**Calculation of phase shift for p+40Ca elastic scattering at low energy**

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**Abstract:** The theoretical analysis of the experimental proton elastic scattering on 40Ca taken in the energy range 9 to 22 MeV has been studied within the framework of the optical model employing Woods-Saxon and its derivative forms for potentials. Using SCAT2000 FORTRAN code and selected set of optical model parameters (OMPs) the contribution of different partial waves to scattering amplitude and cross section are studied. Based on phase shift values the significant number of partial waves in terms of their corresponding angular momenta at a given projectile energy is determined.

[Khalaf AM, Khalifa MM, Solieman AHM, Comsan MNH. **Calculation of phase shift for p+40Ca elastic scattering at low energy**. *Nat Sci* 2015;13(6):144-149]. (ISSN: 1545-0740). <http://www.sciencepub.net/nature>. 9. doi:[10.7537/marsnsj130615.19](http://www.dx.doi.org/10.7537/marsnsj130615.19)

**Keywords:** phase shift; reflection coefficient; partial waves; optical model

**1. Introduction**

Study of nuclear scattering is an important source that provides information about the nuclear structure. Especially the scattering of protons is sensitive to details of the nuclear wave function [1]. Therefore the phase shift of the partial waves, for the elastic scattering nucleon-nucleus interaction, is considered a significant parameter that can be used to study the contribution of partial waves to the scattering amplitude, and hence to the cross section and polarization results [2]. Previously, phase shifts were calculated by Gross et al, for p+40Ca elastic scattering in the range of 30 ≤ Ep ≤ 45 MeV to get simultaneous fit to the polarization and cross-section data with a reasonable set of optical-model parameters[3]. They concluded that the calculated phase shifts gave no indication of elastic scattering resonances in the considered energy range. The phase shifts, for proton elastic scattering on light nuclei in the range of 15 ≤ A ≤ 40 and 14 ≤ Ep ≤ 44 MeV, were used by Fabrici et al. for determining the partial waves that are most significant and responsible for *x*2 improvement of polarization for proton-light nuclei at low energy. They used phase shift analysis for *x*2 minimization for every nucleus and incident energy [4]. In addition, the phase shifts can be used to study anomalous absorption of the scattered partial waves of proton-nucleus interaction by the optical potential [5-6].

In the present work the elastic scattering of P+40Ca will be discussed within optical model in energy range 9 ≤ Ep ≤ 22 MeV. The purpose is the determination of partial waves of most contribution to the scattering amplitude or the cross section values based on the phase shift calculation. Moreover, the reflection coefficients dependence on angular momentum is studied.

**2. Theory**

In nucleon-nucleus elastic scattering problems, Schrodinger equation must include optical potential to get the scattering observables [7-8]. By numerical solution of Schrodinger equation one can obtain the reflection coefficient and phase shift that will be discussed in the next sections. In this section theoretical method of calculation that was used in SCAT2000 code is outlined. The radial wave equation that includes optical potential can be written as:

(1)

where and are the wave number and reduced mass of the system, respectively. E is the center of mass energy is in being the Planck constants, is the angular momentum, is the centrifugal potential, and is the optical potential representing the interaction.

(2)

where , , , , , and are the potential depths of the real volume, real surface, imaginary volume, imaginary surface, real spin orbit, and imaginary spin orbit terms, respectively. The terms, , and are the Wood- Saxon and its derivative forms that can be defined as:

, .

(3)

.

, (4)

,

. (5)

(6)

(7)

where (represents the constant of radius parameter and ( represents the diffuseness parameter. Considering the incident proton as a point charge and the target as a uniformly charged sphere with radius , the coulomb energy has the form.

(8)

where is the Thomas constant, total angular momentum, spin of incident projectile, and atomic number of target and projectile and is the constant of coulomb radius parameter. Equation (1) was solved numerically by using Cowell method [7]. The obtained result of the numerical solution is the transmission coefficients defined as:

. (9)

where is the reflection coefficient.

(10)

Expressing complex phase shifts in terms of real and imaginary parts, then

(11)

where is the phase amplitude coefficient. If , the intensity of the outgoing wave is equal to that of the incoming wave and reflection is complete. However there are non-elastic processes (absorbed wave functions), the intensity of the outgoing elastically scattered wave must be less than that of the incident wave so that [9-11].

The value of phase shift () vector magnitude is:

(12)

where

**3. Optical model parameters**

Three OMPs sets are used to calculate the elastic scattering phase shifts for p+40Ca interaction. Two of them are of Koning [12] and Xiaohua [13] which are obtained from literature by using the reference input parameters library (RIPL-2) [14]. These parameter sets are extracted from (RIPL-3) by FORTRAN code (MOM) [15]. This code is available on the web site of IAEA. The other set is our OMPs set which is obtained by fitting to get better agreement with experimental data. This set is presented in tables 1. The results of three OMPs angular distribution were discussed in details in previous paper [16**]**.

Table 1. OMPs set of our work that was used for fitting the experimental data [16]

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| E  (MeV) | (MeV) | (MeV) | | (MeV) | (MeV) | | (MeV) | (MeV) |
| 9.860 | 49.413 | 2.400 | | 0.525 | 2.610 | | 5.000 | -0.044 |
| 10.370 | 49.112 | 2.332 | | 0.544 | 2.801 | | 5.000 | -0.045 |
| 11.420 | 48.492 | 2.112 | | 0.586 | 3.199 | | 5.000 | -0.048 |
| 12.440 | 47.890 | 1.898 | | 0.628 | 3.587 | | 5.000 | -0.052 |
| 13.950 | 46.999 | 1.581 | | 0.694 | 4.161 | | 5.000 | -0.061 |
| 14.520 | 46.663 | 1.461 | | 0.720 | 4.378 | | 5.000 | -0.065 |
| 15.570 | 46.044 | 1.240 | | 0.770 | 4.777 | | 5.000 | -0.074 |
| 15.970 | 45.810 | 1.151 | | 0.789 | 4.929 | | 5.000 | -0.077 |
| 16.570 | 45.454 | 1.030 | | 0.820 | 5.157 | | 5.000 | -0.083 |
| 17.570 | 44.864 | 0.820 | | 0.870 | 5.537 | | 5.000 | -0.094 |
| 18.570 | 44.274 | 0.613 | | 0.922 | 5.917 | | 5.000 | -0.110 |
| 19.570 | 43.678 | 0.398 | | 0.977 | 6.300 | | 5.000 | -0.120 |
| 20.570 | 43.094 | 0.190 | | 1.032 | 6.677 | | 5.000 | -0.130 |
| 21.680 | 42.440 | 0.000 | | 1.093 | 7.100 | | 5.000 | -0.150 |
| (fm) = 1.246 | | |  | | |  | | |
|  | | |  | | |  | | |
|  | | |

**4. Results and Discussion**

The reflection coefficients are plotted in figure 1. The figure represents the value of versus angular momentum *l* at energies 9.86, 15.57, and 21.57 MeV.



Figure 1. Reflection coefficients () for Koning, Xiaohua, and Our OMPs as function of angular momentum (*l*) at different energy, the plus sign refers to at *j = l+*1/2and minus sign refers to at *j = l-*1/2.



Figure 2. Phase shifts () for Koning, Xiaohua, and Our OMPs as function of angular momentum (*l*) at different energy, the plus sign refers to at *j = l+*1/2and minus sign refers to at *j = l-*1/2.

The minus sign refers to the spin down of the incident protons, i.e. . The plus sign refers to the spin up of the incident protons, i.e. The reflection coefficient as mentioned is the ratio of the reflected partial waves to the total incident partial waves. The value of the reflection coefficient is restricted between . As seen in figure 1 the magnitudes of , at for three OMPs, display large fluctuation between 0.1 and 0.8 at small values of angular momentum *l* . This is due to an anomalous absorption of the partial waves [5-6]. Then the fluctuation of becomes small at MeV and the absorbed partial waves become larger as E increases. Finally at MeV the fluctuation of becomes smaller with value between 0.1 and 0.3 at small value of *l*, this means large absorption of partial waves is occurred, then it tends to unity i.e. ≈ 1 at large value of *l* indicating that total reflection of partial waves occurred. If the value of the nucleus will become black and it can absorb all incident partial waves. However this cannot occur due to the strong reflection by the centrifugal potential.[10-11].

In order to investigate the contribution of the partial waves the value of phase shifts for each partial wave should be calculated. The calculated phase shifts that are plotted in figure 2 can be used to determine the significant partial waves. For fixed energy, the number of partial waves that can contribute to the cross section is calculated by SCAT2000 according to this relation [7].

& j = 1, 2, 3 (13)

where and are the radius parameter and diffuseness respectively. The index j refers to the types of the real volume, imaginary volume, and imaginary surface potentials. This relation determines the number of significant partial up to i.e. is the upper limit of contribution of partial waves to cross sections.

According to the phase shifts analysis in figure 2, the significant number of partial waves can be cut at which is defined as the angular momentum *l* at which the value of its phase shift equals the maximum value of at a given energy. In such case all partial waves that have can be ignored. The difference between of SCAT2000 calculations and of phase shift calculation is plotted in figure 3. The number of significant partial waves at 9.86 MeV equals 6 and 8 at 21.68 MeV as shown in figure 3. The values of total cross section according to and are shown in tables 2.

Table 2. Numerical values of cross section calculated for and partial waves

|  |  |  |
| --- | --- | --- |
| E (MeV) | for | for |
| 9.860 | 1671.100 | 1671.000 |
| 13.950 | 1740.400 | 1740.200 |
| 17.570 | 1698.000 | 1697.900 |
| 21.680 | 1720.000 | 1719.980 |



Figure 3. Number of partial waves at and

On the other hand, the behavior of vector magnitude of phase shifts for the three OMPs are studied to know the dependence of the phase shift on energy and the differences between the OMPs sets as shown in figure 4. The behavior of phase shifts, for at MeV in addition to at 9.86, 19.58, and 21.68 MeV, displays broad peaks. These peaks may be attributed to the effect of the interaction that takes place in the interior region of the nucleus, where low partial have high contribution. The similarity of phase shifts behavior for Xiaohua and our OMPs may be attributed to the similarity of our OMPs and those of Xiaohua. While as directed outword of the nuclear surface at higher ,,…… partial waves, the behavior of this partial waves become smoother [17-18].



Figure 4. Phase shifts (), that are calculated by Koning, Xiaohua, and Our OMPs, as function of incident proton energy for number of partial waves (*l* = 0, 1, 2, 3,4 and s = ± 1/2). Plus and minus signs mean that at *j = l+*1/2and at *j = l-*1/2

**5. Conclusion**

Three OMPs sets were used to calculate reflection coefficients and phase shifts of elastic scattering for p+40Ca. The calculations were performed by means of SCAT2000. The calculated phase shifts were used to study of the contribution of partial waves to cross section. According to the results of phase shift analysis the significant number of partial waves to be taken in calculation has an upper limit , in contrary to SCAT 2000 in which calculations are performed up to . Our used scheme to determine the based on phase shift approach yealds cross sections that agree well with those of SCAT2000 based on . Using means less number of partial waves and hence reducing the calculation time.

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6/13/2015