

---

---

## Natural Properties in a Micro Drill Cutting into Bones

Bo Wun Huang<sup>1</sup>, Huang Kuang Kung<sup>1</sup>, Kuan-Yu Chang<sup>2</sup>, Po Kai Hsu<sup>1</sup>, Jung-Ge Tseng<sup>1\*</sup>

<sup>1</sup> Graduate Institute of Mechatronics Engineering, Cheng Shiu University, Kaohsiung 833, Taiwan, R.O.C.<sup>2</sup> Mackay Memorial Hospital Taitung Branch, Taitung, Taiwan, ROC

Received March 10, 2009

---

**Abstract:** The natural frequency in a micro drill into bones was investigated in this article. During actual service, the spinning speed subjected to some small fluctuation in a drilling process can't keep perfect constant, so the micro drill may be broken in this process. Therefore, the natural properties of this micro drill must understand. For the sake of increasing demand for better quality and higher production rate, the dynamic properties in a drilling bone process must be to pay much attention. In this article, a pre-twisted beam is used to simulate the drill. The drilling force is measured by using drilling bone experiment. The effects of spinning speed and pre-twisted angle of the drill in dynamic properties are considered to study. [Life Science Journal. 2009; 6(4): 28 – 33] (ISSN: 1097 – 8135)

**Keywords:** Micro drilling, Bones, Natural properties.

---

### 1. Introduction

For surgical operation, microsurgery is popularly employed to remedy some sickness, because it is more safe and rapid fully recover from an illness. However, a little microsurgery is used to remedy the brain illness. For this, a micro drilling for cranial bones, microsurgery for cranial bones, is considered to study. Drilling is frequently employed for metal cutting operation. In the manufacturing, the drilling operation is conducted on an extensive variety of machine tool, which includes drilling machine, milling machine, machining center and so forth. A precisely drilled hole leads to a high quality product and its accuracy is based upon the drilling process<sup>[1, 2]</sup>. A point has to be particularly mentioned, the most of holes location errors, reaming and fracture of drill may occur at this process, when the drill is exactly drilling into a work piece. To improve the performance and capability of the drilling, it is necessary to understand the dynamic characteristics of process for drilling. Hence, due to drilling process, a time dependent instability problem of a drill is consider to present. In this present investigation, the dynamic instability caused by rotating speed and thrust force is also considered to study in this drilling process.

In a real drilling process, keeping a spinning speed perfect constant is almost impossible because of the drilling speed subjected to some small fluctuation. Theoretically, at some specified rotation speed, this small speed fluctuation may lead the system to a dynamic unstable condition. Most of the studies about instability in a system focus on time independent problem<sup>[3-6]</sup>. Only a few studies on the time dependent instability in the drilling process have been conducted. Even if the instability lead to undesirable effects such as chatter and drill breakage etc., It is found that traditional analysis of drilling has focused on drill itself, such as<sup>[7-10]</sup>. Many papers<sup>[6, 8, 11, 12]</sup>, reported the vibration in a pre-twisted beam, which was modeled as a drill. The effects of pre-twisted angle, spinning speed on vibration in a drill had been presented. Buckling load and natural frequency of a drill bit also had been investigated, as<sup>[13-15]</sup>.

Previous researchers have studied mathematical models of complex drill bit to estimate natural frequency or cutting property. The effects of complex geometry or cutting chip on cutting and dynamic property of drill bit were paid attentions to research. Even a variation in geometry or symmetry error can make a very strong influence on cutting and dynamic property of a drill, such as<sup>[16-19]</sup>. Such models can provide competently useful information to design a drill bit. Some

---

\*Corresponding Author: [huangbw@csu.edu.tw](mailto:huangbw@csu.edu.tw)

investigators turn their interest to the instability of a drill, as [12, 20, 21]. Results indicate that the effects of spinning speed, pre-twisted angle and axial force may change the dynamic instability.

So far, a study dynamic property in the micro drilling with bones has not found by authors yet. The aim of this paper is to consider the dynamic property in micro drilling bones process to present.

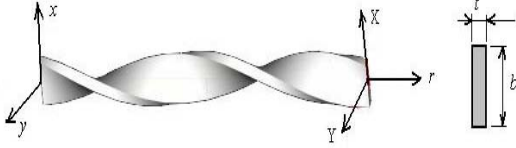


Fig. 1. The schematic diagram of the body for drill

## 2 Theory and Formulations

The effect of complex cross-section of drill has been studied [22]. A little difference between the complex and rectangular cross-section on the dynamic characteristic of a drill has been found. For the sake of convenience, in this article, the simple pre-twisted beam with a rectangular cross-section is employed to simulate a drill. The drill, a cantilever pre-twisted beam, with a spinning speed  $\Omega$  is illustrated in Fig. 1. The length of the drill is  $L$ .  $t$  and  $b$  are used to denote the thickness and breadth of the drill respectively. In this study, the deflection components  $v(r, t)$  and  $u(r, t)$  denote the two transverse flexible deflection of the drill.

The equation of motion of a spinning drill is displayed as

$$\begin{aligned} & \frac{\partial^2}{\partial r^2} \left( EI_{yy} \frac{\partial^2 u}{\partial r^2} + EI_{xy} \frac{\partial^2 v}{\partial r^2} \right) - \frac{\partial}{\partial r} \left( \bar{I}_{yy} \frac{\partial^3 u}{\partial t^2 \partial r} + \bar{I}_{xy} \frac{\partial^3 v}{\partial t^2 \partial r} \right) \\ & + \mu \frac{\partial^2 u}{\partial t^2} - 2\mu\Omega \frac{\partial v}{\partial t} - \mu\Omega^2 u - \mu\dot{\Omega} v = 0 \end{aligned} \quad (1a)$$

$$\begin{aligned} & \frac{\partial^2}{\partial r^2} \left( EI_{xx} \frac{\partial^2 v}{\partial r^2} + EI_{xy} \frac{\partial^2 u}{\partial r^2} \right) - \frac{\partial}{\partial r} \left( \bar{I}_{xx} \frac{\partial^3 v}{\partial t^2 \partial r} + \bar{I}_{xy} \frac{\partial^3 u}{\partial t^2 \partial r} \right) \\ & + \mu \frac{\partial^2 v}{\partial t^2} + 2\mu\Omega \frac{\partial u}{\partial t} - \mu\Omega^2 v + \mu\dot{\Omega} u = 0 \end{aligned} \quad (1b)$$

where a symbol prime (') denotes a partial derivative with respect to  $r$ . In this equation,  $E$  and  $\mu$  are the

Young's modulus and mass per unit length respectively, and  $I_{xx}$ ,  $I_{yy}$  and  $I_{xy}$  are the moments of area. Consider the drill to be pre-twisted with a uniform twist angle  $\beta$ , and then the area moments of inertia at the position  $r$  can be derived as

$$I_{xx} = I_{xx} \cos^2 \left( \frac{r}{L} \beta \right) + I_{yy} \sin^2 \left( \frac{r}{L} \beta \right) \quad (2a)$$

$$I_{yy} = I_{xx} \sin^2 \left( \frac{r}{L} \beta \right) + I_{yy} \cos^2 \left( \frac{r}{L} \beta \right) \quad (2b)$$

$$I_{xy} = (I_{yy} - I_{xx}) \sin \left( \frac{r}{L} \beta \right) \cos \left( \frac{r}{L} \beta \right) \quad (2c)$$

where

$$I_{xx} = \frac{b t^3}{12} \quad (3a)$$

$$I_{yy} = \frac{b^3 t}{12} \quad (3b)$$

, and  $\bar{I}_{xx}$ ,  $\bar{I}_{xy}$ ,  $\bar{I}_{yy}$  are the moments of mass.

Boundaries of the drill can be displayed as

$$u_s = v_s = u'_s = v'_s = 0, \quad \text{at } r = 0 \quad (4a)$$

$$u''_s = v''_s = u'''_s = v'''_s = 0, \quad \text{at } r = L \quad (4b)$$

The purpose of this investigation is to present the dynamic property in a micro drilling with bones. In a mathematical sense, a thrust force can be employed to simulate this drilling process. Considering the axial cutting force, the equation of motion can be rewrite as

$$\begin{aligned} & \frac{\partial^2}{\partial r^2} \left( EI_{yy} \frac{\partial^2 u}{\partial r^2} + EI_{xy} \frac{\partial^2 v}{\partial r^2} \right) - \frac{\partial}{\partial r} \left( \bar{I}_{yy} \frac{\partial^3 u}{\partial t^2 \partial r} + \bar{I}_{xy} \frac{\partial^3 v}{\partial t^2 \partial r} \right) \\ & + P \frac{\partial^2 u}{\partial r^2} + \mu \frac{\partial^2 u}{\partial t^2} - 2\mu\Omega \frac{\partial v}{\partial t} - \mu\Omega^2 u - \mu\dot{\Omega} v = 0 \end{aligned} \quad (5a)$$

$$\begin{aligned} & \frac{\partial^2}{\partial r^2} \left( EI_{xx} \frac{\partial^2 v}{\partial r^2} + EI_{xy} \frac{\partial^2 u}{\partial r^2} \right) - \frac{\partial}{\partial r} \left( \bar{I}_{xx} \frac{\partial^3 v}{\partial t^2 \partial r} + \bar{I}_{xy} \frac{\partial^3 u}{\partial t^2 \partial r} \right) \\ & + P \frac{\partial^2 v}{\partial r^2} + \mu \frac{\partial^2 v}{\partial t^2} + 2\mu\Omega \frac{\partial u}{\partial t} - \mu\Omega^2 v + \mu\dot{\Omega} u = 0 \end{aligned} \quad (5b)$$

where  $P$  is the drilling bones load.

The solutions for the above eigenvalue problem are expressed as

$$u(r, t) = \sum_{i=1}^m p_i(t) \phi_i(r) \quad (6a)$$

$$v(r, t) = \sum_{i=1}^m q_i(t) \phi_i(r) \quad (6b)$$

where  $\phi_i(r)$  are the comparison functions of equations (5a) and (5b), and  $p_i(t)$ ,  $q_i(t)$  are the corresponding weighting coefficients, which are to be

determined. Five exact solutions of a uniform cantilever beam,  $\phi_i(r)$  in transverse direction are used to discrete the equations of motion.

An application of the Galerkin's method, the equations of motion can be derived in matrix form as

$$[M] \begin{Bmatrix} \ddot{p} \\ \ddot{q} \end{Bmatrix} + 2\Omega_0 [G] \begin{Bmatrix} \dot{p} \\ \dot{q} \end{Bmatrix} + \{ [K]_a + P[K]_b + \Omega_0^2 [K]_c \} \begin{Bmatrix} p \\ q \end{Bmatrix} = 0 \tag{7}$$

where

$$[M] = \begin{bmatrix} [M]^1 + [M]^2 & [M]^4 \\ [M]^4 & [M]^1 + [M]^3 \end{bmatrix} \tag{8a}$$

$$[G] = \begin{bmatrix} [0] & -[M]^1 \\ [M]^1 & [0] \end{bmatrix} \tag{8b}$$

$$[K]_a = \begin{bmatrix} [K]^1 & [K]^4 \\ [K]^4 & [K]^2 \end{bmatrix} \tag{8c}$$

$$[K]_b = \begin{bmatrix} -[K]^3 & [0] \\ [0] & -[K]^3 \end{bmatrix} \tag{8d}$$

$$[K]_c = \begin{bmatrix} -[M]^1 & [0] \\ [0] & -[M]^1 \end{bmatrix} \tag{8e}$$

To emphasize, these matrices can be illustrated to go into details as follows,

$$M_{ij}^1 = \int_0^1 \mu \phi_i(\bar{r}) \phi_j(\bar{r}) d\bar{r} \tag{9a}$$

$$M_{ij}^2 = \frac{\bar{I}_{yy}}{L^2} \int_0^1 \frac{d\phi_i(\bar{r})}{d\bar{r}} \frac{d\phi_j(\bar{r})}{d\bar{r}} d\bar{r} \tag{9b}$$

$$M_{ij}^3 = \frac{\bar{I}_{xx}}{L^2} \int_0^1 \frac{d\phi_i(\bar{r})}{d\bar{r}} \frac{d\phi_j(\bar{r})}{d\bar{r}} d\bar{r} \tag{9c}$$

$$M_{ij}^4 = \frac{\bar{I}_{xy}}{L^2} \int_0^1 \frac{d\phi_i(\bar{r})}{d\bar{r}} \frac{d\phi_j(\bar{r})}{d\bar{r}} d\bar{r} \tag{9d}$$

$$K_{ij}^1 = \frac{EI_{yy}}{L^4} \int_0^1 \frac{d^2\phi_i(\bar{r})}{d\bar{r}^2} \frac{d^2\phi_j(\bar{r})}{d\bar{r}^2} d\bar{r} \tag{9e}$$

$$K_{ij}^2 = \frac{EI_{xx}}{L^4} \int_0^1 \frac{d^2\phi_i(\bar{r})}{d\bar{r}^2} \frac{d^2\phi_j(\bar{r})}{d\bar{r}^2} d\bar{r} \tag{13f}$$

$$K_{ij}^3 = \frac{1}{L^2} \int_0^1 \frac{d\phi_i(\bar{r})}{d\bar{r}} \frac{d\phi_j(\bar{r})}{d\bar{r}} d\bar{r} \tag{13g}$$

$$K_{ij}^4 = \frac{EI_{xy}}{L^4} \int_0^1 \frac{d^2\phi_i(\bar{r})}{d\bar{r}^2} \frac{d^2\phi_j(\bar{r})}{d\bar{r}^2} d\bar{r} \tag{13h}$$

The eigenvalue problem equation can be rewritten as.

$$[A] \{ \dot{V} \} + [B] \{ V \} = 0 \tag{14}$$

### 3 Results and Discussion

The dynamic properties in a micro drilling bones process were investigated in this study.  $D=1.0mm$ ,  $d=0.3mm$ ,  $L = 5.5mm$ ,  $t = 0.24mm$ ,  $b = 0.5mm$  and  $\beta = 1800^\circ$  ( $5.712 rad/mm$ ). In this work, the sinbone bone replacement, Purzer produced, is employed to study instead of real bone.



Fig. 2. Experiment setup in a micro drill with bones

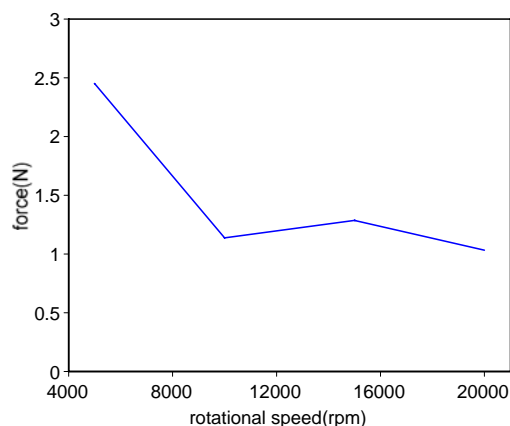


Fig. 3. The variation in drilling force with different rotating speed.

Figure 2 shows the experiment setup in a micro drill

with bones. In this figure, the sinbone bone replacement drilled by a micro drill is found. The variation in drilling force with different rotating speed is illustrated in Fig. 3. The rotating speed from 4000 rpm to 20000 rpm is considered in this work. Experiment analysis shows that the drilling force may decreased as the rotating speed is increased. In this investigation, the rotating speed 15000 rpm is employed to study the natural properties of a micro drill with a bone. As the rotating speed 15000 rpm considered, the drilling force 1.286 N can be obtained.

Figure 4 displays the natural frequency of a micro drill drilling into a bone. Only the lower mode is considered in this work. In this figure, the first order mode natural frequency 50315 Hz and the second order mode natural frequency 53453 Hz are found. The effect of rotating speed on natural frequency of a micro drill with a bone is studied. Figure 5 shows the natural frequency of a micro drill drilling a bone with different rotating speed. The first order mode natural frequencies may be depressed and second order mode natural frequencies may increase as the rotating speed is increased. Finally, the pretwisted angle effect is considered to study. The effect of pretwisted angle on natural frequency of a micro drill with a bone is displayed in Fig. 6. Numerical analysis shows that the first and second order mode natural frequencies will toward a higher frequency domain if the pretwisted angle is increased.

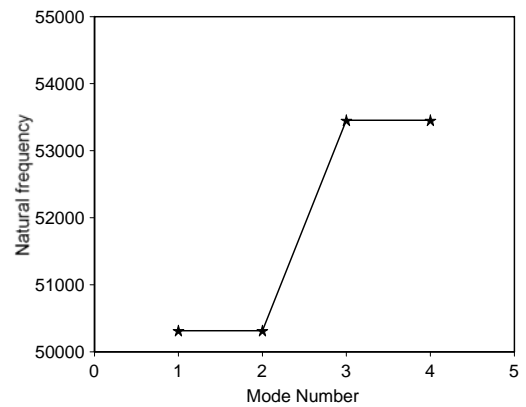


Fig. 4. Natural frequencies of a micro drill drilling a bone.

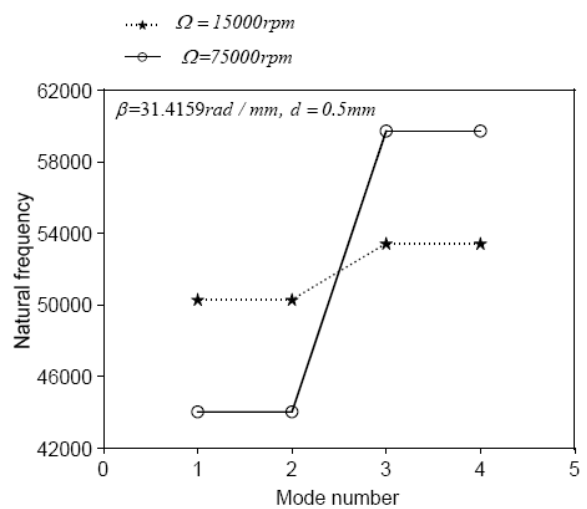


Fig. 5. Natural frequencies in a micro drilling bone process with different rotating speed.

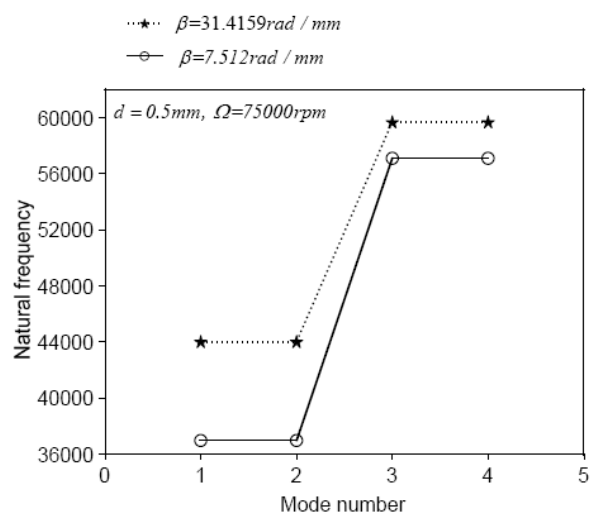


Fig. 6. Natural frequencies in a micro drilling bone process with different pretwisted angle.

#### 4 Conclusions

The dynamic property of a micro drill with a bone was investigated. The major conclusions drawn from the analysis and numerical results obtained in this study are summarized as follows:

- (1) The analysis results indicate that a study of the dynamic drilling characteristics is necessary to improve drilling into a bone performance and capabilities, especially for high speed drilling.
- (2) Experiment analysis shows the drilling force will be depressed as the rotating speed is increased if a micro drill drills into a bone.
- (3) The effects of spinning speed and pre-twisted angle drastically change dynamic properties of a micro drill drilling a bone.

#### Acknowledgements

The financial support by the Ministry of Education, Republic of China..

#### References

1. Katz Z, Poustie A. On the hole quality and drill wandering relationship. *International Journal of Advanced Manufacturing Technology*, 2001, 17(4): 233-237
2. Gupta K, Ozdoganlar OB, Kapoor SG, DeVor RE. Modeling and prediction of hole profile in drilling. Part 1: modeling drill dynamics in the presence of drill alignment errors. *ASME, Journal of Manufacturing Science and Engineering*, 2003, 125(1): 6-13
3. Hsu CS. On a Restricted Class of Coupled Hill's Equations and Some Applications. *ASME Journal of Applied Mechanics*, 1961: 551-557.
4. Nayfeh AH, Mook DT. *Nonlinear Oscillation*, 1979, New York: John Wiley
5. Young TH. Dynamic Response of A Pretwisted, Tapered Beam with Non-constant Rotating Speed. *Journal of Sound and Vibration*, 1991, 150(3): 435-446
6. Liao CL, Huang BW. Parametric Resonance of a spinning Pretwisted Beam with Time-Dependent Spinning Rate. *Journal of Sound and Vibration*, 1995, 180(1): 47-65
7. Rosard DD. Natural Frequency of Twisted Cantilever Beams. *ASME Paper No. 52-A-15*, 1952
8. Jarrett GW, Warner PC. The Vibration of Rotating Tapered Twisted Beam. *ASME, Journal of Applied Mechanics*, 1953: 381-389
9. Tekinalp O, Ulsoy AG. Modeling and Finite Element Analysis of Drill Bit Vibration. *ASME, Journal of Vibration, Acoustics, Stress, and Reliability in Design*, 1989, 111: 148-155
10. Tekinalp O, Ulsoy AG. Effect of Geometric and Process Parameters in Drill Transverse Vibration. *ASME, Journal of Engineering for Industry*, 1990, 112: 189-194
11. Liao CL, Dang YH. Structural Characteristics of Spinning Pretwisted Orthotropic Beams. *Computer & Structures*, 1992, 45(4): 715-731
12. Liao CL, Huang BW. Parametric Instability of a Pretwisted Beam under Periodic Axial Force. *International Journal of Mechanical Science*. 1995, 37(4): 423-439
13. Magrab E, Gilsinn DE. Bucking loads and natural frequency of twist drills. *Transactions of ASME*, 1984, 106: 196-204
14. Ulsoy AG. A lumped parameter model for the transverse vibration of drill bit. *Control of Manufacturing Processes and Robotic System*, 1989: 15-25
15. Fuji H, Marui E, Ema S. Whirling Vibration in drilling part 3: Vibration analysis in Drilling Workpiece with a Pilot Hole. *ASME, Journal of engineering for Industry*, 1988, 110: 315-321
16. Rincon DM, Ulsoy AG. Complex Geometry, Rotary Inertia and Gyroscopic Moment Effects on Drill Vibrations. *Journal of Sound and Vibration*, 1995, 188(5) 701-715

- 
17. Ren K, Ni J. Analyses of drill flute and cutting angles. *International Journal of Advanced Manufacturing Technology*, 1999, 15(8): 546-553
  18. Spanos PD, Chevallier AM, Politis NP. Nonlinear stochastic drill-string vibrations. *ASME, Journal of Vibration and Acoustics*, 2002, 124(4): 512-518
  19. Hsieh JF, Lin PD. Mathematical model of multiflute drill point. *International Journal of Machine Tools and Manufacture*, 2002, 42(10): 1181-1193
  20. Iwastsubo T, Sugiyama Y, Ishihara K. Stability and Stationary Vibrations of Columns under Periodic Loads. *Journal of Sound and Vibration*, 1972, 23: 245-257.
  21. Lee HP. Buckling and Dynamic Stability of Spinning Pre-twisted Beams under Compressive Axial Loads. *International Journal of Mechanical Sciences*, 1994, 36(11): 1011-1026
  22. Huang BW. Vibration of a high speed drill. *Proc. of National Science Council Research Projects*. 2000, NSC 89-2212-E-230-006, Taiwan. (in Chinese)