-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{n+t-1} & x_{n+t-1}^2 & \cdots & x_{n+t-1}^{t+m-1} \\ 0 & 1 & 2x_0 & \cdots & 2x_0^{t+m-2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 1 & 2x_{m-1} & \cdots & (t+m-1)x_{m-1}^{t+m-2} \end{bmatrix}$$

and;

$$C = \begin{bmatrix} \alpha_{j-1,1} & h\beta_{0,1} & \cdots & h\beta_{\mathrm{m}-1,1} \\ \alpha_{j-1,2} & h\beta_{0,2} & \cdots & h\beta_{\mathrm{m}-1,2} \\ \vdots & \vdots & \cdots & \vdots \\ \alpha_{j-1,\mathrm{m}+1} & h\beta_{0,\mathrm{m}+1} & \cdots & h\beta_{m-1,m+1} \end{bmatrix}$$

Matrices C and D are of dimension (t+m) x (t+m). The columns of

C = D^{-1} yields the continuous coefficients $\alpha_j(x)$ and $\beta_j(x)$. Using the Maple software, we find the Inverse of the matrix D.

Construction of the New Class of Generalized Adams Method

Four Step Block Generalized Adams Method (k=4)

The block GAMS (4) is reformulated by choosing $v = \dfrac{k}{4}$ for k even to have a multiple of four. In developing a class of four step generalized Adams method, k=4 with collocation at the point x_n, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4} and interpolation at x_{n+1}.

$$\bar{y}(x) = \alpha_1(x)\,\mathrm{y}_n + h\left(\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2} + \beta_3(x)f_{n+3} + \beta_4(x)f_{n+4}\right)$$

With the use of Maple 17 software, evaluating at points $x = x_n, x_{n+2}, x_{n+3}, x_{n+4}$ yields the new class of GAMs for k = 4.

$$y_{n+1} - y_n = \frac{1}{720}h\left(251f_n + 646f_{n+1} - 264f_{n+2} + 106f_{n+3} - 19f_{n+4}\right)$$

$$y_{n+2} - y_n = \frac{1}{90}h\left(29f_n + 124f_{n+1} + 24f_{n+2} + 4f_{n+3} - f_{n+4}\right)$$

$$y_{n+3} - y_n = \frac{3}{80}h\left(9f_n + 34f_{n+1} + 24f_{n+2} + 14f_{n+3} - f_{n+4}\right)$$

$$y_{n+4} - y_n = \frac{2}{45}h\left(7f_n + 32f_{n+1} + 12f_{n+2} + 32f_{n+3} + 7f_{n+4}\right)$$

Eight Step Block Generalized Adams Method (k = 8)

The general form of the eight step Adams method is given as:

$$\bar{y}(x) = \alpha_2(x)\,\mathrm{y}_{n+1} + h\begin{pmatrix} \beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2} + \beta_3(x)f_{n+3} + \beta_4(x)f_{n+4} + \beta_5(x)f_{n+5} + \beta_6(x)f_{n+6} \\ +\beta_7(x)f_{n+7} + \beta_8(x)f_{n+8} \end{pmatrix}$$

Evaluating at $x = x_n, x_{n+1}, x_{n+3}, x_{n+4}, x_{n+5}, x_{n+6}, x_{n+7}, x_{n+8}$ to yield the following new discrete method of a class of GAMs k = 8.

$$y_n - y_{n+1} = -\frac{1}{3628800}\left(1070017f_n + 4467094f_{n+1} - 4604594f_{n+2} + 5595358f_{n+3} - 5033120f_{n+4} + 3146338f_{n+5} - 1291214f_{n+6} + 312874f_{n+7} - 33953f_{n+8}\right)h$$

$$y_{n+2}-y_{n+1}=-\frac{1}{3628800}\left(33953f_n-1375594f_{n+1}-3244786f_{n+2}+1752542f_{n+3}\right.$$
$$\left.-1317280f_{n+4}+755042f_{n+5}-294286f_{n+6}+68906f_{n+7}-7297f_{n+8}\right)h:$$

$$y_{n+3}-y_{n+1}=-\frac{1}{113400}h\left(833f_n-39874f_{n+1}-152596f_{n+2}-27478f_{n+3}\right.$$
$$\left.-15130f_{n+4}+11162f_{n+5}-4756f_{n+6}+1166f_{n+7}-127f_{n+8}\right)$$

$$y_{n+4}-y_{n+1}=-\frac{1}{44800}h\left(369f_n-16202f_{n+1}-57618f_{n+2}-34434f_{n+3}-33440f_{n+4}\right.$$
$$\left.+9666f_{n+5}-3438f_{n+6}+778f_{n+7}-81f_{n+8}\right)$$

$$y_{n+5}-y_{n+1}=-\frac{1}{14175}h\left(107f_n-5026f_{n+1}-18724f_{n+2}-9232f_{n+3}-19270f_{n+4}\right.$$
$$\left.-4402f_{n+5}-244f_{n+6}+104f_{n+7}-13f_{n+8}\right)$$

$$y_{n+6}-y_{n+1}=-\frac{5}{145152}h\left(245f_n-10546f_{n+1}-37210f_{n+2}-22090f_{n+3}\right.$$
$$\left.-32800f_{n+4}-30070f_{n+5}-13606f_{n+6}+1010f_{n+7}-85f_{n+8}\right)$$

$$y_{n+7}-y_{n+1}=-\frac{1}{1400}h\left(9f_n-482f_{n+1}-1908f_{n+2}-774f_{n+3}-2090f_{n+4}\right.$$
$$\left.-774f_{n+5}-1908f_{n+6}-482f_{n+7}+9f_{n+8}\right)$$

$$y_{n+8}-y_{n+1}=-\frac{7}{518400}h\left(1169f_n-31882f_{n+1}-74578f_{n+2}-105154f_{n+3}\right.$$
$$\left.-7840f_{n+4}-155134f_{n+5}-6958f_{n+6}-116662f_{n+7}-21361f_{n+8}\right)$$

Order and Error constant of the New Block Methods

A Class of the Generalized Adams Block Method k=4

The class of GAMs has the following orders and error constants

Method	Order	Error Constant
y_{n+4}	6	$\frac{-8}{945}$
y_{n+3}	5	$\frac{3}{160}$
y_{n+2}	5	$\frac{1}{90}$
y_n	5	$\frac{3}{160}$

A Class of the Generalized Adams Block Method k=8

The class of GAMs has the following orders and error constants

Method	Order	Error Constant
y_{n+8}	9	$-\frac{8183}{1036800}$
y_{n+7}	10	$\frac{425}{290304}$
y_{n+6}	9	$\frac{-425}{290304}$
y_{n+5}	9	$\frac{-13}{14175}$
y_{n+4}	9	$\frac{-113}{89600}$
y_{n+3}	9	$\frac{-13}{14175}$
y_{n+2}	9	$\frac{-425}{290304}$
y_n	9	$\frac{-8183}{1036800}$

Absolute Stability Regions of a Class of Generalized Adams Methods

The absolute stability of the newly constructed GAMs for the linear multistep methods is plotted using Ehigie (2014) by reformulating the methods as general linear methods. The regions of absolute stability of the newly constructed GAMs are as shown below

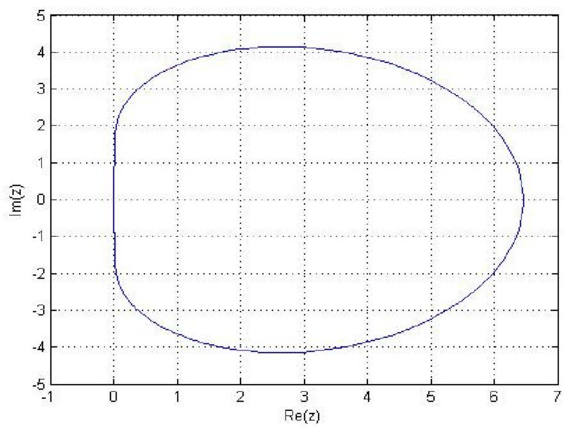

Fig 1. Absolute stability regions of the newly constructed GAMs k = 4

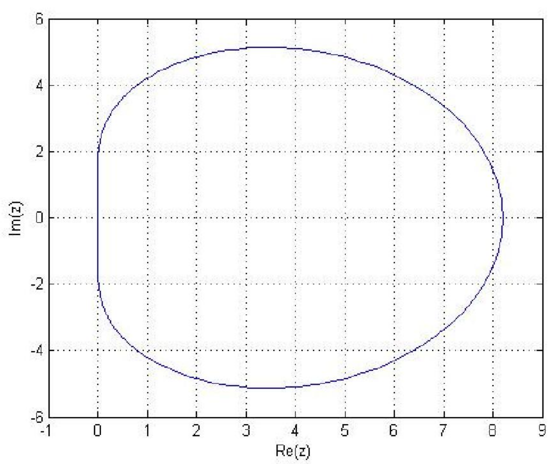

Fig 2. Absolute region of the newly constructed GAMs k = 8

The regions in Fig1 and Fig2 shows that the newly constructed GAMs for k=4 and 8 are A-stable since their regions of absolute stability contains the whole of the left-hand half plane Dahlquist, G. (1956).

Numerical Experiments

in this chapter, the performance of the new block generalized Adams method for k=4,8 is tested on some numerical examples. The results are compared with the exact solution and its performance is weighed against the performance of the conventional Adams for k=4 and 8.

Example 1:

$$y_1' = 998y_1 + 1998y_2$$
$$y_2' = 999y_1 - 1999y_2$$
$$y_1(0) = 1, y_2(0) = 1$$
$$h = \frac{1}{1000}, 0 \le x \le 0.02$$

exact solution:
$$y_1 = 4e^{-x} - 3e^{-1000x}$$
$$y_2 = -2e^{-x} + 3e^{-1000x}$$

Example 2:

$$y_1' = -8y_1 + 7y_2$$
$$y_2' = 42y_1 - 43y_2$$
$$y_1(0) = 1, y_2(0) = 8$$
$$h = \frac{1}{1000}, 0 \le x \le 0.02$$

exact solution:
$$y_1 = 2e^{-x} - e^{-50x}$$
$$y_2 = 2e^{-x} + 6e^{-50x}$$

Example 3:

$$y_1' = -1002y_1 + 1000y_2^2$$
$$y_2' = y_1 - y_2(1 + y_2)$$
$$y_1(0) = 1, y_2(0) = 1$$
$$h = \frac{1}{1000}, 0 \le x \le 0.02$$

exact solution:
$$y_1 = e^{-2x}$$
$$y_2 = e^{-x}$$

Table 2.1: Solution for example 1 using a new class of GAMs k=4

X	Y1 Exact	Y2 Exact	Y1 Numerical	Y2 Numerical
0.001	2.892363675	-0.960397341	2.898019288	-0.900018293
0.002	3.586002145	-1.589998147	3.585667927	-1.589663937
0.003	3.838656777	-1.844647786	3.841043916	-1.847034922
0.004	3.929085040	-1.937069062	3.923513230	-1.931497250
0.005	3.959836076	-1.969811117	3.957900414	-1.967875456
0.006	3.968635599	-1.980599671	3.967874787	-1.979838866
0.007	3.969362126	-1.98331324	3.969132877	-1.983083986
0.008	3.967121271	-1.983057442	3.966906825	-1.982842996
0.009	3.963791286	-1.981710529	3.963714695	-1.981633941
0.010	3.960063135	-1.979963467	3.960033967	-1.979934308
0.011	3.956191010	-1.978070453	3.956181300	-1.978060740
0.012	3.952268419	-1.976124993	3.952262226	-1.976118801
0.013	3.948329759	-1.974161489	3.948327528	-1.974159259
0.014	3.944387682	-1.972192594	3.944386832	-1.972191752
0.015	3.94044684	-1.970222961	3.940446548	-1.970222668
0.016	3.936508942	-1.968254302	3.936508786	-1.968254147
0.017	3.932574614	-1.966287245	3.932574558	-1.966287190
0.018	3.928644084	-1.964322019	3.928644050	-1.964321997
0.019	3.924717432	-1.962358707	3.924717422	-1.962358694
0.020	3.920794687	-1.960397341	3.920794687	-1.960397341

Table 2.2: Solution For Example 1 Using A New Class Of Gams K=8

X	Y1 Exact	Y2 Exact	Y1 Numerical	Y2 Numerical
0.001	2.892363675	-0.960397341	2.892732578	-0.894731573
0.002	3.586002145	-1.589998147	3.586066722	-1.590062719
0.003	3.838656777	-1.844647786	3.838707837	-1.844698840
0.004	3.929085040	-1.937069062	3.929086265	-1.937070279
0.005	3.959836076	-1.969811117	3.959854480	-1.969829510
0.006	3.968635599	-1.980599671	3.968613240	-1.980577302
0.007	3.969362126	-1.98331324	3.969434008	-1.983385100
0.008	3.967121271	-1.983057442	3.966699667	-1.982635833
0.009	3.963791286	-1.981710529	3.963636361	-1.981555597
0.010	3.960063135	-1.979963467	3.960006116	-1.979906444
0.011	3.956191010	-1.978070453	3.956170046	-1.978049488
0.012	3.952268419	-1.976124993	3.952260702	-1.976117268
0.013	3.948329759	-1.974161489	3.948326936	-1.974158663
0.014	3.944387682	-1.972192594	3.944386637	-1.972191549
0.015	3.94044684	-1.970222961	3.940446516	-1.970222615
0.016	3.936508942	-1.968254302	3.936508623	-1.968253977
0.017	3.932574614	-1.966287245	3.932574501	-1.966287124
0.018	3.928644084	-1.964322019	3.928644050	-1.964321980
0.019	3.924717432	-1.962358707	3.924717429	-1.962358699
0.020	3.920794687	-1.960397341	3.920794688	-1.960397334

Table 2.3: Solution For Example 2 Using A New Class Of Gams K=4

X	Y1 Exact	Y2 Exact	Y1 Numerical	Y2 Numerical
0.001	1.046771576	7.705377547	1.046771577	7.705377542
0.002	1.091166579	7.425028505	1.091166580	7.425028509
0.003	1.133301015	7.158256849	1.133301014	7.158256846
0.004	1.173285226	6.904400498	1.173285226	6.904400496
0.005	1.211224175	6.662829657	1.211224175	6.662829651
0.006	1.247217707	6.432945252	1.247217709	6.432945252
0.007	1.281360796	6.214177424	1.281360796	6.214177420
0.008	1.313743784	6.005984106	1.313743785	6.005984106
0.009	1.344452606	5.807849668	1.344452607	5.807849665
0.010	1.373569007	5.619283625	1.373569011	5.619283627
0.011	1.401170748	5.439819420	1.401170748	5.439819415
0.012	1.427331790	5.269013243	1.427331790	5.269013242
0.013	1.452122493	5.106442931	1.452122494	5.106442928
0.014	1.475609785	4.951706912	1.475609786	4.951706914
0.015	1.497857326	4.804423195	1.497857327	4.804423193
0.016	1.518925676	4.664228425	1.518925678	4.664228422
0.017	1.538872437	4.530776960	1.538872438	4.530776958
0.018	1.557752405	4.403740023	1.557752407	4.403740020
0.019	1.575617700	4.28280487	1.575617702	4.282804862
0.020	1.592517906	4.167673994	1.592517909	4.167673993

Table 2.4: Solution For Example 2 Using A New Class Of Gams K=8

X	Y1 Exact	Y2 Exact	Y1 Numerical	Y2 Numerical
0,001	1,046771576	7,705377547	1,046771577	7,705377553
0,002	1,091166579	7,425028505	1,091166581	7,425028506
0,003	1,133301015	7,158256849	1,133301015	7,158256856
0,004	1,173285226	6,904400498	1,173285226	6,904400501
0,005	1,211224175	6,662829657	1,211224175	6,662829664
0,006	1,247217707	6,432945252	1,247217708	6,432945250
0,007	1,281360796	6,214177424	1,281360795	6,214177431
0,008	1,313743784	6,005984106	1,313743785	6,005984113
0,009	1,344452606	5,807849668	1,344452605	5,807849671
0,010	1,373569007	5,619283625	1,373569008	5,619283631
0,011	1,401170748	5,439819420	1,401170746	5,439819424
0,012	1,427331790	5,269013243	1,427331787	5,269013245
0,013	1,452122493	5,106442931	1,452122493	5,106442936
0,014	1,475609785	4,951706912	1,475609786	4,951706911
0,015	1,497857326	4,804423195	1,497857327	4,804423201
0,016	1,518925676	4,664228425	1,518925678	4,664228430
0,017	1,538872437	4,530776960	1,538872435	4,530776968
0,018	1,557752405	4,403740023	1,557752406	4,403740029
0,019	1,575617700	4,28280487	1,575617701	4,282804873
0,020	1,592517906	4,167673994	1,592517906	4,167674001

Table 2.5: Solution For Example 3 Using A New Class Of Gams K=4

X	Y1 Exact	Y2 Exact	Y1 Numerical	Y2 Numerical
0,001	0,9980019987	0,9990004998	0,9987325935	0,9999992686
0,002	0,9960079893	0,9980019987	0,9982711140	0,9999977317
0,003	0,9940179641	0,9970044955	0,9980961824	0,9999959071
0,004	0,9920319148	0,9960079893	0,9980355959	0,9999939665
0,005	0,9900498337	0,9950124792	0,9980064491	0,9999919926
0,006	0,9880717129	0,9940179641	0,9979933488	0,9999900007
0,007	0,9860975443	0,9930244429	0,9979859496	0,9999880012
0,008	0,9841273201	0,9920319148	0,9979808242	0,9999859973
0,009	0,9821610324	0,9910403788	0,9979763215	0,9999839909
0,010	0,9801986733	0,9900498337	0,9979721349	0,9999819821
0,011	0,9782402351	0,9890602788	0,9979680565	0,9999799712
0,012	0,9762857098	0,9880717129	0,9979640201	0,9999779583
0,013	0,9743350896	0,9870841350	0,9979599925	0,9999759433
0,014	0,9723883668	0,9860975443	0,9979559678	0,9999739264
0,015	0,9704455335	0,9851119396	0,9979519400	0,9999719074
0,016	0,9685065821	0,9841273201	0,9979479096	0,9999698864
0,017	0,9665715046	0,9831436846	0,9979438758	0,9999678634
0,018	0,9646402935	0,9821610324	0,9979398387	0,9999658385
0,019	0,9627129409	0,9811793622	0,9979357961	0,9999638114
0,020	0,9607894392	0,9801986733	0,9979317501	0,9999617824

Table 2.6: Solution For Example 3 Using A Class Of Gams K=8

X	Y1 Exact	Y2 Exact	Y1 Numerical	Y2 Numerical
0.001	0.9980019987	0.9990004998	0.9987361496	0.9999992651
0.002	0.9960079893	0.9980019987	0.9982708393	0.9999977320
0.003	0.9940179641	0.9970044955	0.9980977543	0.9999959056
0.004	0.9920319148	0.9960079893	0.9980318381	0.9999939703
0.005	0.9900498337	0.9950124792	0.9980051371	0.9999919939
0.006	0.9880717129	0.9940179641	0.9980051371	0.9999900012
0.007	0.9860975443	0.9930244429	0.9979857498	0.9999880014
0.008	0.9841273201	0.9920319148	0.9979809659	0.9999859972
0.009	0.9821610324	0.9910403788	0.9979763740	0.9999839908
0.010	0.9801986733	0.9900498337	0.9979721536	0.9999819821
0.011	0.9782402351	0.9890602788	0.9979680646	0.9999799712
0.012	0.9762857098	0.9880717129	0.9979640203	0.9999779583
0.013	0.9743350896	0.9870841350	0.9979599939	0.9999759433
0.014	0.9723883668	0.9860975443	0.9979559668	0.9999739264
0.015	0.9704455335	0.9851119396	0.9979519419	0.9999719074
0.016	0.9685065821	0.9841273201	0.9979479071	0.9999698864
0.017	0.9665715046	0.9831436846	0.9979438741	0.9999678635
0.018	0.9646402935	0.9821610324	0.9979398381	0.9999658385
0.019	0.9627129409	0.9811793622	0.9979357965	0.9999638115
0.020	0.9607894392	0.9801986733	0.9979317492	0.9999617824

Error Analysis

In this section, we see the performance of the modified class of generalized Adams Method for step numbers k=4 and 8 in comparison to a class of conventional Adams methods of step numbers k=4 and 8. The absolute error using the two is displayed in the following tables.

Table 2.7: The Absolute Error For Example 1

X	Y1 Error Modified Gams K=4	Y1 Error Conventional Gams K=4	Y2 Error Modified Gams K=4	Y2 Error Conventional Gams K=4
0.001	5.66E-03	5.66E-03	6.04E-02	6.04E-02
0.002	3.34E-04	3.34E-04	3.34E-04	3.34E-04
0.003	2.39E-03	2.39E-03	2.39E-03	2.39E-03
0.004	5.57E-03	5.57E-03	5.57E-03	5.57E-03
0.005	1.94E-03	1.94E-03	1.94E-03	1.94E-03
0.006	7.61E-04	7.61E-04	7.61E-04	7.61E-04
0.007	2.29E-04	2.29E-04	2.29E-04	2.29E-04
0.008	2.14E-04	2.14E-04	2.14E-04	2.14E-04
0.009	7.66E-05	7.66E-05	7.66E-05	7.66E-05
0.010	2.92E-05	2.91E-05	2.92E-05	2.92E-05
0.011	9.71E-06	9.69E-06	9.71E-06	9.70E-06
0.012	6.19E-06	6.18E-06	6.19E-06	6.19E-06
0.013	2.23E-06	2.21E-06	2.23E-06	2.22E-06
0.014	8.50E-07	8.22E-07	8.42E-07	8.32E-07
0.015	2.92E-07	2.67E-07	2.93E-07	2.77E-07
0.016	1.56E-07	1.37E-07	1.55E-07	1.47E-07
0.017	5.60E-08	3.00E-08	5.50E-08	4.60E-08
0.018	3.40E-08	4.00E-09	2.20E-08	9.00E-09
0.019	1.00E-08	2.10E-08	1.30E-08	7.00E-09
0.020	0.00E+00	2.40E-08	0.00E+00	1.10E-08

Table 2.8: The absolute error for example 1

X	Y1 Error Modified Gams K=8	Y1 Error Conventional Gams K=8	Y2 Error Modified Gams K=8	Y2 Error Conventional Gams K=8
0.001	3.69E-04	3.69E-04	6.57E-02	6.57E-02
0.002	6.46E-05	6.46E-05	6.46E-05	6.46E-05
0.003	5.11E-05	5.10E-05	5.11E-05	5.10E-05
0.004	1.23E-06	1.22E-06	1.22E-06	1.21E-06
0.005	1.84E-05	1.84E-05	1.84E-05	1.84E-05
0.006	2.24E-05	2.24E-05	2.24E-05	2.24E-05
0.007	7.19E-05	7.19E-05	7.19E-05	7.19E-05
0.008	4.22E-04	4.22E-04	4.22E-04	4.22E-04
0.009	1.55E-04	1.55E-04	1.55E-04	1.55E-04
0.010	5.70E-05	5.70E-05	5.70E-05	5.70E-05
0.011	2.10E-05	2.10E-05	2.10E-05	2.10E-05
0.012	7.72E-06	7.74E-06	7.72E-06	7.73E-06
0.013	2.82E-06	2.85E-06	2.83E-06	2.84E-06
0.014	1.04E-06	1.06E-06	1.05E-06	1.06E-06
0.015	3.24E-07	3.49E-07	3.46E-07	3.51E-07
0.016	3.19E-07	3.48E-07	3.25E-07	3.33E-07
0.017	1.13E-07	1.41E-07	1.21E-07	1.31E-07
0.018	3.40E-08	6.60E-08	3.90E-08	5.30E-08
0.019	3.00E-09	4.60E-08	8.00E-09	2.60E-08
0.020	1.00E-09	4.30E-08	7.00E-09	2.50E-08

Table 2.9: The absolute error for example 2

X	Y1 Error M0dified Gams K=4	Y1 Error Conventional Gams K=4	Y1 Error M0dified Gams K=4	Y1 Error Conventional Gams K=4
0.001	1.00E-09	1.00E-09	5.00E-09	2.00E-09
0.002	1.00E-09	0.00E+00	4.00E-09	3.00E-09
0.003	1.00E-09	1.00E-09	3.00E-09	6.00E-09
0.004	0.00E+00	0.00E+00	2.00E-09	1.00E-09
0.005	0.00E+00	0.00E+00	6.00E-09	2.00E-09
0.006	2.00E-09	1.00E-09	0.00E+00	3.00E-09
0.007	0.00E+00	0.00E+00	4.00E-09	2.00E-09
0.008	1.00E-09	2.00E-09	0.00E+00	0.00E+00
0.009	1.00E-09	2.00E-09	3.00E-09	3.00E-09
0.010	4.00E-09	3.00E-09	2.00E-09	1.00E-09
0.011	0.00E+00	2.00E-09	5.00E-09	4.00E-09
0.012	0.00E+00	4.00E-09	1.00E-09	3.00E-09
0.013	1.00E-09	4.00E-09	3.00E-09	4.00E-09
0.014	1.00E-09	4.00E-09	2.00E-09	0.00E+00
0.015	1.00E-09	3.00E-09	2.00E-09	2.00E-09
0.016	2.00E-09	4.00E-09	3.00E-09	3.00E-09
0.017	1.00E-09	5.00E-09	2.00E-09	3.00E-09
0.018	2.00E-09	5.00E-09	3.00E-09	2.00E-09
0.019	2.00E-09	3.00E-09	3.00E-09	0.00E+00
0.020	3.00E-09	6.00E-09	1.00E-09	5.00E-09

Table 2.10: The absolute error for example 2

X	Y1 Error Modified Gams K=8	Y1 Error Conventional Gams K=8	Y2 Error Modified Gams K=8	Y2 Error Conventional Gams K=8
0.001	1.00E-09	3.00E-09	6.00E-09	3.00E-09
0.002	2.00E-09	1.00E-09	1.00E-09	5.00E-09
0.003	0.00E+00	3.00E-09	7.00E-09	6.00E-09
0.004	0.00E+00	4.00E-09	3.00E-09	5.00E-09
0.005	0.00E+00	4.00E-09	7.00E-09	1.20E-08
0.006	1.00E-09	1.00E-09	2.00E-09	1.10E-08
0.007	1.00E-09	2.00E-09	7.00E-09	3.00E-09
0.008	1.00E-09	2.00E-09	7.00E-09	8.00E-09
0.009	1.00E-09	5.00E-09	3.00E-09	8.00E-09
0.010	1.00E-09	3.00E-09	6.00E-09	9.00E-09
0.011	2.00E-09	4.00E-09	4.00E-09	3.00E+00
0.012	3.00E-09	4.00E-09	2.00E-09	1.10E-08
0.013	0.00E+00	3.00E-09	5.00E-09	6.00E-09
0.014	1.00E-09	7.00E-09	1.00E-09	1.00E-08
0.015	1.00E-09	2.00E-09	6.00E-09	8.00E-09
0.016	2.00E-09	7.00E-09	5.00E-09	1.00E-08
0.017	2.00E-09	6.00E-09	8.00E-09	1.00E-08
0.018	1.00E-09	8.00E-09	6.00E-09	1.20E-08
0.019	1.00E-09	4.00E-09	8.00E-09	7.00E-09
0.020	0.00E+00	5.00E-09	7.00E-09	9.00E-09

Table 2.11: The absolute error for example 3

X	Y1 Error Modified Gams K=4	Y1 Error Conventional Gams K=4	Y2 Error Modified Gams K=4	Y2 Error Conventional Gams K=4
0.001	7.31E-04	7.31E-04	9.99E-04	9.99E-04
0.002	2.26E-03	2.26E-03	2.00E-03	2.00E-03
0.003	4.08E-03	4.08E-03	2.99E-03	2.99E-03
0.004	6.00E-03	6.00E-03	3.99E-03	3.99E-03
0.005	7.96E-03	7.96E-03	4.98E-03	4.98E-03
0.006	9.92E-03	9.92E-03	5.97E-03	5.97E-03
0.007	1.19E-02	1.19E-02	6.96E-03	6.96E-03
0.008	1.39E-02	1.39E-02	7.95E-03	7.95E-03
0.009	1.58E-02	1.58E-02	8.94E-03	8.94E-03
0.010	1.78E-02	1.78E-02	9.93E-03	9.93E-03
0.011	1.97E-02	1.97E-02	1.09E-02	1.09E-02
0.012	2.17E-02	2.17E-02	1.19E-02	1.19E-02
0.013	2.36E-02	2.36E-02	1.29E-02	1.29E-02
0.014	2.56E-02	2.56E-02	1.39E-02	1.39E-02
0.015	2.75E-02	2.75E-02	1.49E-02	1.49E-02
0.016	2.94E-02	2.94E-02	1.58E-02	1.58E-02
0.017	3.14E-02	3.14E-02	1.68E-02	1.68E-02
0.018	3.33E-02	3.33E-02	1.78E-02	1.78E-02
0.019	3.52E-02	3.52E-02	1.88E-02	1.88E-02
0.020	3.71E-02	3.71E-02	1.98E-02	1.98E-02

Table 2.12: The absolute error for example 4.3

X	Y1 Error Modified Gams K=8	Y1 Error Conventional Gams K=8	Y2 Error Modified Gams K=8	Y2 Error Conventional Gams K=8
0.001	7.34E-04	7.34E-04	9.99E-04	9.99E-04
0.002	2.26E-03	2.26E-03	2.00E-03	2.00E-03
0.003	4.08E-03	4.08E-03	2.99E-03	2.99E-03
0.004	6.00E-03	6.00E-03	3.99E-03	3.99E-03
0.005	7.96E-03	7.96E-03	4.98E-03	4.98E-03
0.006	9.93E-03	9.92E-03	5.97E-03	5.97E-03
0.007	1.19E-02	1.19E-02	6.96E-03	6.96E-03
0.008	1.39E-02	1.39E-02	7.95E-03	7.95E-03
0.009	1.58E-02	1.58E-02	8.94E-03	8.94E-03
0.010	1.78E-02	1.78E-02	9.93E-03	9.93E-03
0.011	1.97E-02	1.97E-02	1.09E-02	1.09E-02
0.012	2.17E-02	2.17E-02	1.19E-02	1.19E-02
0.013	2.36E-02	2.36E-02	1.29E-02	1.29E-02
0.014	2.56E-02	2.56E-02	1.39E-02	1.39E-02
0.015	2.75E-02	2.75E-02	1.49E-02	1.49E-02
0.016	2.94E-02	2.94E-02	1.58E-02	1.58E-02
0.017	3.14E-02	3.14E-02	1.68E-02	1.68E-02
0.018	3.33E-02	3.33E-02	1.78E-02	1.78E-02
0.019	3.52E-02	3.52E-02	1.88E-02	1.88E-02
0.020	3.71E-02	3.71E-02	1.98E-02	1.98E-02

Conclusion

Classes of linear multistep collocation methods for the solution of initial value problems of general first order ordinary differential equations have been developed in this research. The collocation technique yielded A-stable 4 and 8-step GAMs with constant coefficients. Results from the numerical solutions of Stiff and non-linear IVPs shows that these classes of methods are good for the solution of Stiff ODEs and performed relatively better upon comparison with the conventional Adams Schemes for step number k=4 and for step number k=8.

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