Mathematical Modeling of Salt Water Transport and its Control in Groundwater

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ABSTRACT: A mathematical model is presented for simulating two-dimensional sub-surface transport of salt-water considering the porous medium to be homogeneous and isotropic under the influence of constant seepage velocity. The transport equation is solved numerically by using finite difference method of Alternate Direction Implicit (ADI) scheme. Model simulations were carried out when sub-surface barrier is either absent or present. The effectiveness of the sub-surface barrier to arrest salinity movement is studied by choosing its locations at $X_1 = 1.2$ and $X_1 = 2.8$. The time evolution of the zone of salt-water concentration is studied for two years, four years, six years and ten years period for all the situations. It is observed that due to the barrier location at $X_1 = 2.8$, the salinity plume at the upper boundary is restricted in comparison to the non-barrier case. However, due to barrier, location at $X_1 = 1.2$ salinity intrusion is controlled significantly. [Nature and Science. 2006;4(4):32-39].

Keywords: Mathematical model; Simulation; ADI scheme; Salt water intrusion; Subsurface transport

INTRODUCTION

Intrusion of salt-water is a major problem in the coastal regions of all over the world. The occurrence of saline water intrusion in groundwater represents a special category of pollution, making groundwater unfit for human consumption as well as for industrial use. As these processes are of sub-surface nature, it is often difficult to assess the extent of contamination. Todd (1974)[4] has studied salt water intrusion in an aquifer and its control by considering various alternatives with respect to the flow field. Suitable sub-surface barrier was proposed by Todd (1974)[4] which includes cement grout, sheet pile or pulled clay etc. to arrest salt-water intrusion. Mahesha and Babu (2002)[3] have studied the effectiveness of sub-surface barrier on salt-water intrusion for sudden draw down conditions. Although, determination of interface by knowing the flow field is an important aspect under draw down condition, the roll of concentration gradient for mixing in relatively calm water at great depth cannot be over emphasized.

Latinopoulos *et al.* (1988)[2] have obtrained analytical solution for chemical transport in two-dimensional aquifers. Das *et al.* (2000)[1] have studied solute transport in porous media with first order chemical reaction for various disposal schemes by using numerical method.

The present study describes the evolution of salt-water concentration in a homogeneous, isotropic aquifer by assuming constant seepage velocity. This study would by useful in making initial estimation on the extent of aquifer contamination when *in situ* information is lacking and also for choosing suitable location of sub-surface barrier to arrest salinity intrusion.

MATHEMATICAL FORMULATION

The transport of conservative type of material through a homogeneous, isotropic plane aquifer of unit thickness under constant seepage velocity can be described by advective-dispersive differential equation in Cartesian coordinate system as :

$$D_x \frac{\partial^2 f}{\partial x^2} + D_y \frac{\partial^2 f}{\partial y^2} - V \frac{\partial f}{\partial x} - S \frac{\partial f}{\partial t} = 0 \qquad (1)$$

where, x – axis is in the direction of the flow and y – axis is along the depth of the aquifer. The schematic diagram of sub-surface transport process is shown in Figure1. Assuming the depth of the aquifer (L) is orthogonal to the natural groundwater flow direction, a finite line source of length ‘a’ aligned along the y-axis is allowed to enter into the model from left boundary. D_x and D_y being longitudinal and transverse dispersion coefficients respectively, U being the constant seepage velocity and R being retardation factor. The corresponding initial and boundary conditions can be described as –

$$f(x, y, 0) = f_i$$

$$f(0, y, t) = f_{input} \quad y \leq a, \ t > 0 \qquad (2)$$

$$= f_i \quad y > a, t < 0 \qquad (3)$$

$$\underset{y \to L}{\mathrm{Lt}} \frac{\partial f}{\partial y} = 0 \qquad (4)$$

$$\underset{x \to \infty}{\mathrm{Lt}} \frac{\partial f}{\partial f} = 0 \qquad (5)$$

where, f_i is the initial concentration of the aquifer and f_{input} is the boundary concentration. An impervious sub-surface barrier of unit thickness extending upto length ‘a’ from impermeable bottom is considered at a distance x_1 from the left boundary. The following internal boundary condition is imposed for impervious barrier.

$$\underset{x \to x_1}{\mathrm{Lt}} \frac{\partial f}{\partial x} = 0, y \leq a, \ t > 0 \qquad (6)$$

Writing the governing equation (1) in non-dimensional form as –

$$D_1 \frac{\partial F}{\partial X^2} + D_t \frac{\partial F}{\partial Y^2} - \frac{\partial F}{\partial X} - S \frac{\partial F}{\partial T} = 0 \qquad (7)$$

where, the dimensional quantities are given by –

$$X = \frac{x}{L}; Y = \frac{y}{L}; A = \frac{a}{L}; T = \frac{tU}{L}; F = \frac{f}{f_0}; F_1 = \frac{f_1}{f_0}; F_{input} \frac{F_{input}}{f_0}; D_1 = \frac{D_x}{LV}; D_1 = \frac{D_y}{LV}$$

Here, f_0 is the concentration of sea salinity. The corresponding initial and boundary conditions can be written in the dimensionless form as –

$$F(X,Y,0) = F_1 \qquad (8)$$

$$F(0,Y,T) = F_{input} \qquad Y \leq A,\ T \geq 0 \qquad (9)$$

$$= F_1 \qquad Y \leq A,\ T \geq 0$$

$$\underset{Y \to 0}{\text{Limit}} \frac{\partial F}{\partial Y} = 0 \qquad (10)$$

$$\underset{Y \to \infty}{\text{Limit}} \frac{\partial F}{\partial Y} = 0 \qquad (11)$$

The governing equation (7) described above is solved by using initial and boundary conditions (8) – (11) and employing finite difference scheme, specifically by using ADI method.

NUMERICAL COMPUTATION

In order to solve the governing equation (7) numerically, a well known. Alternate Direction Implicit (ADI) scheme of finite difference technique is adopted. This is based on implicit approach of the Douglas Rachford difference scheme. The details of the scheme and its finite difference representation can be obtain from Das *et al.* (2000)[1]. To get an insight in to the dispersion process, numerical computations have been made by assigning the values of various parameters as $D_l = 0.5$, $D_t = 0.05$, $A = 0.6$. Initially the aquifer is free from salinity i.e., $F_i = 0$. The boundary salinity F_{input} is taken as unity due to non-dimensional formulation with respect to constant sea salinity i.e. $f_0 = 34$ ppt, considered in this study.

The model area covers non-dimensional distance varying from 0 to 5.0 along the length of the aquifer and 0 to 1.0 along the depth of the aquifer. A square grid cell with $\Delta X = \Delta Y = 0.1$ is considered for which the total number of grid cells along (X, Y) directions, become (50, 10). The model simulations were carried out for 3 years, 6 years, 9 years and 12 years period either due to the absence or presence of the sub-surface barrier. Further, effectiveness of the barrier is investigated by considering two locations for which non-dimensional distances are $X_1 = 2.8$ and $X_1 = 1.2$, which is nearly middle of the aquifer and closer to the left boundary respectively.

RESULTS AND DISCUSSION

To begin with, the model simulations were carried for non-barrier case. Figure 2(a) – 2(d) show the evolution of the zone of salt-water concentration for 3 years, 6 years, 9 years and 12 years respectively. To analyze the result, monitoring concentration has been chosen as 0.05, which is 5% of the input concentration, beyond which groundwater would not be useful, assumed in this study. It can be observed that the concentration plume progresses upto a distance of 24 grids (X = 2.5) along the longitudinal direction and 7 grids (Y = 0.7) along vertical direction (Figure 2a). The contour lines with higher values indicate higher concentration. At the end of four years simulation, concentration plume touches at the upper boundary, and the interface moves upto 36 grids along longitudinal direction (Figure 2b). The salt-water plume further progresses at the end of nine years simulation and covers nearly 85% the area of the aquifer (Figure 2c). Finally, after ten years of simulation whole groundwater becomes completely saline (Figure 2d).

To understand the effectiveness of the barrier and its location, the barrier is considered at a distance X_1 = 2.8 which is nearly middle of the aquifer length. Figure 3(a) – 3(d) show the evolution of the zone of salt-water concentration plume with various years of simulation. Figure 3a shows that the salinity contours progresses without any disturbance at the end of 3 years, similar to the non-barrier case. After 6 years of simulation it is observed that the interface just touches the barrier and dispersion takes place towards the upper boundary (Figure 3b). At the end of 9 years simulation, the corresponding concentration values are found to be lower in comparison to the non-barrier case. The concentration plume moves upto a distance of 37 grids as shown in Figure 3(c). Figure 3(d) shows that the contour with 0.05 attaches completely to the upper boundary. There is an overall reduction of salinity due to the presence of the barrier.

Figure 4(a) – 4(d) shows the movement of concentration plume when an impervious barrier is considered at a distance (X_1 = 1.2) from the left boundary. It can be seen that due to closeness of the barrier from left boundary, salt-water intrusion is arrested significantly (Figure 4a). However, due to the presence of the barrier the saline water disperses in the vertical direction due to concentration gradient. After four years of simulation, the concentration plume attaches to the upper boundary and spreads upto nearly 10 grids (Figure 4b). At the end of 9 years simulation, the spreading of salinity plume further increases at the upper boundary and covers upto 20 grids (Figure 4c). Figure 4(d) shows that the interface could move upto 39 grids along horizontal direction. Therefore, any possible withdrawal of groundwater after a longitudinal distance of 1.2 near the bed would indicate salinity, whereas, at higher depth it still remains unaffected. At the end of 12 years of simulation, the corresponding concentration values are found to be lower in comparison with earlier cases, indicating significant reduction in salinity. Model results indicate that the barrier closer to the sea face boundary reduces the salinity intrusion considerably.

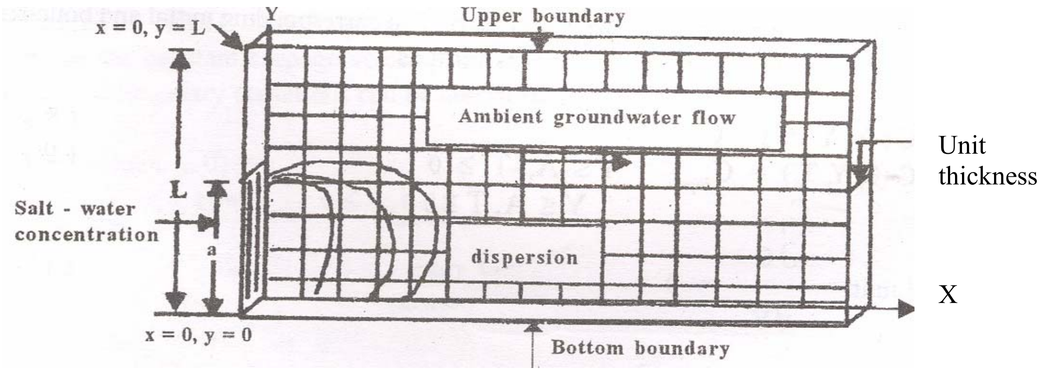

Figure 1. Schematic cross sectional representation of subsurface transport in an aquifer

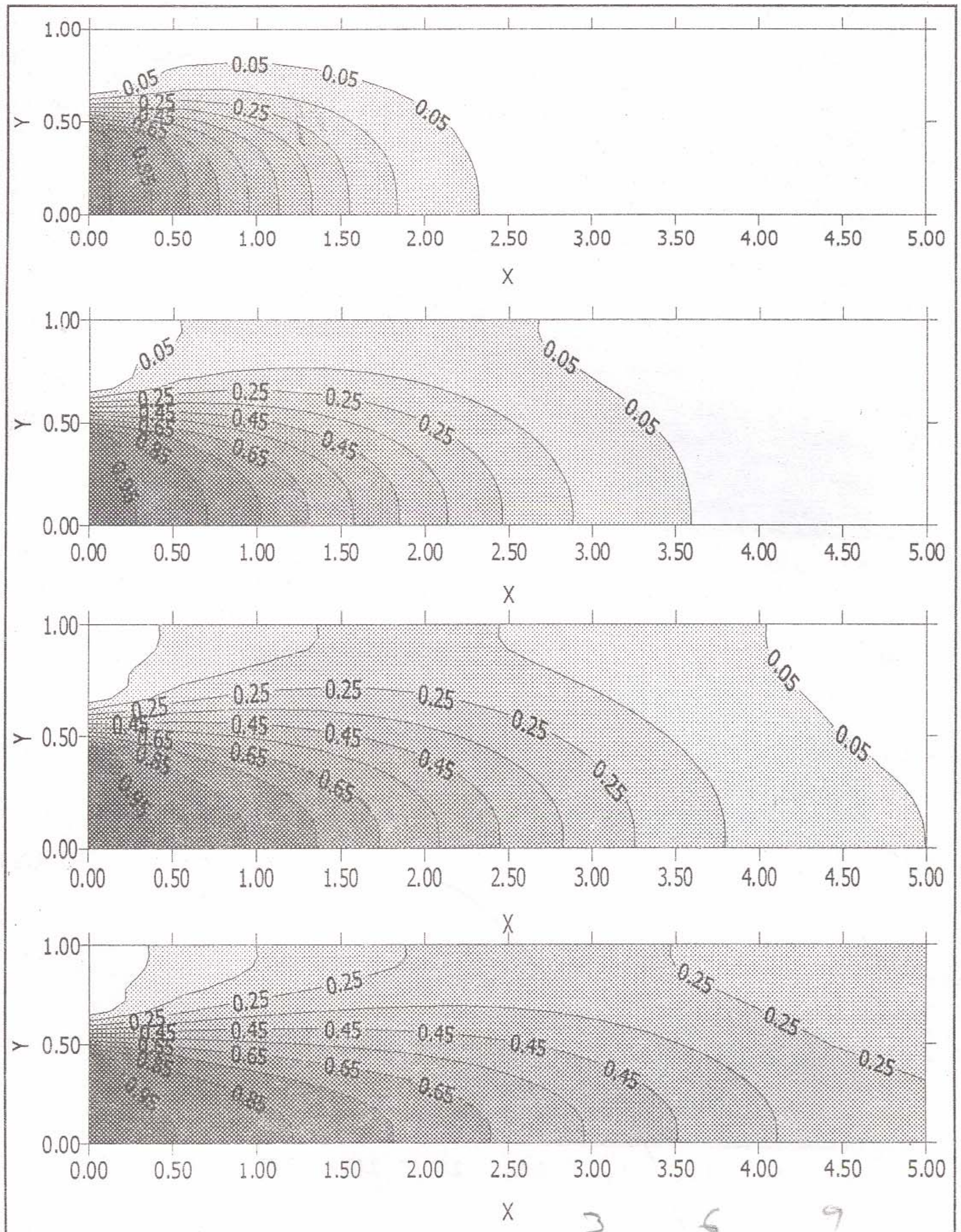

Fig. 2. **Evolution of the zone of salt-water concentration for (a) ~~two~~ 3 years (b) ~~four~~ 6 years (c) ~~six~~ 9 years and (d) ~~ten~~ years simulation (without barrier).**

Figure 2. Evolution of the zone of salt-water concentration for (a) 3 years (b) 6 years (c) 9 years and (d) 12 years simulation (without barrier)

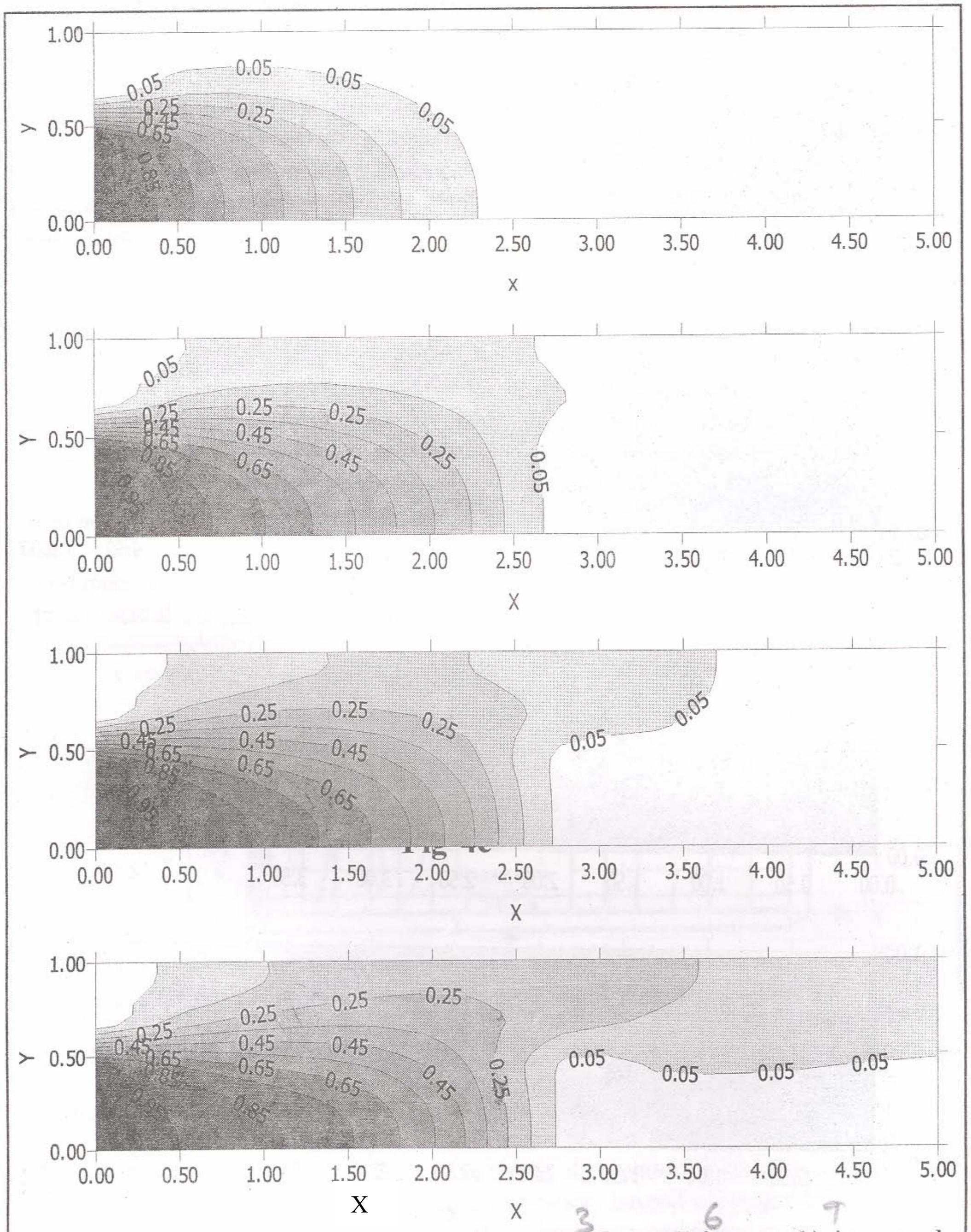

Fig. 3. **Evolution of the zone of salt-water concentration for (a) ~~two~~ 3 years (b) ~~four~~ 6 years (c) ~~six~~ 9 years and (d) ~~ten~~ years simulation (with barrier located at X = 2.7).**

Figure 3. Evolution of the zone of salt-water concentration for (a) 3 years (b) 6 years (c) 9 years and (d) 12 years simulation (without barrier located at X = 2.8)

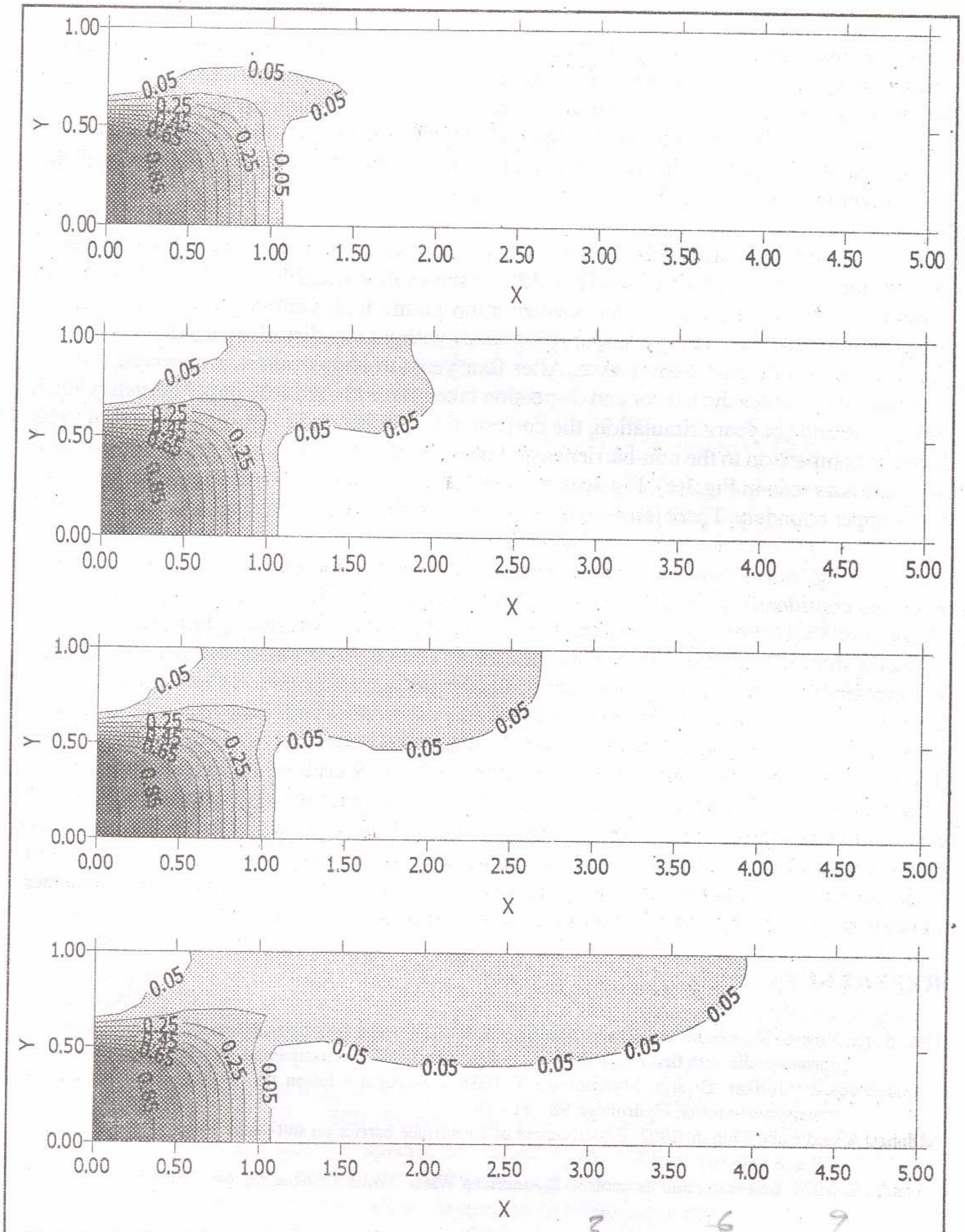

Fig.4. **Evolution of the zone of salt-water concentration for (a) ~~two~~ 3 years (b) ~~four~~ 6 years (c) ~~six~~ 9 years and (d) ~~ten~~ 12 years simulation (with barrier located at X = 1.1).**

Figure 4. Evolution of the zone of salt-water concentration for (a) 3 years (b) 6 years (c) 9 years and (d) 12 years simulation (without barrier located at X = 1.1)

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