

# Connectance and Reliability Computation of Wireless Body Area Networks using Signal Flow Graphs

Ali Peiravi

DEPARTMENT OF ELECTRICAL ENGINEERING, SCHOOL OF ENGINEERING, FERDOWSI UNIVERSITY OF MASHHAD, MASHHAD, IRAN

ali\_peiravi@yahoo.com

**Abstract:** Ambulatory monitoring and health care using wireless body area networks is an active area of applied research. In this study, a new method is presented to compute the connectance and reliability of wireless body area networks based on an improved approach to the topological analysis of the network's digraph using Mason's rule. The procedure outlined is simple and fast and may be used to compute quantitative measures of a wireless body area network's reliability. [Life Science Journal 2010;7(2):52-56]. (ISSN: 1097-8135).

**Keywords:** Connectance, reliability, wireless body area network, signal flow graph

## Introduction

The general network topology used for wireless body area networks is the star topology with the sensor nodes sending their data to a central processing node for data processing and/or fusion. Reliability of these networks is very important since they deal with human life. Reported applications to date have had performance and reliability problems. In this paper, a novel approach to compute the connectance and reliability of wireless body area networks is proposed.

Wireless body area networks are distributed systems deployed on a patient's body with wireless connectivity. They are characterized by scarce energy and computation power of individual nodes. Sensing physical quantities on various locations in the body and sending the sensed information to a central node and then possibly to a remote place via a sink node

through wireless means are two primary jobs of a wireless body area network and are usually performed for remote patient monitoring and care. These jobs must be performed with a high level of reliability because in such applications major decisions regarding human life depend on the information received from the wireless body area network. A major characteristic of wireless body area networks is that the branches formed from the sensor nodes to the sink are usually unidirectional.

Wireless body area networks (WBANs) promise ambulatory health monitoring for extended periods of time and near real-time updates of patients' medical records through the Internet or intranet. Jovanov et al. (2006) presented a WBAN as shown in Figure 1 utilizing a common off-the-shelf wireless sensor platform with a ZigBee-compliant radio interface and an ultra low-power microcontroller. The standard platform interfaces to custom sensor boards that are equipped with accelerometers for motion monitoring and a bioamplifier for electrocardiogram or electromyogram monitoring. They used TinyOS operating system to develop the software modules for on-board processing, communication, and network synchronization.

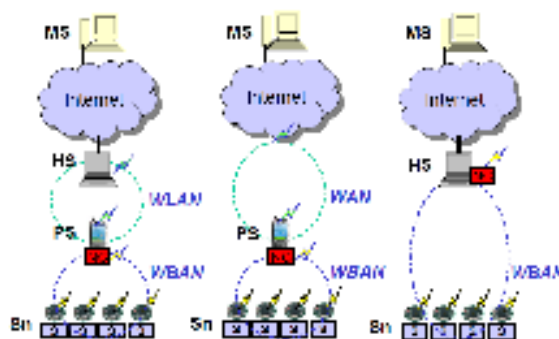


Figure 1. The Wireless Body Area Network (WBAN) for ambulatory monitoring adopted from Jovanov et al. (2006)

## Motivation and Earlier Works

The main motivation of this study is to obtain the connectance and reliability of wireless body area networks. In order to compute these measures, one needs to establish an approach to quickly determine the tie-sets between the sensor nodes and the base station where the processing is done. The general trend in wireless body area networks is that the branches are directed with the flow of signals usually from the sensor nodes through the hubs and towards the base stations. Systems that result in undirected graphs can also be handled by the approach presented herein merely by replacing each undirected branch with a pair of oppositely directed branches. However, the typical sparsity of the connection matrix that makes the proposed approach so attractive would vanish in many such instances.

Much of the reported work on network reliability has made use of the minimal cutset of the underlying graphs. For example, one may cite the work of Hakimi (1983). However, that approach is restrictive since the critical elements of the network are not easily identified.

There have also been methods presented to compute reachability in social networks. Luce and Perry

(1949) applied matrix methods to the analysis of experimental data concerning group structure indicating relationships which can be depicted by line diagrams such as sociograms. They introduced the concepts of n-chain and clique, which have simple relationships to the powers of certain matrices. Using them it is possible to determine the group structure by methods which are both faster and more certain than less systematic methods.

Signal flow graphs were introduced by Mason (1953) to model the electrical behavior of amplifiers by introducing a topological structure with a set of algebraic linear equation. Later on Mason (1956) presented a topological formula for the solution of the equations. Coates (1959) extended this approach and presented the association of a linear graph called a flow graph with a set of linearly independent algebraic equations.

Desoer (1960) presented an independent derivation of the optimum formula for the gain of a flow graph by starting from the definition of a determinant and using a few of its elementary properties and reported on a simpler and independent derivation of Coates' important result.

Chen (1965) showed that a general network determinant and its cofactors can be evaluated by means of "directed trees" and "directed two-trees". He stressed that the products of such directed trees and directed two-trees will automatically give the correct signs. In case of passive networks that contain no magnetic coupling, the digraph associated with the node-admittance matrix reduces to the original network; the directed trees and directed two-trees reduce to ordinary trees and two-trees, respectively.

Interest in the reliability of wireless sensor networks has increased in recent years even though studies on the reliability of protocols used in WSN have a longer history. Bein et al. (2005) explored the reliability issues in multi-fusion sensor networks by presenting Markov models of the reliability using different types of sensors and spares that replace sensors when failed and compared these models in terms of reliability, cost and MTTF.

AboElFotouh et al. (2006) defined a reliability measure for wireless sensor networks that considers the aggregate flow of sensor data into a sink node (gateway or cluster head). Given an estimation of the data generation rate and the failure probability of each sensor, they formulated the reliability measure.

Purohit et al. (2008) addressed the issue of quantifying reliability for a wireless sensor network. They adopted the reliability block diagram approach and modeled the hardware and software modules of the wireless sensor network as a series-parallel structure. The drawback of their proposed approach is that software reliability estimation is quite different from hardware reliability estimation and cannot be treated the same way. Moreover, many of the modules which they model as independent components in series in the RBD are not actually so and may have some degree of dependency and redundancy.

Vasar et al. (2009) proposed Markov models for the reliability analysis of the wireless sensor networks with an emphasis on fault tolerant systems. They also presented a comparison between systems using dedicated replacements and universal replacements for defective nodes.

Peiravi (2009) proposed the RBD approach for the computation of the reliability of WBANs using existing databases such as manufacturer's data, the MIL-HDBK 217F or the NPRD and EPRD for estimates of the failure rates of the various parts that make up the network. In this paper, a new approach to computing connectance and reliability of wireless body area networks is proposed based on the earlier works of Mason (1956), Coates (1959), Desoer (1960) and Chen (1965) who developed signal flow graph based solutions of linear algebraic equations. The method is advanced further and is easy to use. It yields a fast method of computing the tie-sets in wireless body area networks that may be used to compute the reliability of WBAN's.

### Signal Flow Graph Modeling of WBAN

There is an increasing interest in the application of Wireless Body Area Networks (WBANs) in recent years since they may be used to develop patient monitoring systems with flexibility and mobility for patients. The network topology that is generally used in wireless sensor networks for such ambulatory studies is of the star configuration [1].

Signal flow graphs were first introduced by Mason (1953), and were later modified by Coates (1959) to investigate the behavior of complex networks and control systems, particularly those involving feedback loops. The flow graph methodology is used in this study to devise some very useful connectivity properties of a digraph associated with a wireless body area network.

A wireless body area network may be modeled by a digraph  $G(N,B)$  with  $n = |N|$  and  $b = |B|$ . To each node of a WBAN represented by a digraph  $G$  we assign a signal  $y_i$ . The branch from node  $i$  to node  $j$  is imbued with a transmission property. Let us call it a transmittance or connectance  $c_{ij}$ . The node signals  $y_i$  may be of an arbitrary nature and magnitude as long as the ratio of any two of them such as  $y_j/y_i$  has some physical or topological significance. If  $y_i$  exists at node  $i$ , and node  $i$  is connected to node  $j$  through a branch, then the presence of  $y_i$  is felt as a contribution  $c_{ij}y_i$  to  $y_j$  at node  $j$ . This relationship does not in any way affect the originating signal  $y_j$  unless feedback is present that does not exist in wireless body area networks. Fig. 2 shows the concept of branch connectance and how the signals at the sensor nodes are transmitted through the branches to the upper level nodes.

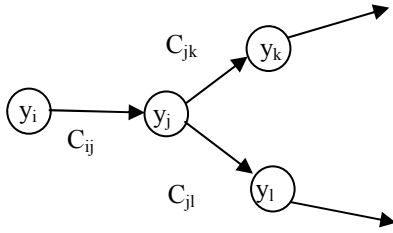


Fig 2. The concept of node to node connectance

If the connectances are assigned binary values of  $c_{ij} = 0$  or  $c_{ij} = 1$  depending on whether or not a branch exists that directly connects node  $i$  to  $j$ , then the graph can be thought of as a Boolean switching system. This representation is adequate when studying the connectivity and reliability properties of wireless sensor networks. In such cases, the absence of a direct branch between nodes  $i$  and  $j$  is indicated by the presence of a zero in place of  $c_{ij}$  in the graph's connection matrix.

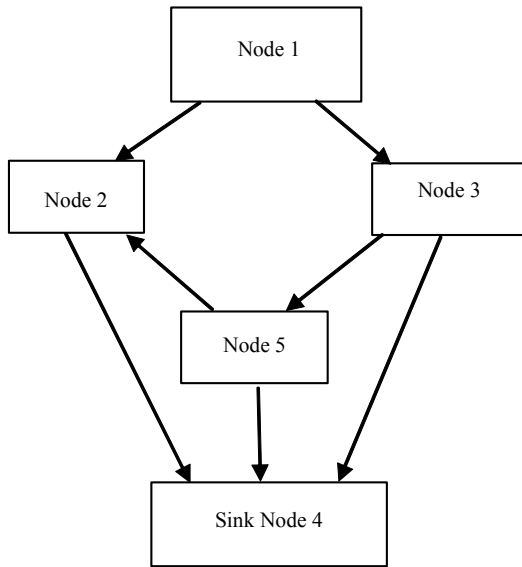


Fig 3 The schematic diagram of simple wireless body area network with node 2 as a cluster head and node

The signals at the nodes may be computed from (1) through (3) as follows:

$$y_j = c_{ij} y_i \tag{1}$$

$$y_k = c_{jk} y_j = c_{ij} c_{jk} y_i \tag{2}$$

$$y_l = c_{jl} y_j = c_{ij} c_{jl} y_i \tag{3}$$

We can find an equivalent connectance from node  $i$  to node  $k$ , or from node  $i$  to node  $l$  simply from (4) and (5):

$$y_k / y_i = c_{ij} c_{jk} \tag{4}$$

$$y_l / y_i = c_{ij} c_{jl} \tag{5}$$

The interconnections among the nodes of a wireless body area network may be represented by a connection matrix  $C = [c_{ij}]$ . The elements  $c_{ij}$  of the connection matrix represent the existence of a direct connection between nodes  $i$  and  $j$ . The value used for  $c_{ij}$  is usually taken to be either a zero implying no direct connections, or a 1 in case there is a connection. One may also assume other values for this parameter to indicate more sophisticated features of the link from  $i$  to  $j$ . For example, one may use  $c_{ij} = R_{ij}$  to denote the reliability of the direct link connecting node  $i$  to node  $j$ , or  $c_{ij} = f_{ij}$  to denote the capacity of the flow limit in the direct link from node  $i$  to node  $j$ .

For example, let us consider the simple wireless body area network shown in Fig. 3. The digraph for this network is shown in Fig. 4 with the connectances marked on the branches. The connection matrix  $C$  for this network may be written as (6):

$$C = \begin{bmatrix} 0 & C_{12} & C_{13} & 0 & 0 \\ 0 & 0 & C_{23} & C_{24} & 0 \\ 0 & 0 & 0 & C_{34} & C_{35} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & C_{52} & 0 & C_{54} & 0 \end{bmatrix} \tag{6}$$

The fact that row 4 contains all zeros indicates that node 4 is a sink in this network. Similarly the zeros in column 1 indicate that node 1 is a purely source node and that it does not relay the data from other nodes.

**Computing Tie-Set Connectances**

Consider the digraph of a simple wireless body area network as shown in Fig. 3. By substituting the transmittance values of the branches of the WBAN in the connection matrix, we get the following matrix as (7):

$$C = \begin{bmatrix} 0 & A & B & 0 & 0 \\ 0 & 0 & C & D & 0 \\ 0 & 0 & 0 & E & H \\ 0 & 0 & 0 & 0 & 0 \\ 0 & F & 0 & G & 0 \end{bmatrix} \tag{7}$$

The signal flow graph equations for the wireless body area network shown in Fig 4 take the following form as (8):

$$\begin{aligned} y_2 &= Ay_1 + Fy_5 \\ y_3 &= By_1 + Cy_2 \\ y_4 &= Dy_2 + Ey_3 + Gy_5 \\ y_5 &= Hy_3 \end{aligned} \tag{8}$$

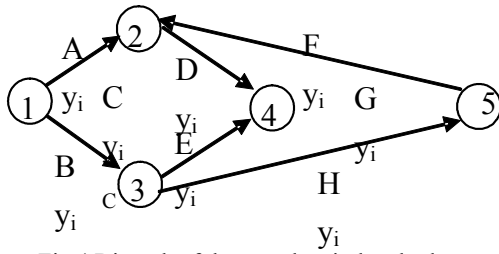


Fig 4 Digraph of the sample wireless body area network with node 4 as the sink

These equations can easily be ordered and rearranged as follows in (9):

$$\begin{aligned}
 -Ay_1 + y_2 - Fy_5 &= 0 \\
 -By_1 + y_3 - Cy_2 &= 0 \\
 -Dy_2 + y_4 - Gy_5 &= 0 \\
 -Hy_3 + y_5 &= 0
 \end{aligned} \tag{9}$$

Among the five signals  $y_1, y_2, y_3, y_4, y_5$  appearing in the four equations above, one is assumed to be given. Let it be  $y_1$  for the moment and let us find the connectivity between node 1 and the rest of the nodes in the network. This may be found from the ratios  $y_2/y_1, y_3/y_1, y_4/y_1$  and  $y_5/y_1$ . If any of these ratios is non-zero it indicates that the associated node is reachable from node 1. Thus we obtain full reachability information about the network. For non-zero ratios, the transmittance to the sink node is obtained which may also be used to find the tie-sets or path connectances.

Cramer's rule is proposed as a means of obtaining this information in this paper. The determinant of (9) may be computed as follows:

$$\det = \begin{vmatrix} +1 & 0 & 0 & -F \\ -C & +1 & 0 & 0 \\ -D & -E & +1 & -G \\ 0 & -H & 0 & +1 \end{vmatrix} = 1 - FCH \tag{10}$$

The term  $FCH = c_{52}c_{23}c_{35}$  in the determinant above represents a cyclic path from node 5, to node 2, to node 3 and then back to node 5 and should be assigned an equivalent connectance equal to zero.

Then we can compute the ratio  $y_2/y_1$  as follows:

$$y_2/y_1 = \frac{1}{\det} \begin{vmatrix} +A & 0 & 0 & -F \\ +B & +1 & 0 & 0 \\ 0 & -E & +1 & -G \\ 0 & -H & 0 & +1 \end{vmatrix} = A + BHF \tag{11}$$

Similarly one can obtain the tie set connectances from 3 to 1, 4 to 1 and 5 to 1 as follows:

$$\begin{aligned}
 y_3/y_1 &= B + AC \\
 y_4/y_1 &= AD + BE = ACE + BHG = ACHG + BHFD \\
 y_5/y_1 &= BH + ACH
 \end{aligned} \tag{12}$$

It is noteworthy that the ratios above yield the equivalent connectances between the associated node pairs. Also, these connectances indicate all the path sets that exist. Using this simple procedure, one may find all the path sets between any pairs of nodes of a wireless body area network. Given the above information, one may easily use series-parallel rules of reliability to compute the pair-wise reliabilities of the WBAN. For example, the tie-set from node 1 to 2 is indicated as A+BHF. This means that the reliability of connection from node 1 to node 2 may be computed based on the equivalent RBD as shown in Fig. 5.

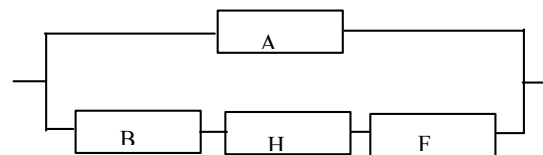


Fig 5 The equivalent reliability block diagram for

Given that we have now computed all the connectances in the network, we can compute R using the usual series-parallel rules of reliability calculations. For example, the reliability of having a connection between nodes 1 and 2 is as in (13):

$$R_{12}(t) = 1 - (1 - R_A(t))(1 - R_B(t)R_H(t)R_F(t)) \tag{13}$$

The reliabilities of each component j may be computed using estimates of the component failure rates as mentioned before using (14)

$$R_j = e^{-\lambda_j t} \tag{14}$$

All other pair-wise reliabilities may be computed in a similar fashion.

### Conclusions

Although a lot of progress has been reported in the application of WBAN, a lot more work should be done in improving the performance and reliability of wireless body area networks used for health monitoring and care. In this paper, a simple and innovative approach for the calculation of the connectance and reliability of wireless body area networks is presented based on a modification of the signal flow graph methodology. The application of the approach to the digraph of a sample wireless body area network is presented to indicate the speed and efficiency of the proposed approach.

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