Merits and Demerits of Boundary Element Methods For Incompressible Fluid Flow Problems

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Abstract:

In this paper, the merits and demerits of boundary element methods (BEMs) for incompressible fluid flow problems are described. BEMs are gaining popularity due to their applications in the vast fields of science and technology and it is also being applied for calculating the solution of incompressible fluid flow problems. Every method has its merits and demerits The efficiency as well as accuracy of a method can be easily checked for a certain problem by its merits as well as demerits for the solution of that problem. So the performance of BEMs in the present case is judged by giving its merits and demerits in details. [Journal of American Science 2009; 5(6):57-61]. (ISSN: 1545-1003).

Keywords:

Merits and Demerits, Boundary Element Methods, Incompressible Flow, CFD.

1. Introduction

Boundary element method (BEM) is a modern numerical technique, which can play an important role for the development of science and technology and it is used for the solution of linear partial differential equations, which are transformed into boundary integral forms. Such forms are considered as exact solutions of the governing partial differential equations. The term' boundary elements' opened eyes within the department of civil engineering at Southampton University, United Kingdom (Brebbia,1978). In this method, the external boundary of a body is discretized into a number of segments over which the function under discussion can vary much in the ways as in finite elements. (Brebbia,1978). In literature, these methods existed under different names such as 'panel methods',' surface singularity methods',' boundary integral equation methods' or 'boundary integral solutions'. BEMs are more popular amongst computational community than 'domain type' like finite element method (FEM) and finite difference method (FDM),etc (Hirt et al.,1978, Markatos,1983; Demuran et al.,1982 and Ecer,1982) due to its character of reducing the dimensionality of a boundary value problem represented by linear partial differential equations. Thus an equation governing a three-dimensional domain is transformed into over its surface and likewise a two-dimensional domain on its boundary contour only. Such reduction in dimensions results in a smaller system of equations, which leads to reduction in data and thus reduction in computational efforts as well as in time. BEMs are well suited to problems with complicated and unbounded regions. It has been applied to obtain the computational solution of a large number of physical problems BEMs are applied in diverse topics as stress analysis, heat transfer and electromagnetic theory, potential theory, fracture mechanics, fluid mechanics, elasticity, elastostatics and elastodynamics etc (Muhammad et al.,2009). Nowadays BEMs are being used in biomedical and environmental problems like circulation of blood and urine in human body and prediction of weather. So, such method can be useful in the numerical biomechanical analysis of systemic circulation. Thus, it can be helpful in diagnosing and in treatment of diseases. In this way, merits of

BEMs can be brought in the service of humanity. BEMs have been classified as direct and indirect techniques, which depends on whether the functions used in derivatives are physical quantities or fictitious density functions (Becker.A.A.).The equation of direct method can be formulated using either as an approach based on Green's function (Lamb,1932, Milne-Thomson,1968 and Kellogg,1929) or a particular case of the weighted residual methods(Brebbia and Walker,1980).The equation of indirect method can be derived from that of direct method. In the early 1980, a surge in research activities on BEMs occurred and this technique found its way in the field of fluid mechanics (Gaul.L., Kogl.M, Wagner.M,2003).In stead of achieving successes in the field of hydrodynamics, there are cases in which computations of BEMs still pose serious challenges(Vaz et al,2003). The flow fields were calculated around three-dimensional bodies by using a lower-order indirect method (Hess and Smith,1962;1967). The direct method was applied to calculate the potential flow problems (Morino et al ,1975).

2. General Mathematical Formulation of Boundary Element Method

The general mathematical formulation of boundary element method can be obtained by using the differential equation

$$L(\phi) = a$$

or

$$L(\phi) - a = 0 \text{ in } \varsigma \qquad (1)$$

Where L is an arbitrary linear differential operator with constant coefficients, 'ϕ' is the field variable and 'a' is an arbitrary source distribution inς. The weighted form equation (1) is as follows:

$$\int_{\varsigma} \left(L(\phi) - a\right) \psi \, d\varsigma = 0 \qquad (2)$$

$$\int_{\varsigma} L^{*}(\psi).\phi \, d\varsigma + \int_{\Gamma} \left(G(\phi) . S^{*}(\psi) - S(\phi) . G^{*}(\psi)\right) d\Gamma - \int_{\varsigma} a\phi^{*} \, d\varsigma = 0 \qquad (3)$$

By choosing the fundamental solution, the first integral in equation (3) can be eliminated due to the sifting property of the Dirac distribution and equation(3) reduces to

$$\phi(i) = \int_{\Gamma} \left(G(\phi) . S^{*}(\psi) - S(\phi) . G^{*}(\psi)\right) d\Gamma - \int_{\varsigma} a\phi^{*} \, d\varsigma \qquad (4)$$

Where 'i' is an arbitrary point within ς and equation (4) only holds for the point 'i' within ς. When the point is moved to the boundary in a special limiting process, the boundary integral equation (BIE) can be obtained (Gaul.L, and Kogl.M, Wagner,2003). In fact, the concept of boundary integral equation laid down the foundation stone of boundary element method (BEM)

3. Merits and Demerits of Boundary Element Methods: (1)

(a) Merits:

(i) Discretization Only On The Boundary:

In BEMs, one only has to discretize the boundary not the entire flow field. In this way, the dimension of the physical problems under consideration is effectively reduced by one order. That is a three-dimensional problem is transformed into two-dimensional one over the boundary and likewise a two-dimensional problem on the boundary contour only. Thus the problem becomes simple and easy to solve.

(ii) Economical And Time-Saving:

Since the discretization in BEMs takes place only on the boundary for fluid flow problems. The system of equations for such problem is much smaller. Consequently, the amount of data is thus significantly very small and many hours are not spent in preparing and checking the data. So BEM is economical and time-saving.

(iii) Less Number Of Nodes And Elements:

The number of nodes and elements used in BEMs for the solution of a flow problem is needed more than in 'domain' type methods with the same standard of efficiency and accuracy.

(iv) Well-Suited To Infinite And Semi-Infinite Flow Fields:

BEMs are well-suited to infinite and semi-infinite flow fields. In such case, the dimensionality of a flow problem is considerably reduced by one. Thus three-dimensional and two-dimensional flow fields can be reduced by one order. Therefore, in flow past a body, the governing equation for an infinite domain is reduced to one over the finite boundary.

(v) Useful To Laminar Flows:

BEMs are useful for the solution of laminar flow problems. Since all the engineering problems are of turbulent nature, it does not mean that BEMs are totally failure in turbulent flows and even then it can be applied to simple channel flows.

(vi) Suitable To Complex Flow Problems:

BEMs are more suitable to complex flow problems. Because they reduce the size of complicated flow problems into the simple ones which can be solved easily.

(vii) Applicable To Potential Flow Problems:

BEMs can be applied in the best ways to potential flow problems. Such problems of a great significance in computational dynamics (CFD) can be easily tackled by this numerical technique.

(viii) Flow Problems Involving Small Non-Linearity:

Though BEMs are successfully applied to flow problems involving linearity, but it is also applied to fluid flow problems in which non-linearity is of small type.

(ix) Variable Or Unidentified Flow Fields:

BEMs are also useful in flow problems where the flow fields are variable or unidentified types such as problems with free surface flows.

(x) Capability For A Complete Solution:

BEMs posses a unique capability for complete solutions of fluid flow problems in the form of boundary values only.

(xi) Applicable Easily To Incompressible Flow Problems:

BE techniques handle incompressible flow problems effectively, so it can be applied to problems involving incompressible type fluids.

(xii) Flow Field Around Bodies Of Complex Geometry:

Such methods are very useful for calculating the flow fields around bodies of complex configurations.

(b) Demerits:

(i) Non-Homogeneous And Non-Linear Flow Problems:

BEMs are not successfully applied to non - homogeneous and non-linear fluid flow problems. However, these are used to turbulent flow of minor nature. Therefore, these are not useful to apply in the non-linear computations of partial and super-cavitating flows on hydrofoils and marine propellers.

(ii) Flow Problems Where Formulations Impossible:

In flow problems, whose mathematical formulation is impossible, BEMs cannot be applied successfully.

(iii) Mathematical Complexity:

Mathematical complexity is also a great hurdle in the way of implementation of BEMs to flow problems. Engineers are not mostly familiar with the mathematics used in BEMs. So they cannot work properly in this field . To overcome this difficulty, the BE techniques should be included in engineering courses and more books on this topic should be available in the market.

(iv) New Practical Applications:

BEMs have relatively new practical applications in fluid flow problems. their roots are not so deep in such field and it will take time for BEMs to be matured and more useful.

(v) Non-Symmetric And Fully Populated Solution Matrix:

The resulting solution matrix in BEMs is non-symmetric and fully populated with non-zero coefficients, which needs a large space in the computer core memory. This makes situation even more severe.

(vi) Requirement For Knowledge Of A Suitable Fundamental Solution:

For finding he solution of fluid flow problems in BEMs, it is required to have knowledge of suitable fundamental solution. Otherwise it will be difficult to tackle the relevant problems.

4. Conclusion

In this paper, the merits and demerits of boundary element methods (BEMs) for incompressible fluid flow problems have been presented. Like other numerical schemes, BEMs have also merits and demerits. These demerits are less in comparison to merits. Therefore in spite of its some demerits, they are progressing rapidly and their applications in different fields of science and technology are increasing day by day. That is why these numerical schemes are becoming more and more popular amongst the computational community of recent world. Such methods can be very useful in modeling bodies of complex geometry such as airplanes, road vehicles, space shuttle, ships, etc.

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