COMPARISON OF DIRECT AND INDIRECT BOUNDARY ELEMENT METHODS FOR THE FLOW PAST A CIRCULAR CYLINDER WITH CONSTANT ELEMENT APPROACH

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ABSTRACT

In this paper, a comparison of direct and indirect boundary element methods is applied for calculating the flow past(i.e. velocity distribution) a circular cylinder with constant element approach. To check the accuracy of the method, the computed flow velocity is compared with the analytical solution for the flow over the boundary of a circular cylinder.

KEYWORDS

Boundary element methods, Flow past, Velocity distribution, Circular cylinder, Constant element

INTRODUCTION

From the time of fluid flow modeling, it had been struggled to find the solution of a complicated system of partial differential equations (PDE) for the fluid flows which needed more efficient numerical methods. With the passage of time, many numerical techniques such as finite difference method, finite element method, finite volume method and boundary element method etc. came into beings which made possible the calculation of practical flows. Due to discovery of new algorithms and faster computers, these methods were evolved in all areas in the past. These methods are CPU time and storage hungry. One of the advantages is that with boundary elements one has to discretize the entire surface of the body, whereas with domain methods it is essential to discretize the entire region of the flow field. The most important characteristics of boundary element method are the much smaller system of equations and considerable reduction in data which is prerequisite to run a computer program efficiently. Furthermore, this method is well–suited to problems with an infinite domain. From above discussion, it is concluded that boundary element method is a time saving, accurate and efficient numerical technique as compared to other numerical techniques which can be classified into direct boundary element method and indirect boundary element method. The direct method takes the form of a statement which provides the values of the unknown variables at any field point in terms of the complete set of all the boundary data. Whereas the indirect method utilizes a distribution of singularities over the boundary of the body and computes this distribution as the solution of integral equation.

VELOCITY DISTRIBUTION

Let a circular cylinder be of radius 'a' with center at the origin and let the onset flow be the uniform stream with velocity U in the positive direction of the x-axis as shown in figure (1).

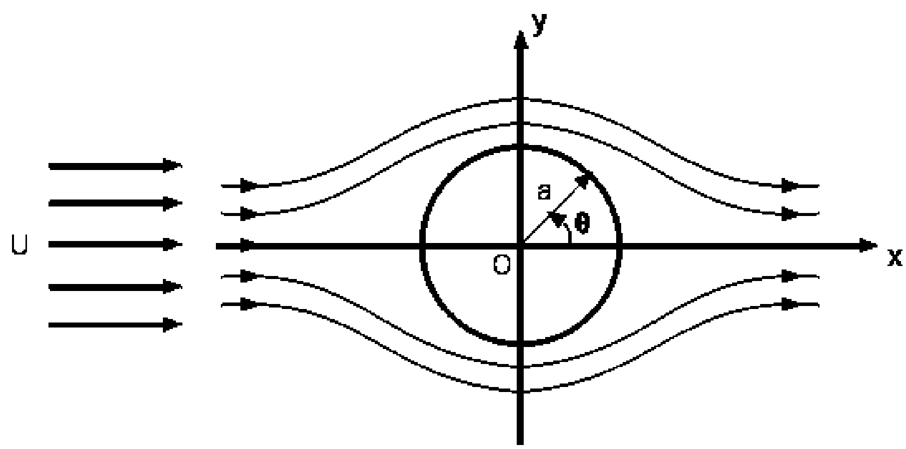


Figure (1)

The magnitude of the exact velocity distribution over the boundary of the circular cylinder is given by

$$|\vec{V}| = 2aU \sin\theta \qquad (1)$$

where θ is the angle between the radius vector and the positive direction of the x–axis.

Now the condition to be satisfied on the boundary of the circular cylinder is

$$\hat{n} \bullet \vec{V} = 0 \qquad (2)$$

where $\hat{n}$ is the unit normal vector to the boundary of the cylinder.

Since the motion is irrotational,

$$\vec{V} = -\nabla\Phi$$

where Φ is the total velocity potential. Thus equation (2) becomes

$$\hat{n} \bullet (-\nabla\Phi) = 0$$

or
$$\frac{\partial \Phi}{\partial n} = 0 \qquad (3)$$

Now the total velocity potential Φ is the sum of the perturbation velocity potential and the velocity potential of the uniform stream $\phi_{u.s.}$

i.e. $$\Phi = \phi_{u.s} + \phi_{c.c} \tag{4}$$

or $$\frac{\partial \Phi}{\partial n} = \frac{\partial \phi_{u.s}}{\partial n} + \frac{\partial \phi_{c.c}}{\partial n}$$

Which on using equation (3) becomes

$$\frac{\partial \phi_{c.c}}{\partial n} = -\frac{\partial \phi_{u.s}}{\partial n} \tag{5}$$

But the velocity potential of the uniform stream is

$$\phi_{u.s} = -U\,x \tag{6}$$

$$\frac{\partial \phi_{u.s}}{\partial n} = -U\frac{\partial x}{\partial n}$$

$$= -U\left(\hat{n} \bullet \hat{i}\right) \tag{7}$$

Thus from equations (5) and (7),

$$\frac{\partial \phi_{c.c}}{\partial n} = U\left(\hat{n} \bullet \hat{i}\right) \tag{8}$$

or $$\frac{\partial \phi_{c.c}}{\partial n} = U\frac{x}{\sqrt{x^2 + y^2}} \tag{9}$$

Equation (9) is the boundary condition which must be satisfied over the boundary of the circular cylinder.

Now for the approximation of the boundary of the circular cylinder, the coordinates of the extreme points of the boundary elements can be generated within the computer program as follows:

Divide the boundary of the circular cylinder into m elements in the clockwise direction by using the formula

$$\theta_k = \frac{(m+3) - 2\,k}{m}\pi\,, \quad k = 1, 2, \ldots, m \tag{10}$$

Then the coordinates of the extreme points of these m elements are

Calculated from

$$\left.\begin{array}{l} x_k = a\cos\theta_k \\ y_k = a\sin\theta_k \end{array}\right\} \quad k = 1, 2, \ldots, m \tag{11}$$

Take $m = 8$ and $a = 1$.

In the case of constant boundary elements where there is only one node at the middle of the element and the potential ϕ and the potential derivative $\frac{\partial \phi}{\partial n}$ are constant over each element and equal to the value at the middle node of the element.

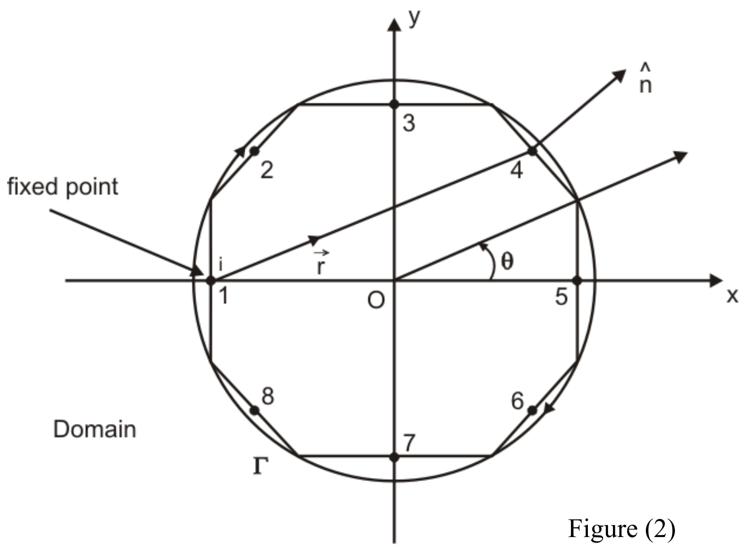


Figure (2)

Figure (2) shows the discretization of the circular cylinder of unit radius into 8 constant boundary elements. The coordinates of the middle node of each boundary element are given by

$$\left.\begin{aligned} x_m &= \frac{x_k + x_{k+1}}{2} \\ y_m &= \frac{y_k + y_{k+1}}{2} \end{aligned}\right\} \quad k\,,\ m = 1, 2, \ldots, 8 \qquad (12)$$

And therefore the boundary condition (9) in this case takes the form

$$\frac{\partial \phi_{c.c}}{\partial n} = U \frac{x_m}{\sqrt{x_m^2 + y_m^2}}$$

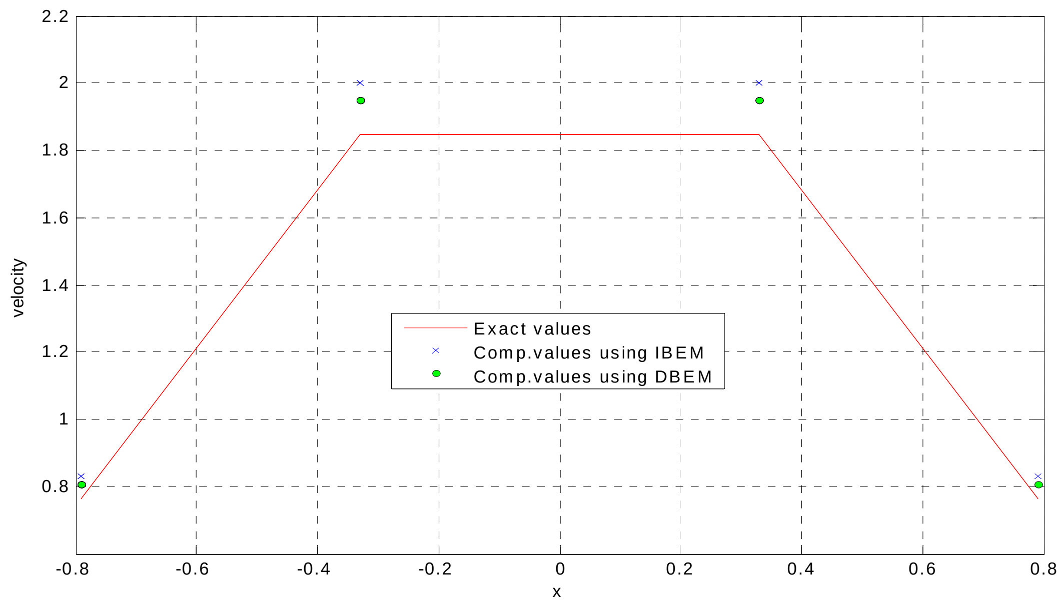
The velocity U of the uniform stream is also taken as unity.

The following table shows the comparison of the direct and indirect boundary element methods for computed and analytical velocity distributions over the boundary of a circular cylinder for 8 constant boundary elements.

TABLE (1)

Element	x-Coordinate	y-Coordinate	Computed Velocity Using DBEM	Computed Velocity Using IBEM	Analytical Velocity
1	-.79	.33	.80718E+00	.82884E+00	.76537E+00
2	-.33	.79	.19487E+01	.20010E+01	.18478E+01
3	.33	.79	.19487E+01	.20010E+01	.18478E+01
4	.79	.33	.80718E+00	.82884E+00	.76537E+00
5	.79	-.33	.80718E+00	.82884E+00	.76537E+00
6	.33	-.79	.19487E+01	.20010E+01	.18478E+01
7	-.33	-.79	.19487E+01	.20010E+01	.18478E+01
8	-.79	-.33	.80718E+00	.82884E+00	.76537E+00

GRAPH 1



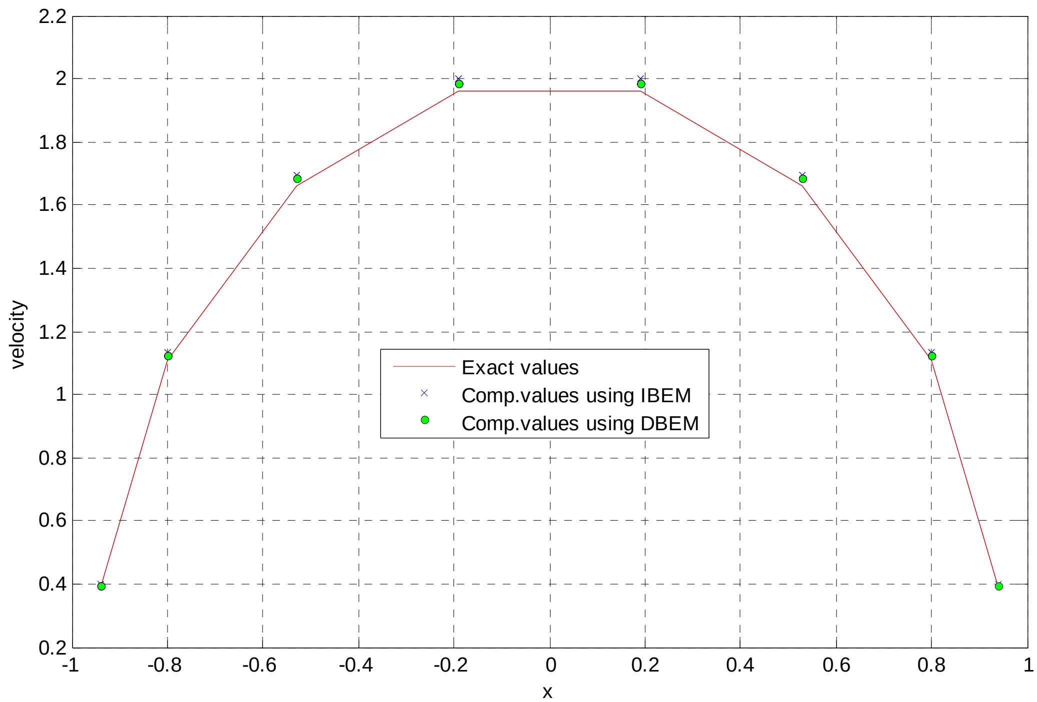
Graph 1: Comparison of computed and analytical velocity distributions over the boundary of a circular cylinder using 8 boundary elements with constant variation.

The improvement gained by using 16 constant boundary elements can be seen from the following table.

TABLE (2)

Element	x-Coordinate	y-Coordinate	Computed Velocity Using DBEM	Computed Velocity Using IBEM	Analytical Velocity
1	-.94	.19	.39524E+00	.39785E+00	.39018E+00
2	-.80	.53	.11256E+01	.11330E+01	.11111E+01
3	-.53	.80	.16845E+01	.16956E+01	.16629E+01
4	-.19	.94	.19870E+01	.20001E+01	.19616E+01
5	.19	.94	.19870E+01	.20001E+01	.19616E+01
6	.53	.80	.16845E+01	.16956E+01	.16629E+01
7	.80	.53	.11256E+01	.11330E+01	.11111E+01
8	.94	.19	.39524E+00	.39785E+00	.39018E+00
9	.94	-.19	.39524E+00	.39785E+00	.39018E+00
10	.80	-.53	.11256E+01	.11330E+01	.11111E+01
11	.53	-.80	.16845E+01	.16956E+01	.16629E+01
12	.19	-.94	.19870E+01	.20001E+01	.19616E+01
13	-.19	-.94	.19870E+01	.20001E+01	.19616E+01
14	-.53	-.80	.16845E+01	.16956E+01	.16629E+01
15	-.80	-.53	.11256E+01	.11330E+01	.11111E+01
16	-.94	-.19	.39524E+00	.39785E+00	.39018E+00

GRAPH 2



Graph 2: Comparison of computed and analytical velocity distributions over the boundary of a circular cylinder using 16 boundary elements with constant variation.

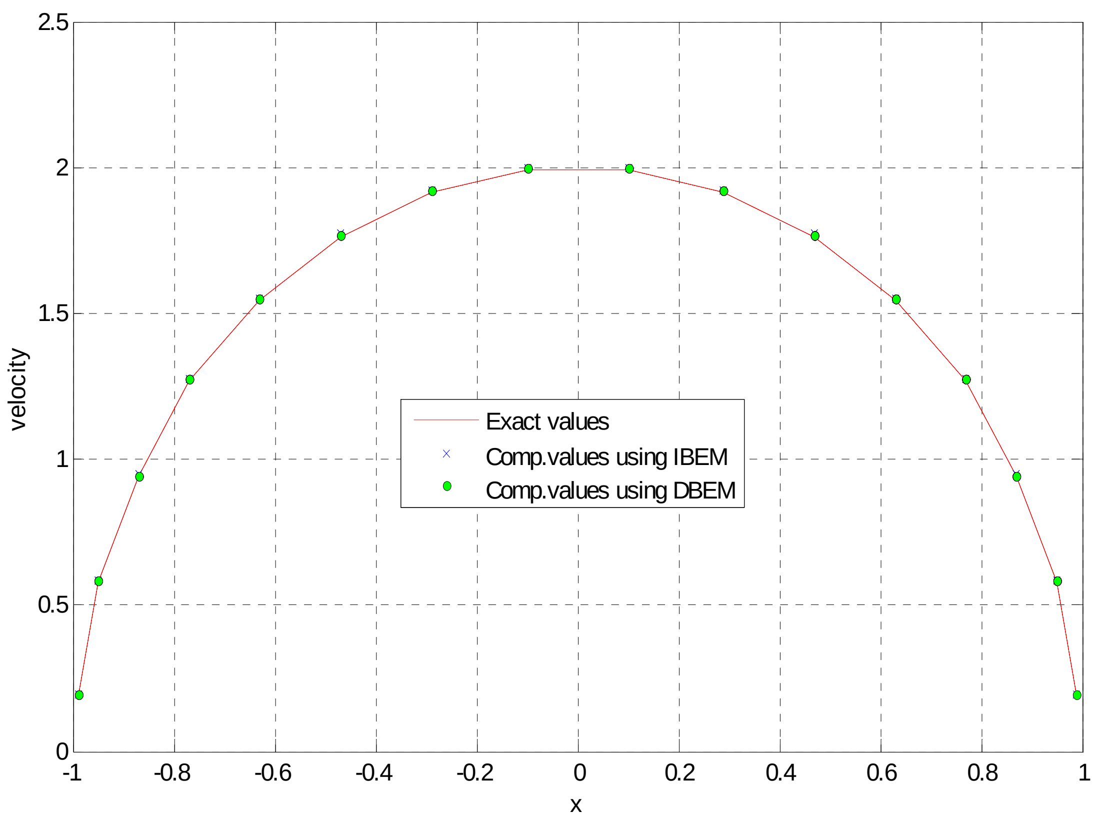
The improvement gained by using 32 constant boundary elements can be seen from the following table.

TABLE (3)

Element	x-Coordinate	y-Coordinate	Computed Velocity Using DBEM	Computed Velocity Using IBEM	Analytical Velocity
1	-.99	.10	.19667E+00	.19699E+00	.19604E+00
2	-.95	.29	.58244E+00	.58339E+00	.58057E+00
3	-.87	.47	.94583E+00	.94738E+00	.94279E+00
4	-.77	.63	.12729E+01	.12750E+01	.12688E+01
5	-.63	.77	.15510E+01	.15535E+01	.15460E+01
6	-.47	.87	.17695E+01	.17724E+01	.17638E+01
7	-.29	.95	.19200E+01	.19232E+01	.19139E+01
8	-.10	.99	.19968E+01	.20001E+01	.19904E+01
9	.10	.99	.19968E+01	.20001E+01	.19904E+01
10	.29	.95	.19200E+01	.19232E+01	.19139E+01
11	.47	.87	.17695E+01	.17724E+01	.17638E+01
12	.63	.77	.15510E+01	.15535E+01	.15460E+01
13	.77	.63	.12729E+01	.12750E+01	.12688E+01
14	.87	.47	.94583E+00	.94738E+00	.94279E+00
15	.95	.29	.58244E+00	.58339E+00	.58057E+00
16	.99	.10	.19667E+00	.19699E+00	.19603E+00

17	.99	-.10	.19667E+00	.19699E+00	.19603E+00
18	.95	-.29	.58243E+00	.58339E+00	.58057E+00
19	.87	-.47	.94583E+00	.94738E+00	.94279E+00
20	.77	-.63	.12729E+01	.12750E+01	.12688E+01
21	.63	-.77	.15510E+01	.15535E+01	.15460E+01
22	.47	-.87	.17695E+01	.17724E+01	.17638E+01
23	.29	-.95	.19200E+01	.19232E+01	.19139E+01
24	.10	-.99	.19968E+01	.20001E+01	.19904E+01
25	-.10	-.99	.19968E+01	.20001E+01	.19904E+01
26	-.29	-.95	.19200E+01	.19232E+01	.19139E+01
27	-.47	-.87	.17695E+01	.17724E+01	.17638E+01
28	-.63	-.77	.15510E+01	.15535E+01	.15460E+01
29	-.77	-.63	.12729E+01	.12750E+01	.12688E+01
30	-.87	-.47	.94583E+00	.94738E+00	.94279E+00
31	-.95	-.29	.58244E+00	.58339E+00	.58057E+00
32	-.99	-.10	.19666E+00	.19698E+00	.19604E+00

GRAPH 3



Graph 3: Comparison of computed and analytical velocity distributions over the boundary of a circular cylinder using 32 boundary elements with constant variation.

CONCLUSION

A direct and indirect boundary element methods have been applied for the calculation of flow past a circular cylinder. The calculated flow velocities obtained using these methods are compared with the analytical solutions for flow over the boundary of a circular cylinder . It is found that the results obtained with the direct boundary element method for the flow past are excellent in agreement with the analytical results for the body under consideration.

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