Indirect Boundary Element Method for Calculation of Compressible Flow past a Symmetric Aerofoil with Linear Element Approach Using Doublet Distribution Alone

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Abstract:
In this paper, an indirect boundary element method is applied to calculate the compressible flow past a symmetric aerofoil. The velocity distribution for the flow over the surface of the symmetric aerofoil has been calculated with linear boundary element approach using doublet distribution alone. To check the accuracy of the method, the computed flow velocity is compared with the exact velocity. The comparison of these results has been given in the tables and graphs. It is found that the computed results are in good agreement with the analytical results.

Keywords: Indirect boundary element method, Compressible flow, Velocity distribution, Symmetric aerofoil, linear element.

1. Introduction

In the past, many numerical techniques such as finite difference method, finite element method, and boundary element method etc. came into being making possible to solve various practical fluid flow problems. Boundary element method has received much attention from the researchers due to its various advantages over the other domain methods. One of the advantages is that with boundary elements one has to discretize only the surface of the body, whereas with domain methods it is essential to discretize the entire region of the flow field. Moreover, this method is well–suited to problems with an infinite domain. The boundary element method can be classified into two categories i.e. direct and indirect. The direct method takes the form of a statement which provides the values of the unknown variables at any field point in terms of the complete set of all the boundary data. On the other hand, the indirect method utilizes a distribution of singularities over the boundary of the body and computes this distribution as the solution of integral equation. The equation of indirect method can be derived from that of direct method. (Lamb, 1932; Milne-Thomson, 1968, Kellogge, 1929 and Brebbia and Walker, 1980). The indirect method has been used in the past for flow field calculations around arbitrary bodies (Hess and Smith, 1967; Muhammad, 2008, Luminita, 2008, Mushtaq, 2008, 2009 & 2010). Most of the work on fluid flow calculations using boundary element methods has been done in the field of incompressible flow. Very few attempts have been made on flow field calculations using boundary element methods in the field of compressible flow. In this paper, the indirect boundary element method has been used for the solution of compressible flows around a symmetric aerofoil with linear element approach using doublet distribution alone.

2. Mathematical Formulation

We know that equation of motion for two – dimensional, steady, irrotational, and isentropic flow is

$$(1 - Ma^2)\frac{\partial^2 \Phi}{\partial X^2} + \frac{\partial^2 \Phi}{\partial Y^2} = 0 \qquad (1)$$

where Ma is the Mach number and Φ is the total velocity potential of the flow. Here X and Y are the space coordinates.

Using the dimensionless variables, $x = X$, $y = \beta Y$, where $\beta = \sqrt{1 - Ma^2}$, equation (1) becomes

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

or $\nabla^2 \Phi = 0 \qquad (2)$

which is Laplace's equation.

3. Symmetric Aerofoil

The Joukowski transformation

$$z = \zeta + \frac{a^2}{\zeta} \tag{3}$$

transforms the circle shown in figure (1) in the ζ – plane on to symmetric aerofoil in the z-plane.

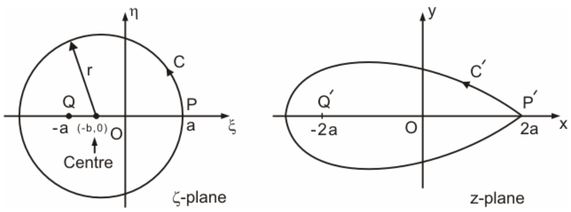

Figure 1

4. Flow Past a Symmetric Aerofoil

Consider the flow past a symmetrical aerofoil and let the onset flow be the uniform stream with velocity U in the positive direction of the x – axis as shown in figure (2) .

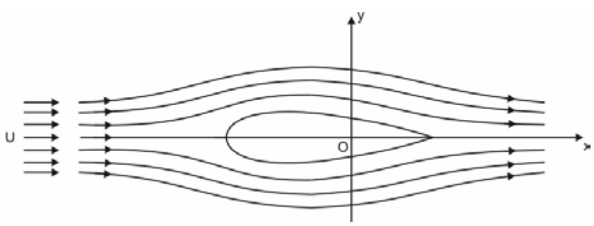

Figure 2: Flow past a symmetric aerofoil.

Exact Velocity

The magnitude of the exact velocity distribution over the boundary of a symmetric aerofoil is given by Chow[3] as

$$V = U \left| \frac{1 - \left(\dfrac{r}{z-b}\right)^2}{1 - \left(\dfrac{a}{z}\right)^2} \right|$$

where r = radius of the circular cylinder,
a = Joukowski transformation constant
and b = a – r = x-coordinates of the centre of the circular cylinder

In Cartesian coordinates, we have

$$V = U \; \frac{\sqrt{\left[\left\{(x-b)^2+y^2\right\}^2 - r^2\left\{(x-b)^2-y^2\right\}\right]^2 + 4r^4y^2(x-b)^2}}{\left[(x-b)^2+y^2\right]^2} \times \frac{\sqrt{\left[(x^2+y^2)^2 - a^2(x^2-y^2)\right]^2 + 4a^4x^2y^2}}{(x^2+y^2)^2 - 2a^2(x^2-y^2) + a^4}$$

Boundary Conditions

Now the condition to be satisfied on the boundary of a symmetric aerofoil is

$$\vec{V} \cdot \hat{n} = 0 \tag{4}$$

where $\hat{n}$ is the unit normal vector to the boundary of the aerofoil .

Since the motion is irrotational

$$\vec{V} = -\nabla \Phi$$

where Φ is the total velocity potential . Thus equation (4) becomes

$$(-\nabla \Phi) \cdot \hat{n} = 0$$

or

$$\frac{\partial \Phi}{\partial n} = 0 \tag{5}$$

Now the total velocity potential Φ is the sum of the perturbation velocity potential $\phi_{s.a}$ where the subscript s . a stands for symmetric aerofoil and the velocity potential of the uniform stream $\phi_{u.s}$.

i.e.

$$\Phi = \phi_{u.s} + \phi_{s.a} \tag{6}$$

or

$$\frac{\partial \Phi}{\partial n} = \frac{\partial \phi_{u.s}}{\partial n} + \frac{\partial \phi_{s.a}}{\partial n} \tag{7}$$

From equations (5) and (7) , we get

$$\frac{\partial \phi_{s.a}}{\partial n} + \frac{\partial \phi_{u.s}}{\partial n} = 0$$

or

$$\frac{\partial \phi_{s.a}}{\partial n} = -\frac{\partial \phi_{u.s}}{\partial n} \tag{8}$$

But the velocity potential of the uniform stream , given in Milne – Thomson [6], Shah [7], is

$$\phi_{u.s} = -Ux \tag{9}$$

$$= -U\frac{\partial x}{\partial n}$$

$$= -U(\hat{n} \cdot \hat{i}) \tag{10}$$

Thus from equations (8) and (10), we get

$$\frac{\partial u_{s.a}}{\partial n} = U(\hat{n} \cdot \hat{i}) \tag{11}$$

Now from the figure (3)

$$\vec{A} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

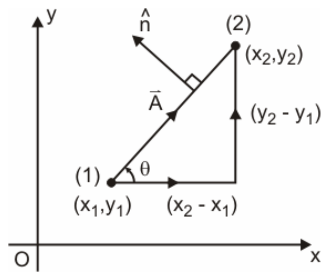

Figure 3

Therefore the unit vector in the direction of the vector $\vec{A}$ is given by

$$\vec{A} = \frac{(x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

The outward unit normal vector $\hat{n}$ to the vector $\vec{A}$ is given by

$$\hat{n} = \frac{-(y_2 - y_1)\,\hat{n} + (x_2 - x_1)\,\hat{j}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Thus $$\hat{n}\,.\,\hat{i} = \frac{(y_1 - y_2)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \qquad (12)$$

From equations (11) and (12) , we get

$$\frac{\partial \phi_{s.a}}{\partial n} = U \frac{(y_1 - y_2)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \qquad (13)$$

Equation (13) is the boundary condition which must be satisfied over the boundary of a symmetric aerofoil.

Equation of Indirect Boundary Element Method

The equation of indirect boundary element method for two–dimensional flow in the case of doublet distribution alone [Muhammad,2008 & Mushtaq, 2009 & 2010] is :

$$-c_i\,\Phi_i + \frac{1}{2\pi} \int_{\Gamma - i} \Phi \frac{\partial}{\partial n}\left(\log \frac{1}{r}\right) d\Gamma + \phi_\infty = -(\phi_{u.s})_i \qquad (14)$$

where c_i = 0 when ‘i’ is within R'

= 1 when ‘i’ is within R

$= \frac{1}{2}$ when ‘i’ is on S and S is smooth

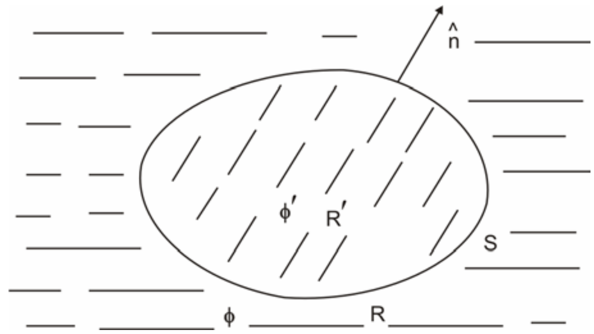

Figure 4

Matrix Formulation

Let the boundary of the region be discretized into m linear elements , then equation (14) can be written as

$$-c_i\,\Phi_i + \sum_{j=1}^{m} \left[\frac{1}{2\pi} \int_{\Gamma_j - i} \Phi \frac{\partial}{\partial n}\left(\log \frac{1}{r}\right) d\Gamma \right] + \phi_\infty = -(\phi_{u.s})_i \qquad (15)$$

where $\Gamma_j - i$ is the length of the element ‘j’ excluding the point ‘i’ .

For the linear boundary element approach, the number of nodes will be more than the number of elements . Suppose that m is the number of nodes in this case . Since Φ varies linearly over the element , its value at any point can be defined in terms of the nodal values and the two shape functions N_1 , N_2 , that is

$$\Phi = N_1 \Phi_1 + N_2 \Phi_2 = \left[N_1 \;\; N_2 \right] \begin{Bmatrix} \Phi_1 \\ \Phi_2 \end{Bmatrix} \qquad (16)$$

where $N_1 = \frac{1}{2}(1 - \delta)$,

and $N_2 = \frac{1}{2}(1 + \delta)$, $-1 \le \delta \le 1$

The integrals along the element ‘j’ i.e.

$$\frac{1}{2\pi} \int_{\Gamma_j - i} \Phi \frac{\partial}{\partial n}\left(\log \frac{1}{r}\right) d\Gamma$$

can be written as

$$\frac{1}{2\pi} \int_{\Gamma_j - i} \Phi \frac{\partial}{\partial n}\left(\log \frac{1}{r}\right) d\Gamma$$

$$= \frac{1}{2\pi} \int_{\Gamma_{j-i}} \left[N_1 \; N_2 \right] \frac{\partial}{\partial n}\left(\log \frac{1}{r} \right) d\Gamma \begin{Bmatrix} \Phi_1 \\ \Phi_2 \end{Bmatrix}$$

$$= \left[h^1_{ij} \;\; h^2_{ij} \right] \begin{Bmatrix} \Phi_1 \\ \Phi_2 \end{Bmatrix} \qquad (17)$$

where $h^1_{ij} = \frac{1}{2\pi} \int_{\Gamma_{j-i}} \Phi_1 \frac{\partial}{\partial n}\left(\log \frac{1}{r} \right) d\Gamma$,

$$h^2_{ij} = \frac{1}{2\pi} \int_{\Gamma_{j-i}} \Phi_2 \frac{\partial}{\partial n}\left(\log \frac{1}{r} \right) d\Gamma$$

or $$h^k_{ij} = \frac{1}{2\pi} \int_{\Gamma_{j-i}} k \frac{\partial}{\partial n}\left(\log \frac{1}{r} \right) d\Gamma \qquad (18)$$

$$k = 1, 2$$

The h^k_{ij} are influence coefficients during the interaction between the point 'i' under consideration and a particular node k on an element 'j'.

To write the equation (15) corresponding to the node 'i' the contributions from all elements associated with the node 'i' are to be added into one term, defining the nodal coefficients. This will give the following equation

$$- c_i \Phi_i + \left[\hat{H}_{i1} \hat{H}_{i2} \, \ldots\ldots\ldots \, \hat{H}_{im} \right] \begin{Bmatrix} \Phi_1 \\ \Phi_2 \\ . \\ . \\ . \\ \Phi_m \end{Bmatrix} + \phi_\infty = - (\phi_{u.s.})_i \qquad (19)$$

where $\hat{H}_{ij}$ term is the sum of the contributions from all the adjoining elements of the node 'i'. Hence equation (19) represents the assembled equation for node 'i' and can be written as

$$- c_i \Phi_i + \sum_{j=1}^{m} \hat{H}_{ij} \Phi_j + \phi_\infty = - (\phi_{u.s})_i \qquad (20)$$

or $$\sum_{j=1}^{m} H_{ij} + \phi_\infty = - (\phi_{u.s})_i \qquad (21)$$

where $H_{ij} = \begin{cases} \hat{H}_{ij} & \text{when } i \neq j \\ \hat{H}_{ij} - c_i & \text{when } i = j \end{cases}$

When all nodes are taken into consideration, equation (21) is M x (M + 1) system of equations. Which can put in the matrix form in case of linear element as

$$\left[H \right] \left\{ \underline{U} \right\} = \left\{ \underline{R} \right\} \qquad (22)$$

where as usual $[H]$ is a matrix of influence coefficients, $\{ \underline{U} \}$ is a vector of unknown total potentials Φ_i and $\{ \underline{R} \}$ on the R.H.S. is a known vector whose elements are the negative of the values of the velocity potential of the uniform stream at the nodes on the region of the body. Note that $\{ U \}$ in equation (22) has (M + 1) unknowns $\Phi_1, \Phi_2, \ldots, \Phi_m, \phi_\infty$. To solve precisely this system of equations, the value of Φ at some position must be specified. For convenience ϕ_∞ is chosen as zero. Thus M x (M + 1) system reduces to an M x M system of equations which can be solved as before but now the diagonal coefficients of [H] will be found by

$$H_{ii} = - \sum_{\substack{j=1 \\ j \neq i}} H_{ij} - 1 \qquad (23)$$

Process of Discretization

Now for the discretization of the boundary of the symmetric aerofoil, the coordinates of the extreme points of the boundary elements can be generated within computer programme using Fortran language as follows:

Divide the boundary of the circular cylinder into m elements in the clockwise direction by using the formula.

$$\theta_k = \left[(m + 2) - 2k \right] \frac{\pi}{m},$$

$$k = 1, 2, \ldots\ldots, m \qquad (24)$$

Then the extreme points of these m elements of circular cylinder are found by

$$\xi_k = - b + r \cos \theta_k$$

$$\eta_k = r \sin \theta_k$$

Now by using Joukowski transformation in equation (3), the extreme points of the symmetric aerofoil are

$$x_k = \xi_k\left(1+\frac{a^2}{\xi_k^2+\eta_k^2}\right)$$

$$y_k = \eta_k\left(1-\frac{a^2}{\xi_k^2+\eta_k^2}\right)$$

where $k = 1, 2, \ldots\ldots, m$.

The coordinates of the middle node of each boundary element are given by

$$\left.\begin{aligned} x_m &= \frac{x_k + x_{k+1}}{2} \\ y_m &= \frac{y_k + y_{k+1}}{2} \end{aligned}\right\}$$

$$k, m = 1, 2, \ldots\ldots, n \qquad (25)$$

and therefore the boundary condition (13) in this case takes the form

$$\frac{\partial \phi_{s.a}}{\partial n} = U\frac{(y_1)_m-(y_2)_m}{\sqrt{\left[(x_2)_m-(x_1)_m\right]^2+\left[(y_2)_m-(y_1)_m\right]^2}} \qquad (26)$$

The following tables show the comparison of computed and analytical velocity distribution over the boundary of a symmetric aerofoil for 8, 16, 32, and 64 linear boundary elements for r = 1.1, a = 0.1 and Ma = 0.7.

Table 1: The comparison of the computed velocity with exact velocity over the boundary of a symmetric aerofoil using 8 linear boundary elements.

ELEMENT	X	Y	$R=\sqrt{X^2+Y^2}$	VELOCITY	EXACT VELOCITY
1	-1.94	.39	1.98	.72096E+00	.83769E+00
2	-1.39	.94	1.68	.17375E+01	.20086E+01
3	-.62	.93	1.12	.17174E+01	.20216E+01
4	-.01	.38	.38	.75534E+00	.70748E+00
5	-.01	-.38	.38	.75534E+00	.70748E+00
6	-.62	-.93	1.12	.17174E+01	.20216E+01
7	-1.39	-.94	1.68	.17375E+01	.20086E+01
8	-1.94	-.39	1.98	.72096E+00	.83768E+00

Table 2: The comparison of the computed velocity with exact velocity over the boundary of a symmetric aerofoil using 16 linear boundary elements.

ELEMENT	X	Y	$R=\sqrt{X^2+Y^2}$	VELOCITY	EXACT VELOCITY
1	-2.06	.21	2.07	.38546E+00	.39565E+00
2	-1.90	.60	1.99	.10974E+01	.11264E+01
3	-1.60	.89	1.84	.16415E+01	.16923E+01
4	-1.22	1.05	1.61	.19341E+01	.20055E+01
5	-.79	1.05	1.32	.19293E+01	.20115E+01
6	-.40	.89	.98	.16249E+01	.16985E+01
7	-.10	.58	.59	.10539E+01	.10938E+01
8	.11	.20	.23	.42683E+00	.30764E+00
9	.11	-.20	.23	.42683E+00	.30764E+00
10	-.10	-.58	.59	.10539E+01	.10938E+01
11	-.40	-.89	.98	.16249E+01	.16985E+01
12	-.79	-1.05	1.32	.19293E+01	.20115E+01
13	-1.22	-1.05	1.61	.19341E+01	.20055E+01
14	-1.60	-.89	1.84	.16415E+01	.16923E+01
15	-1.90	-.60	1.99	.10974E+01	.11264E+01
16	-2.06	-.21	2.07	.38546E+00	.39565E+00

Table 3: The comparison of the computed velocity with exact velocity over the boundary of a symmetric aerofoil using 32 linear boundary elements.

ELEMENT	X	Y	$R=\sqrt{X^2+Y^2}$	VELOCITY	EXACT VELOCITY
1	-2.09	.11	2.10	.19581E+00	.19530E+00
2	-2.05	.32	2.08	.57987E+00	.57871E+00
3	-1.97	.51	2.04	.94154E+00	.94085E+00
4	-1.85	.69	1.98	.12669E+01	.12682E+01
5	-1.70	.84	1.90	.15433E+01	.15485E+01
6	-1.52	.96	1.80	.17601E+01	.17706E+01
7	-1.32	1.04	1.68	.19088E+01	.19256E+01
8	-1.11	1.08	1.55	.19836E+01	.20067E+01
9	-.90	1.08	1.41	.19814E+01	.20097E+01
10	-.69	1.04	1.25	.19022E+01	.19331E+01
11	-.49	.96	1.07	.17484E+01	.17783E+01
12	-.31	.84	.89	.15255E+01	.15493E+01
13	-.16	.68	.70	.12408E+01	.12519E+01
14	-.04	.50	.50	.90285E+00	.89212E+00
15	.06	.29	.29	.52553E+00	.47258E+00
16	.15	.09	.17	.23435E+00	.17793E+00
17	.15	-.09	.17	.23435E+00	.17793E+00
18	.06	-.29	.29	.52553E+00	.47258E+00
19	-.04	-.50	.50	.90286E+00	.89212E+00
20	-.16	-.68	.70	.12408E+01	.12519E+01
21	-.31	-.84	.89	.15255E+01	.15493E+01
22	-.49	-.96	1.07	.17484E+01	.17783E+01
23	-.69	-1.04	1.25	.19022E+01	.19331E+01
24	-.90	-1.08	1.41	.19814E+01	.20097E+01
25	-1.11	-1.08	1.55	.19836E+01	.20067E+01
26	-1.32	-1.04	1.68	.19088E+01	.19256E+01
27	-1.52	-.96	1.80	.17601E+01	.17706E+01
28	-1.70	-.84	1.90	.15433E+01	.15485E+01
29	-1.85	-.69	1.98	.12669E+01	.12682E+01
30	-1.97	-.51	2.04	.94154E+00	.94085E+00
31	-2.05	-.32	2.08	.57985E+00	.57871E+00
32	-2.09	-.11	2.10	.19581E+00	.19529E+00

Table 4: The comparison of the computed velocity with exact velocity over the boundary of a symmetric aerofoil using 64 linear boundary elements.

ELEMENT	X	Y	$R=\sqrt{X^2+Y^2}$	VELOCITY	EXACT VELOCITY
1	-2.10	.05	2.10	.98261E-01	.97607E-01
2	-2.09	.16	2.10	.29388E+00	.29139E+00
3	-2.07	.27	2.09	.48665E+00	.48261E+00
4	-2.04	.37	2.07	.67469E+00	.66939E+00
5	-2.00	.47	2.05	.85622E+00	.85001E+00
6	-1.95	.56	2.03	.10295E+01	.10228E+01
7	-1.89	.65	2.00	.11927E+01	.11861E+01
8	-1.82	.74	1.96	.13445E+01	.13383E+01
9	-1.74	.81	1.92	.14832E+01	.14781E+01
10	-1.66	.88	1.88	.16076E+01	.16040E+01
11	-1.57	.94	1.83	.17164E+01	.17148E+01
12	-1.47	.99	1.78	.18086E+01	.18093E+01
13	-1.37	1.03	1.72	.18832E+01	.18867E+01

14	-1.27	1.06	1.66	.19396E+01	.19459E+01
15	-1.17	1.08	1.59	.19772E+01	.19864E+01
16	-1.06	1.09	1.52	.19955E+01	.20075E+01
17	-.95	1.09	1.45	.19945E+01	.20090E+01
18	-.84	1.08	1.37	.19740E+01	.19906E+01
19	-.74	1.06	1.29	.19342E+01	.19524E+01
20	-.63	1.03	1.21	.18756E+01	.18946E+01
21	-.53	.99	1.12	.17985E+01	.18174E+01
22	-.44	.93	1.03	.17036E+01	.17213E+01
23	-.35	.87	.94	.15919E+01	.16072E+01
24	-.27	.80	.85	.14642E+01	.14756E+01
25	-.19	.73	.75	.13216E+01	.13275E+01
26	-.12	.64	.65	.11653E+01	.11637E+01
27	-.06	.55	.55	.99650E+00	.98490E+00
28	-.00	.45	.45	.81656E+00	.79156E+00
29	.04	.34	.35	.62733E+00	.58415E+00
30	.08	.23	.24	.43330E+00	.36688E+00
31	.12	.11	.16	.25449E+00	.18028E+00
32	.17	.03	.17	.13772E+00	.17941E+00
33	.17	-.03	.17	.13772E+00	.17941E+00
34	.12	-.11	.16	.25448E+00	.18028E+00
35	.08	-.23	.24	.43330E+00	.36688E+00
36	.04	-.34	.35	.62733E+00	.58415E+00
37	-.00	-.45	.45	.81657E+00	.79156E+00
38	-.06	-.55	.55	.99650E+00	.98490E+00
39	-.12	-.64	.65	.11653E+01	.11637E+01
40	-.19	-.73	.75	.13216E+01	.13275E+01
41	-.27	-.80	.85	.14642E+01	.14756E+01
42	-.35	-.87	.94	.15919E+01	.16072E+01
43	-.44	-.93	1.03	.17036E+01	.17213E+01
44	-.53	-.99	1.12	.17985E+01	.18174E+01
45	-.63	-1.03	1.21	.18756E+01	.18946E+01
46	-.74	-1.06	1.29	.19342E+01	.19524E+01
47	-.84	-1.08	1.37	.19740E+01	.19906E+01
48	-.95	-1.09	1.45	.19945E+01	.20090E+01
49	-1.06	-1.09	1.52	.19955E+01	.20075E+01
50	-1.17	-1.08	1.59	.19772E+01	.19864E+01
51	-1.27	-1.06	1.66	.19396E+01	.19459E+01
52	-1.37	-1.03	1.72	.18832E+01	.18867E+01
53	-1.47	-.99	1.78	.18086E+01	.18093E+01
54	-1.57	-.94	1.83	.17164E+01	.17148E+01
55	-1.66	-.88	1.88	.16076E+01	.16040E+01
56	-1.74	-.81	1.92	.14833E+01	.14781E+01
57	-1.82	-.74	1.96	.13445E+01	.13383E+01
58	-1.89	-.65	2.00	.11928E+01	.11861E+01
59	-1.95	-.56	2.03	.10295E+01	.10228E+01
60	-2.00	-.47	2.05	.85624E+00	.85001E+00
61	-2.04	-.37	2.07	.67471E+00	.66939E+00
62	-2.07	-.27	2.09	.48660E+00	.48261E+00
63	-2.09	-.16	2.10	.29389E+00	.29139E+00
64	-2.10	-.05	2.10	.98278E-01	.97606E-01

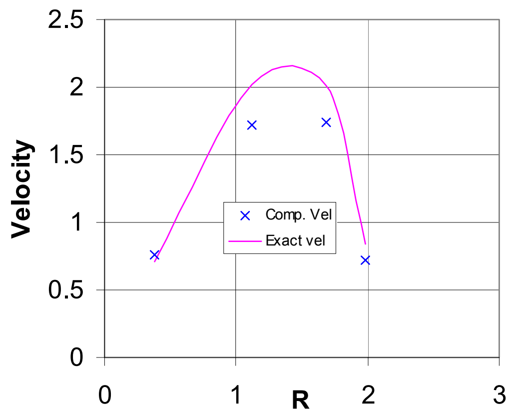

Figure 5: Comparison of computed and analytical velocity distributions over the boundary of a symmetric aerofoil using 16 boundary elements with linear element approach.

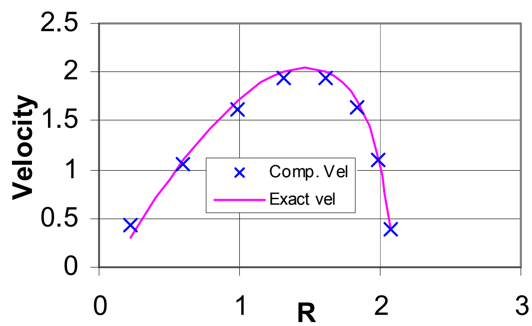

Figure 6: Comparison of computed and analytical velocity distributions over the boundary of a symmetric aerofoil using 16 boundary elements with linear element approach.

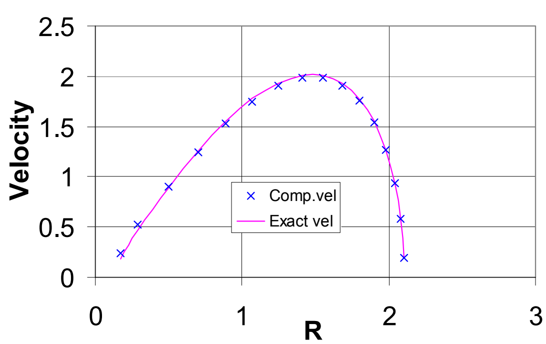

Figure 7: Comparison of computed and analytical velocity distributions over the boundary of a symmetric aerofoil using 32 boundary elements with linear element approach.

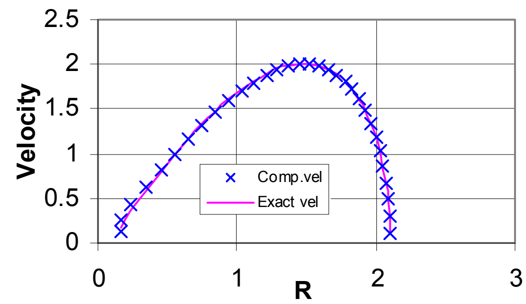

Figure 8: Comparison of computed and analytical velocity distributions over the boundary of a symmetric aerofoil using 64 boundary elements with linear element approach.

5. Conclusion

An indirect boundary element method has been applied for the calculation of compressible flow past a symmetric aerofoil with linear element approach using doublet distribution alone. The calculated flow velocities obtained using this method are compared with the analytical solutions for flow over the boundary of a symmetric aerofoil. It is found from the tables and figures that the computed results obtained by this method are excellent in agreement with the analytical ones for the body under consideration and the accuracy of the result increases due to increase of number of boundary elements.

6. Acknowledgement

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