Application of Homotopy Perturbation Method for the Large Angle period of Nonlinear Oscillator

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Abstract

The homotopy perturbation method is used to determine the period of nonlinear oscillator. The method produces the result even for large amplitude. The result is compared with others in the literature.

Keyword: homotopy perturbation method; nonlinear oscillator; period.

1.0 Introduction

The study of nonlinear oscillators is of great importance to many scientific researchers in various fields. Various methods such as variational iteration methods [2],[4-5], parameter expanding method [1] have been proposed.

The homotopy perturbation method (HPM), proposed first by He J in 1999 for solving differential and integral equations, linear and nonlinear has been the subject of extensive analytical and numerical studies. The method is a coupling of the traditional perturbation method and homotopy in topology. This method has a significant advantage in that it provides an approximate solution to a wide range of nonlinear problems in applied sciences.

In this paper, we apply HPM to obtain the frequency of nonlinear oscillator. The solution obtained is of high accuracy which is valid for the whole solution domain.

2.0. Homotopy Perturbation Method.

We consider a general nonlinear oscillator of the form:

$$mu''+\omega_0^2 u + kf(u,u',u'') = 0 \qquad (1)$$

according to [7], we expand m and ω_0^2 as follows:

$$\omega_0^2 = \omega^2 + p\omega_1 + p^2\omega_2 + ... + p^n\omega_n \qquad (2).$$

$$m = 1 + pm_1 + p^2 m_2 + ... + p^n m_n \qquad (3)$$

Where p is a homotopy parameter, ω_i and m_i are unknown constants to be further determined.

3.0. Mathematical Pendulum

When friction is neglected, the differential equation governing the free oscillation of the mathematical pendulum is given by:

$$u''+\omega^2 \sin u = 0 \qquad (4)$$

$$u(0) = A, u'(0) = 0.$$

where

u is the angle of deviation from the vertical equilibrium position.

$\omega^2 = \frac{g}{l}$; is the acceleration due to gravity and l is the length of the pendulum.

The model look simple if the approximation $\sin u \sim u$ is used, the equation (4) becomes

$$u''+\omega^2 u = 0 \qquad (5)$$

To obtain more accurate result we modify the above equation by putting

$$\sin u \sim u - \frac{u^3}{6} \qquad (6)$$

Substituting for equation (6) in (5) we have;

$$u''+\omega^2 u - \frac{\omega^2}{6} u^3 = 0 \qquad (7)$$

4.0. **Application of HPM**

Equation (7) can be written in the form of equation (1) such that.

$$u''+\omega_0^2 u + ku^3 = 0, k = -\frac{\omega_0^2}{6} \quad \text{.......(8)}$$

Applying equation (2) in equation (8), we have;

$$u''+(\omega^2 + pc_1)u + pku^3 = 0 \quad \text{.......(9)}$$

The basic assumption is that:

$$u(t) = u_0 + pu_1 + p^2u_2 + p^3u_3 + ... + p^n u_n = 0 \quad \text{.......(10)}$$

When $p = 1$ equation (9) becomes

$$u''+(\omega^2 + c_1)u + ku^3 = 0 \quad \text{.......(11)}$$

Comparing equation (8) and (11);

$$\omega_0^2 = \omega^2 + c_1 \quad \text{.......(12)}$$

Substituting equation (10) in equation (9) and equating the coefficients of like powers of p.

$$u_0'' + \omega^2 u_0 = 0, u_0(0) = A, u'(0) = 0 \quad \text{.......(13)}$$

$$u_1'' + \omega^2 u_1 + c_1 u_0 + ku_0^3 = 0, u_0(0) = A, u'(0) = 0 \quad \text{.......(14)}$$

The solution to equation (13) is

$$u_0 = A\cos\omega t \quad \text{.......(15)}$$

Putting equation (15) in equation (14) and eliminating the secular term; we have.

$$c_1 = -\frac{3}{4}kA^2 \quad \text{.......(16).}$$

From equation (12)

$$\omega^2 = \omega_0^2 + \frac{3}{4}kA^2 \quad \text{.......(17)}$$

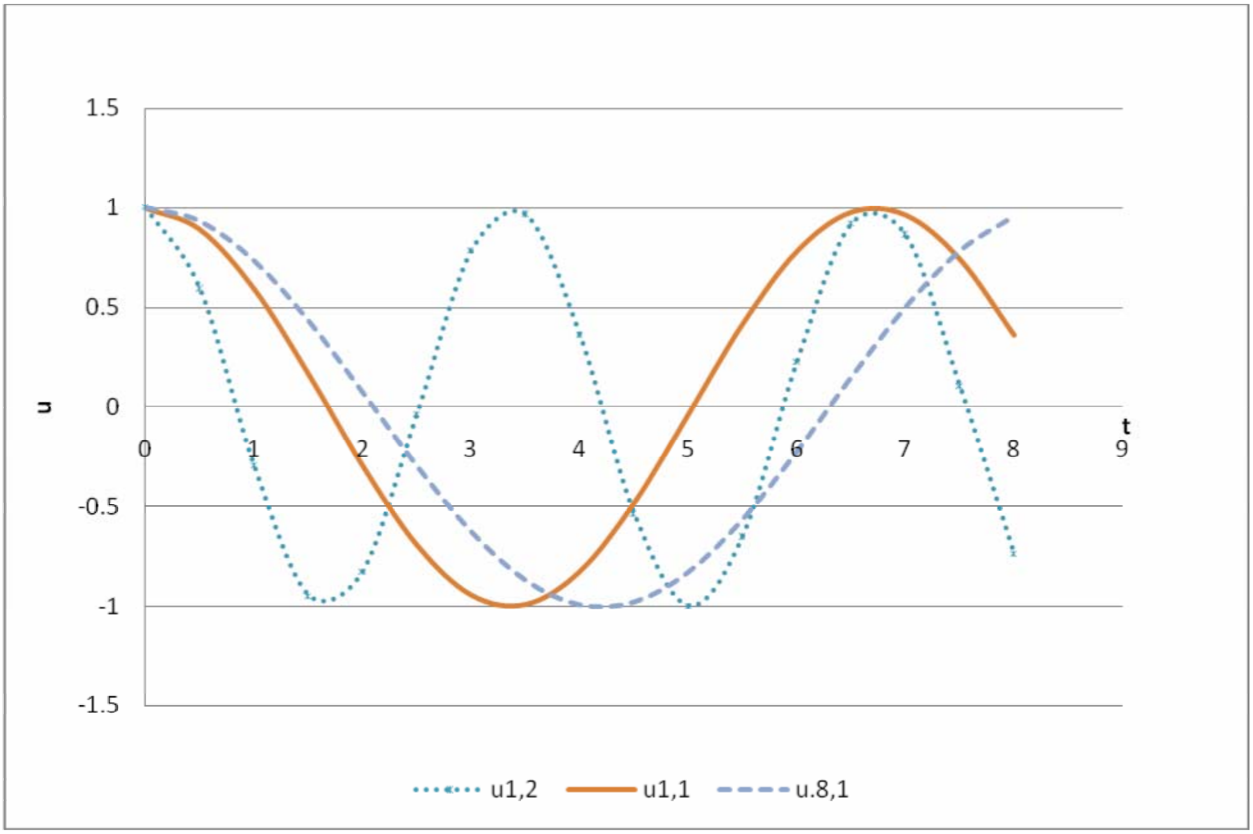
Therefore,

$$\omega^2 = \omega_0^2(1 - \frac{1}{8}A^2) \quad \text{.......(18)}$$

The period (T) is therefore:

$$T = \frac{2\pi}{\omega_0\sqrt{1 - \frac{1}{8}A^2}} \quad \text{.......(19)}$$

This compared favourably with the solution obtain in [6] and the value given in [8].



$for\ u_{0.8,1}\ \ \omega_0 = 0.8, A = 1, \quad for\ u_{1,1}\ \ \omega_0 = 1, A = 1, \quad for\ u_{1,2}\ \ \omega_0 = 1, \quad A = 2$

5.0. **Conclusion**

In this work, the homotopy perturbation method has been successfully applied to find the approximate solutions for the nonlinear system.

It is worth mentioning that the method is capable of reducing the volume of computational work while still maintaining high accuracy of the numerical result. This amounts to the improvement of performance of approach. This method is relatively new and may lead to some novel and innovative applications in solving linear and nonlinear problems.

The method which proved to be a powerful mathematical tool to nonlinear oscillators can be used as a searching tool for the period or frequency of various nonlinear oscillatory systems.

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