Advantages and Disadvantages of Boundary Element Methods For Compressible Fluid Flow Problems

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Abstract:

In this paper, the advantages and disadvantages of boundary element methods (BEMs) for compressible fluid flow problems are presented. BEMS are gaining popularity due to their applications in the vast fields of science and technology and it is also being applied for calculating the solution of compressible fluid flow problems. All techniques have some advantages and disadvantages. The efficiency as well as accuracy of a method can be easily checked for the solution of a certain problem by its advantages as well as disadvantages. So the performance of BEMs in the present case is judged by giving its advantages and disadvantages in details. [Journal of American Science 2010; 6(1): 162-165]. (ISSN: 1545-1003).

Keywords: Advantages and disadvantages, Boundary Element Methods, Compressible Flow, CFD.

1. Introduction

A compressible flow is different from an incompressible one in that the density of fluid does not remain the same. In compressible flow, the equation governing the inviscid compressible flow of a homogeneous fluid were first derived by Euler. Euler considered all the characteristics of the fluid to be continuous functions of time and space. The approach taken by Euler assumes that the fluid is a continuum (Schreier, 1982). In applying the continuum assumption, the care must be taken that the average distance between molecules is small as compared to the scale of the problem under consideration. From the time of fluid flow modeling, it had been struggled to find the solution of a complicated system of partial differential equations (PDE) for the fluid flows which needed more efficient numerical methods. From time to time, many numerical techniques such as finite difference method, finite element method, finite volume method and boundary element method etc. came into beings which made possible the calculation of practical flows. Due to discovery of new algorithms and faster computers, these methods were evolved in all areas in the past such as stress analysis, heat transfer and electromagnetic theory, potential theory, fracture mechanics, fluid mechanics, elasticity, elastostatics and elastodynamics , biological and biomedical problems ,etc. These methods are CPU time and storage hungry. Boundary element method originated within the Department of civil engineering at Southampton University, U.K. (Brebbia, 1978). These methods existed under different names such as 'Panel Method', 'Surface singularity methods' , 'boundary integral equation methods' ,or 'boundary integral solutions'. Now a days , the boundary element method is successfully applied by numerical community. One of the advantages is that with boundary elements one has to discretize the entire surface of the body, whereas with domain methods it is essential to discretize the entire region of the flow field. The most important characteristics of boundary element methods are the much smaller system of equations and considerable reduction in data which is prerequisite to run a computer program efficiently. Moreover, this method is well–suited to problems with an infinite domain. From above discussion, it is concluded that boundary element method is a time saving, accurate and efficient numerical technique as compared to other numerical techniques which can be classified into direct boundary element method and indirect boundary element method which depends on whether the functions used in derivatives are physical quantities or fictitious density functions (Becker, A.A). The direct method takes the form of a statement which provides the values of the unknown variables at any field point in terms of the complete set of all the boundary data. Whereas the indirect method utilizes a distribution of singularities over the boundary of the body and computes this distribution as the solution of integral equation. The equation of direct boundary element method (DBEM) can be formulated using either as an approach based on Green's theorem (Lamb, 1932; Milne-Thomson, 1968 and Kellogge, 1929) or a particular case of the weighted residual methods (Brebbia and Walker, 1980). The equation of indirect method can be derived from that of direct method. In the early 1980, a surge in research activities on BEMs occurred and this technique found its way in the field of fluid mechanics (Gaul et al, 2003). The direct boundary element method was used for flow field calculations around complicated bodies (Morino et al., 1975 Luminita, 2007, Mushtaq, 2008). While the indirect method has been used in the past for flow field calculations surrounding arbitrary bodies (Hess and Smith, 1967; Hess, 1973, Muhammad, 2009, Luminita, 2008, Mushtaq, 2009). Now the boundary element method is being used for the solution of compressible flows around complex configurations .Thus it can be said that the boundary element method is a powerful numerical technique receiving much attention from researchers, engineering community and is becoming popular technique in the computational solution of a number of physical problems.

2. General Mathematical Formulation of Boundary Element Method

Consider the differential equation

$$\mathcal{L}(u) = b \tag{1}$$

or $\mathcal{L}(u) - b = 0$ in Ω

Where $\mathcal{L}$ is an arbitrary linear differential operator with constant coefficients, 'u' is the field variable and 'b' is an arbitrary source distribution in Ω.

In multi–dimensional case, equation (1) can be written as:

$$\int_{\Omega} \left(\mathcal{L}(u) - b\right) w \, d\Omega = 0 \tag{2}$$

In two and three – dimensional problems, the domain integrals are reduced to boundary integrals by using integration by parts and Gauβ' theorem as follows

$$\int_{\Omega} \mathcal{L}^*(u)\, w \, d\Omega + \int_{\Gamma} \left[F(u).S^*(w) - S(u).F^*(w)\right] d\Gamma - \int_{\Omega} \mathcal{L}(u)\, w \, d\Omega = 0 \tag{3}$$

The first integral in equation (3) can be eliminated by using the shifting property of the Dirac distribution and the following formula is obtained

$$u\,(i) = \int_{\Gamma} (F(\,u\,).\,S^{*}\,(w) - S\,(u\,).\,F^{*}(w)\,d\,\Gamma$$

$$- \int_{\Omega} b\,u^{*}\,d\,\Omega \qquad (4)$$

The equation (4) holds only, if 'i' is inside the domain. By shifting the load point to the boundary point in a special limiting process, the boundary integral equation can be obtained in which all unknown field variables have been transformed to the boundary. This equation is the basis of boundary element method .

3. Advantages and Disadvantages of Boundary Element Methods:

(a) Advantages:

(i) Less data:

In BEMs, less data is required to run a program efficiently.

(ii) Less Time:

In BEMs, less time is required for the solution of a problem due to a small system of equation.

(iii) Less Unwanted Information:

In such technique, unwanted information is much less than other numerical techniques.

(iv) Process of discretization:

In BEMs, the process of discretization takes place only on the surface of body so that the system of equations is much smaller.

(v) Not costly:

Since the discretization is only on the surface in boundary element method. Thus amount of data is small. That is why such technique is not costly.

(vi) Open domain:

BEM is well-suited to problems of open domains.

(b) Disadvantages:

(i) Non-Linear Flow Problems:

BEM is not successfully applied to non-linear fluid flow problems.

(ii) Unfamiliar Mathematics

The mathematics used in BEM is unfamiliar to engineering community. So they are not interested in such mathematics.

(iii) Fully populated matrix:

Matrices resulted in BEM are unsymmetric and fully populated. They are not easy to solve.

4. Conclusion

In this paper, the advantages and disadvantages of boundary element methods (BEMs) for compressible fluid flow problems have been presented. Like other numerical methods, BEMs have also advantages and disadvantages. Since advantages of such methods are more than its disadvantages, Therefore it can be successfully applied for compressible fluid flow problems.. it can be very useful in modeling of different types of bodies such as airplanes, space shuttle, etc.

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