Convergence of Periodical Evolutionary Computation

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Abstract: This article study the convergence of the Genetic Evolution Computation with the algorithm that can be described by discrete Markov Chain with constant transfer probabilities matrix, for example, the algorithm with the periodical mutations in crossing, and static strategy. They likely converge at a stable distribution of probabilities if correctly manipulated, although are generally not convergent well.
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1. Introduction

In Genetic Evolutionary Computation the objective of computation is always binary codes that's discrete and is evolved on. It's right to describe the process of evolution as a discrete Markov chain, with the population as a sample in states set in the stochastic process. From one states to the next states the probabilities of transfer is described by a square matrix called Transfer Probabilities Matrix that's denoted by

$$Pn=P_n(S)=P_{n-1}(S)M$$

$P_n(s_i)$ is the probability of state $s_i \in S$ happening in the n-th generation. The matrix of M posses the following property

$$\sum_j\sum_ip_iM_{ij}=\sum p_i,p_i\geq0$$

In fact if M is constant then it described a cycle of crossings combined with mutations.

2. Convergence

The evolutionary operator is S_{M}, which operates on population:

$$X_n=X_{n-1}S_M$$

The problem is to calculate the power of its Transfer Probabilities Matrix

$$M^k$$

and

$$\lim_{k\to \infty}M^k$$

They respectively corresponds the results of population after k generations and the convergence of the population after infinite generations.

Simplify the calculation by Camille Jordan's method \cite{p1}:

$$M=PJP^{-1}$$

J is Jordan's canonical form [1] like

$$J=\left(\begin{array}{cccc}J_0&&&&J_2&&&&\cdot&&&&J_n\end{array}\right)$$

$$J_i=\left(\begin{array}{ccccc}\lambda_i&1&&&&\lambda_i&1&&&&\cdot&\cdot&&&&\lambda_i&1&&&&\lambda_i\end{array}\right)$$

The calculation of M^k is transformed to calculation of J^k, and J^k

is resolved to calculation of direct sum J_i^k. Try to calculate

$$J_i^k$$

$$r_n:=\delta_{i-j+n}$$

D is the dimension of J_i.

$$J_i^k=(\lambda_iE+r_1)^k=\lambda_i^k\sum_{j=0}^{D-1}C_k^j\frac{r_j}{\lambda_i^{j}}$$

It's easy to find if $|\lambda_i|<1$, J_i^k approaches to zero if k approaches to infinity. And easy to find if

$D\neq 1$ an $|\lambda_i|=1$ the factor C_k^{D-1} in above expression would far greater than any others, and the element in the right-up corner of the matrix j_i^k, as $k\to \infty$ it will far surpass all the other calculation it does not participate. This case directs to a invalid transfer probabilities matrix of M^k. The case of $|J_i=\lambda_i|>1$ is similar. We can draw a conclusion that modulus of all the characters of a stochastic matrix are not greater than 1. This results already explicated by G. Rudolph in 1997 (p.55) [2] along with other a few results..

For the character of M: $\lambda=e^{ia}$ the case is that the corresponding character of M^k is e^{iak}.

The solution of recursive is fluctuation. It's suitable to calculate the mean value coupling with the reachable of the calculation of n-th generation, which probability is weighted r^n

$$\frac{1}{1-r}M_m:=\sum_{i=0}^\infty r^iM^i=\frac{1}{1-rM}$$

If make $r\to 1$ one can find in M_k the only dominating are those character $\lambda=1$ as $k\to \infty$, The result is in fact a known constant.

Now to prove the character 1 is always exists. Make

$$|M-E|$$

Easy to find that $\Sigma_i(M-E)_{ij}=0$, Hence M-E has character 0, so M has character 1.

3. Adopted Manipulation

A Feasible way to get a convergent probabilities of population is to merge two populations according to their rates artificially defined. The merging method is choose elements from the two populations randomly according the their artificial rate, which operation corresponds to the weighted addition of probabilities for the two generations.

Obviously the simple recursive evolutionary algorithm that we are used to apply is not well convergent, but after a small variations there exists a few convergent algorithms like these two:

1)

$$\sum_nr^nM^n$$

2)

$$S_1=M,\ S_2=1+rS_1,S_i=1+rMS_{i-1}$$

S_i is proved a well convergent transfer probabilities matrix, its algorithm is described as

1) Choose a population x randomly.
2) Evolute for a generation g_1 to represent M.
3) Merge the generation x,g_1 according to the rate 1:r (to represent S_1).
4) Continue this way. (g_n represents S_n) ... ...
5) Merge the generation x,g_n according to the rate $1:r(1-r^{n})/(1-r)$.

There still are convergent probabilities transfer matrixes like

$$(M+M^2)/2$$

Hence the character $e^{ia},a\neq 0$ of M is transformed to

$$(e^{ia}+e^{i2a})/$$

whose modulus is less than 1. This matrix represents the algorithm that merges the successive two generations of M. It's an ideal algorithm that's convergent with the modulus of characters of its matrix less than 1 if the character not being 1. Its schedule is

1) A population X_0.
2) To get population $X_1=X_0S_M$.
3) Merging X_0,X_1 at equal rate to X_2.
4) To get population $X_3=X_2S_M$.
5) Merging X_2,X_3 to X_4.
6) Calculating $X_5=X_4S_M$ by the evolution procedure.
7) Continuing on in this way.

The eigen vectors of M is always that of $(M+M^2)/2$ though the verse versa is also correct. In the other view angle, though $(M+M^2)/2$ is a different strategy of evolution it servers the same objective. As a fact the convergent part is the eigen part the evolution matrix and it's not always unique but a blended space of eigen vectors according the their same character 1 that belongs to a 1-dimensional irreducible space.

4. Problems

In the view angle of matrix, the main of current convergence results includes those [2] by G. Rudolph . These results form the current opinions on the convergence of evolution computation or neuro-nets. However, his proposal base of convergence is nearly all right except the frequently cited lemma:

Let P be a reducible stochastic matrix, where $C\in R^{m\cdot m}$ is a primitive stochastic matrix and $R,T\neq 0$. Then

$$P^\infty=\lim_{t\to\infty}\left(\begin{array}{cc} C^t&0\sum_{i=0}^{t-1}T^iRC^{t-i}&T^i\end{array} \right).$$

$$=\left(\begin{array}{cc}C^\infty&0R^\infty&0\end{array}\right).$$

is a stable stochastic matrix with $P^\infty=ep^\infty$, where $p^\infty=p^0P^\infty$ is unique regardless of the initial distribution and the limit distribution p^∞ satisfies

$$p_i^\infty>0,\textrm{ for 1\leq i\leq m and p_i^\infty=0}$$

$$for m<i\leq n.}$$

In fact, some the irreducible part of the limit of powers of the matrix is unstable possibly.

The other problem is that, I think the classical merging or selection of populations separately in the course of crossing or mutation is not always leading to stable convergence.

5. Conclusion

From the calculation of this article, It's obvious that the periodical and static strategies of genetic evolutionary computation, is generally not convergent well, however, if considering the cost or correctly manipulated, it likely leads to convergence with a stable probabilities distribution of the states of populations.

References

[1] Daniel T. Finkbeiner II. Introduction to Matrices and Linear Transformations, Third Edition, Freeman, 1978

[2] G. Rudolph. convergence propertyies of evolutionary algorithm. Kovac, Hamburg, 1997.

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