



Studies On Characterization Of Total Domination Number And Chromatic Number Of Fuzzy Graph

*Dr. Satendra Kumar and **Sonu Kumar

* Professor, Department of Mathematics, OPJS University, Churu, Rajasthan (India)

**Research Scholar, Department of Mathematics, OPJS University, Churu, Rajasthan (India)

Email: sonukumar1402@gmail.com

Abstract: In many practical problems, information about the problem is not certain. There is vagueness in the description of objects or in its relationship or in both. For example, in a time tabling problem, the priorities given to the teachers need not be equal. In that situation, we need to design fuzzy graph model for that problem. Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. During the same time researcher introduced various connectedness concepts in graph theory. Researcher has obtained the fuzzy analogues of several basic graph theoretic concepts like bridges, paths, cycles, connectedness and established some of their properties. Fuzzy set theory provides meaningful and powerful representation of measurement of uncertainties, as well as vague concepts expressed in natural languages. Every crisp set is fuzzy set but every fuzzy set is not crisp set. The mathematical embedding of conventional set theory into fuzzy sets is as natural as the idea of embedding the real numbers into complex plane. Researcher introduced domination in fuzzy graph using strong edges. Researcher studied the concept of regular fuzzy graph. The concept of fuzzy line graph was introduced by researcher and the fuzzy labeling graph was introduced by the researcher. Researcher introduced the concept of middle, subdivision & total fuzzy graph and their properties. The concept of Dominator coloring was introduced by the researcher. The fuzzy coloring of a fuzzy graph was defined by the researcher. They defined the fuzzy coloring of the fuzzy graph based on some conditions which is same as crisp coloring.

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1.1 Introduction

The investigation of dominating sets in diagrams was started by Ore and Berge, the domination number; all out domination number are presented by Cockayne and Hedetniemi. A Mathematical system to portray the marvels of vulnerability, all things considered, circumstance is first proposed by L.A. Zadeh in 1965. Research on the hypothesis of fuzzy sets has been seeing an exponential development; both inside science and in its applications. This extends from conventional numerical subjects like rationale, topology, variable based math, examination and so forth consequently fuzzy set hypothesis has risen as potential zone of interdisciplinary research and fuzzy diagram hypothesis is of late intrigue.

The fluffy definition of fluffy charts was proposed by Kaufmann, from the fluffy relations introduced by Zadeh Although Rosenfeld introduced another expounded definition, including fluffy vertex and fluffy edges. A few fluffy analogs of diagram theoretic concepts, for example, ways, cycles connectedness and so on. The idea of domination in fluffy charts was investigated by A. Somasundram, S. Somasundram A. Somasundram introduced the

concepts of free domination, complete domination, associated domination and domination in Cartesian item and structure of fluffy charts.

A few creators have examined the issue of obtaining an upper destined for the entirety of a domination parameter and a diagram theoretic parameter and portrayed the comparing extremely charts.

In, Paulraj Joseph J and Arumugam S proved that they additionally described the class of diagrams for which the upper bound is attained. They additionally proved comparative outcomes for γ and χ . In, Paulraj Joseph J and Mahadevan G, proved that $\gamma_{cc} + \chi \leq 2n-1$ and portrayed the corresponding extremely diagrams.

In, Mahadevan G presented the idea the correlative immaculate control number γ_{cp} and proved that $\gamma_{cp}(G) + \chi \leq 2n-2$, what's more, portrayed the corresponding extremely charts. They additionally acquired the comparative outcomes for the incited corresponding immaculate domination number and chromatic number of a chart. In, S. Vimala and J.S.

Sathya proved that $\gamma_t(G) + \chi(G) = 2n - 5$. They likewise described the class of charts for which the upper bound is achieved.

Spurred by the above outcomes, in this section we get an upper bound for the total of the fluffy absolute domination number and chromatic number and portray the comparing extremely structures of fluffy diagrams of request up to $2n - 6$.

The accompanying preliminary outcomes and documentations are utilized in consequent portrayals:

Previous Results

Theorem I For any connected graph G , $\gamma_{ft}(G) \leq n$

Theorem II For any connected graph G , $\chi(G) \leq \Delta(G) + 1$

The following notations are used in the succeeding theorems.

Notation 1.1 $K_n(P_m)$ denotes the diagram got from K_n by attaching the end vertex of P_m to anybody vertices of K_n .

Example:

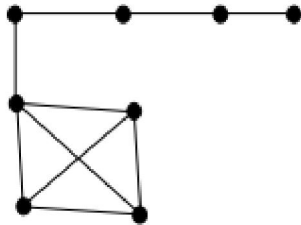


Figure 1.1 edges to any one vertex $K_4(P_5)$

Notation 1.2 $K_n(m_1, m_2, m_3, \dots, m_k)$ denotes the chart got from K_n by joining m_1 edges to any one vertex u_i of K_n , m_2 edges to any one vertex u_j for $i \neq j$ of K_n , m_3 edges to any one vertex u_k for $i \neq j \neq k$ of K_n , m_1, m_2, \dots, m_k edges to all the distinct vertices of K_n .

Notation 1.3 Let G be a connected fluffy chart with m vertices $v_1, v_2, v_3, \dots, v_m$. The graph $G(n_1P_{l_1}, n_2P_{l_2}, n_3P_{l_3}, \dots, n_mP_{l_m})$ where $n_i, l_i \geq 0$ and $1 \leq i \leq m$, is obtained from G by attaching n_1 times a pendant vertex of P_{l_1} on the vertex v_1 , n_2 times a pendant vertex of P_{l_2} on the vertex v_2 and so on.

Example: Let v_1, v_2, v_3, v_4 be the vertices of k_4 the graph $k_4(2P_2, P_3, P_4, P_3)$ is obtained from k_4 by connecting multiple times a pendant vertex of p_2 on v_1 , 1 time a pendant vertex of p_3 on v_2 , 1 time a pendant vertex of p_4 and v_3 and 1 time a pendant vertex of p_3 on v_4 .

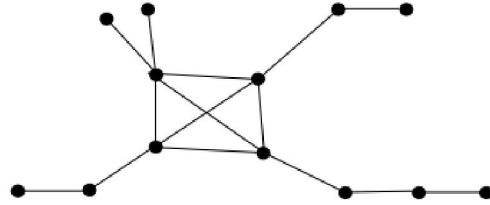


Figure 1.2 pendant vertex $K_4(2P_2, P_3, P_4, P_3)$

Notation 1.4 $C_3(P_n)$ is the graph obtained from C_3 by attaching the pendant edge of P_n to any one vertices of C_3 .

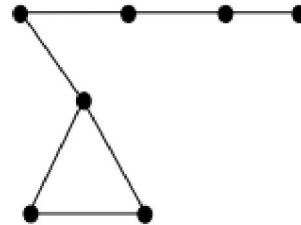
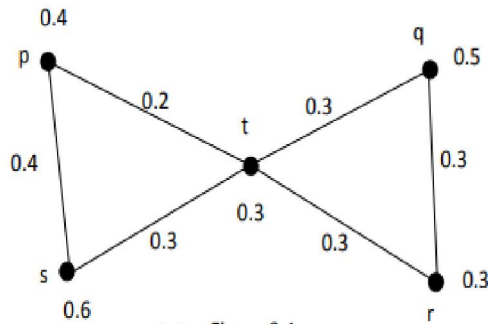


Figure 1.2 pendant vertex $C_3(P_5)$

Definition 1.5: A subset S of V is known as a dominating set in a fuzzy diagram $G(V, \sigma, \mu)$ if every vertex in $\langle V - S \rangle$ is successfully neighboring at least one vertex is S . The base cardinality assumed control over every ruling set in G is known as the mastery number of G and is signified by Y_f .

An overwhelming set is said to be all out ruling set if each vertex in V is viably neighboring at any rate one vertex in S . The base cardinality assumed control over all negligible complete commanding set is known as the absolute mastery number and is meant by Y_{ft}

Example:



for figure $\gamma_{ft}(G) = \{\bar{s}, t\} = 2$

Figure 1.4 hypotheses any place the logical inconsistency

Note: In all the accompanying hypotheses any place the logical inconsistency is happened there is no fluffy chart exists.

Main Results

Theorem 1.6 For any associated solid fluffy chart $\gamma_{ft}(G)+\chi(G)\leq 2n$ a's more, the correspondence holds if and as it were if $G\neq K_1$.

Proof: Let $\gamma_{ft}(G)+\chi(G)\leq n+\Delta+1=n+(n-1)+1\leq 2n$. If $\gamma_{ft}(G)+\chi(G)=2n$ Then the only possible case is $\gamma_{ft}(G)=n$ and $\chi(G)=n$, since $\chi(G)=n, G=K_n$, But for $K_n, \gamma_{ft}(G)=1$ so that $G\neq K_2$

Proof: Assume that $\gamma_{ft}(G)+\chi(G)=2n-1$ this is possible only if $\gamma_{ft}(G)=n$ and $\chi(G)=n-1$ (or) $\gamma_{ft}(G)=n-1$ and $\chi(G)=n$.

Case (i) Let $\gamma_{ft}(G)=n$ and $\chi(G)=n-1$.

Since $\chi(G)=n-1, G$ contains an inner circle K on $n-1$ vertices or doesn't contain a coterie K on $n-1$ vertices. Let G contains an inner circle K on $n-1$ vertices.

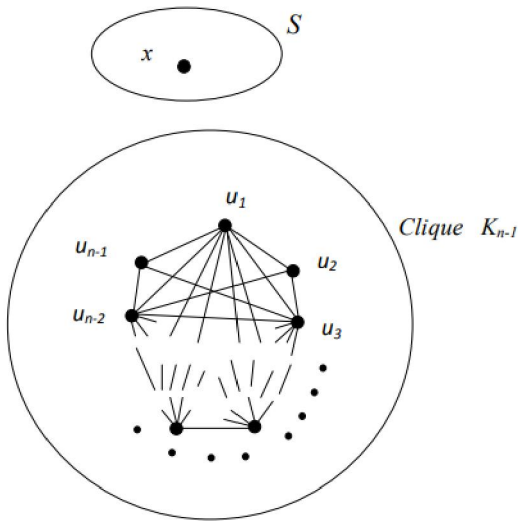


Figure 1.5 fluffy diagram

Let x be a vertex of $G-K_{n-1}$. Since G is associated, the vertex x is nearby one vertex u_i for some i in K_{n-1} . Then $\{u_i\}$ is a Y_{ft} - set so that $\gamma_{ft}(G)=1$ we have $n=1$. Then $x=0$ which is a contradiction.

In the event that G doesn't contain a club K on $n-1$ vertices, at that point it very well may be confirmed that no new fluffy diagram exists.

Since $\chi(G)=n, G=K_n$ But for $K_n, \gamma_{ft}(G)=1$, so that $n=2, x=1$. Hence $G\cong K_2$. Converse is obvious.

Theorem 1.7 For any associated solid fluffy diagram $G, \gamma_{ft}(G)+\chi(G)=2n-2$ if and only if $G\cong K_3$.

Proof: If G is K_3 , then clearly $\gamma_{ft}(G)+\chi(G)=2n-2$. Conversely assume that $\gamma_{ft}(G)+\chi(G)=2n-2$. This is possible only if $\gamma_{ft}(G)=n$ and $\chi(G)=n-2$ or $\gamma_{ft}(G)=n-1$ and $\chi(G)=n-1$ or $\gamma_{ft}(G)=n-2$ and $\chi(G)=n$

Case (i) Let $\gamma_{ft}(G)=n$ and $\chi(G)=n-2$.

Since $X(G)=n-2$ contains a faction K on $n-2$ vertices or doesn't contain a coterie K on $n-2$ vertices. Let G contains an inner circle K on $n-2$ vertices.

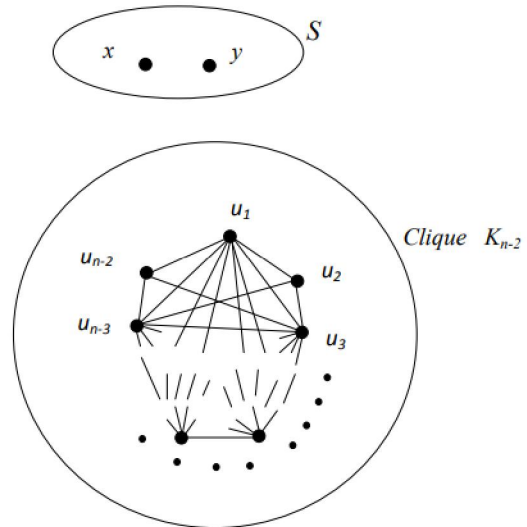


Figure 1.6 coterie K_{n-2} vertices

Let $S=\{x,y\}\in G-K_{n-2}$. Then $\langle S \rangle = K_2$ or K_2

Subcase (a) Let $\langle S \rangle = K_2$

Let x,y be the vertices of K_2 . Since G is associated, if x is adjoining a few u_i of K_{n-2} then $\{x,u_i\}$ is Y_{ft} - set, so that $\gamma_{ft}(G)=2$ and hence $n=2$. But $X(G)=n-2=0$ which is a contradiction.

Subcase (b) Let $\langle S \rangle = \overline{K_2}$.

Let x,y be the vertices of $\overline{K_2}$ Since G is associated, x is nearby a few u_i of K_{n-2} . Then y is adjacent to the same u_i of K_{n-2} . Then $\{u_i\}$ is Y_{ft} - set, so that $\gamma_{ft}(G)=1$ and hence $n=1$. But $X(G)=n-2=0$ which is a logical inconsistency. Leave y alone neighboring u_j of K_{n-2} for $i\neq j$. In this case $\{u_i,u_j\}$ is Y_{ft} -set, so that $\gamma_{ft}(G)=2$ and hence $n=1$. But $\chi(G)=n-2=0$, which is a contradiction.

On the off chance that G doesn't contain an inner circle K on $n-2$ vertices, at that point it very well may be checked that no new fluffy diagram exists.

Case (ii) Let $\gamma_{ft}(G)=n-2$ and $X(G)=n-1$.

Since $X(G)=n-1$ G contains a club K on $n-1$ vertices or doesn't contain a coterie on $n-1$ vertices. Let G contains an inner circle K on $n-1$ vertices.

Let x be a vertex of $G-K_{n-1}$. Since G is associated, x is nearby one vertex u_i for some i in K_{n-1} . Then $\{u_i\}$ is Y_{ft} - set, so that $\gamma_{ft}(G)=1$, we have $n=1$. Then $X=n-1=1$, which is for completely disengaged chart. This is a logical inconsistency.

In the event that G doesn't contain a clique K on $n-1$ vertices, at that point it very well may be confirmed that no new fluffy chart exists.

Case (iii) Let $\gamma_{ft}(G)=n-2$ and $\chi(G)=n$
 Since $\chi(G)=n$, $G=K_n$, But for K_n , $\gamma_{ft}(G)=1$; so
 that $n=3$, $x=3$. Hence $G \cong K_3$.

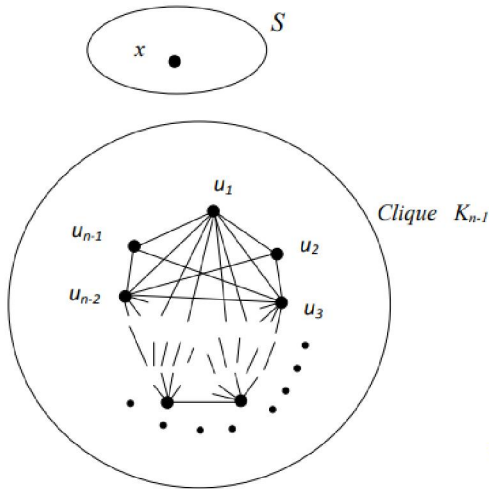


Figure 1.7 coterie on n-1 vertices

Theorem 1.8 For any connected strong fuzzy graph G, $\gamma_{ft}(G)+\chi(G)=2n-3$ if and only if $G \cong P_3, K_4$.

Proof: If $G \cong P_3, K_4$ then clearly $\gamma_{ft}(G)+\chi(G)=2n-3$ conversely assume that $\gamma_{ft}(G)+\chi(G)=2n-3$ this is possible only if $\gamma_{ft}(G)=n$ and $\chi(G)=n-3$ (or) $\gamma_{ft}(G)=n-1$ and $\chi(G)=n-2$ (or) $\gamma_{ft}(G)=n-2$ and $\chi(G)=n-1$ (or) $\gamma_{ft}(G)=n-3$ and $\chi(G)=n$.

Case (i) Let $\gamma_{ft}(G)=n$ and $\chi(G)=n-3$.
 Since $\chi(G)=n-3$, G contains a clique K on n-3 vertices or doesn't contain a clique K on n-3 vertices.
 Let G contains a clique K on n-3 vertices

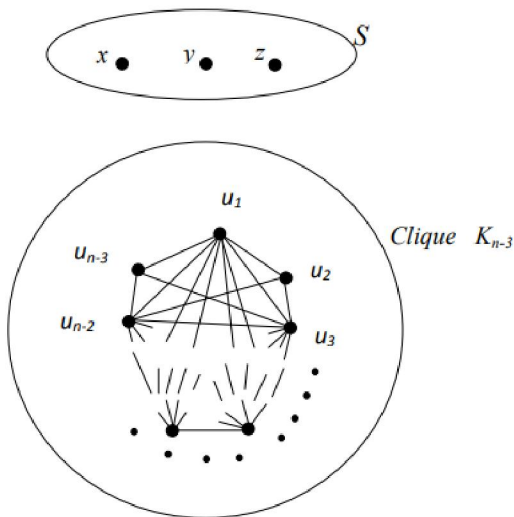


Figure 1.8 clique K on n-3 vertices

Let $S=\{x,y,z\} \in G-K_{n-3}$. Then $\langle S \rangle = K_3, K_3, K_2 \cup K_1, P_3$
Subcase (a) Let $\langle S \rangle = K_3$
 Let x,y,z be the vertices of K_3 . Since G is associated, x is nearby a few u_i of K_{n-3} . Then $\{x,u_i\}$ is Y_{ft} -set so that $Y_{ft}(G)=1$. And hence $n=1$. But $X(G)=n-3=$ negative worth which is a logical inconsistency.

Subcase (b) Let $\langle S \rangle = \overline{K_3}$

Let x,y,z be the vertices of $\overline{K_3}$. Since G is associated, one of the vertices of K_{n-3} say u_i is neighboring all the vertices of S (or) there exists u_i in K_{n-3} which is adjoining x and y and u_j is contiguous z (or) every vertices of S are neighboring diverse vertices of K_{n-3} .

If u_i for some i is adjoining all the vertices of S, at that point $\{u_i\}$ in K_{n-3} is a Y_{ft} -set of G, so that $Y_{ft}(G)=1$ and hence $n=1$. But $X(G)=n-3=$ negative worth, which is a logical inconsistency. Since G is associated u_i for some i is nearby two vertices of S say x and y and z is adjacent to u_j for $i \neq j$ in K_{n-3} , then $\{u_i, u_j\}$ in K_{n-3} is Y_{ft} -set of G, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)=n-3=$ negative value, which is a contradiction.

If u_i for some i is adjacent to x and u_j is adjacent to y and u_k is adjacent to z, then $\{u_i, u_j, u_k\}$ for $i \neq j \neq k$ in K_{n-3} is a Y_{ft} -set of G, so that $Y_{ft}(G)=3$ and hence $n=3$. But $X(G)=n-3=0$, which is a contradiction.

Subcase (c) Let $\langle S \rangle = K_2 \cup K_1$

Let xy be the edge of $K_2 \cup K_1$ and z is the isolated vertex of $K_2 \cup K_1$. Since G is connected, there exists a u_i in K_{n-3} is adjacent to x and z. Then $\{x, u_i\}$ is Y_{ft} -set of G, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)=n-3=$ negative value, which is a contradiction. If z is adjacent to u_j for some $i \neq j$ then $\{x, u_i, u_j\}$ for $i \neq j$ is Y_{ft} -set of G, so that $Y_{ft}(G)=3$ and hence $n=3$. But $X(G)=n-3=0$, which is a contradiction.

Subcase (d) Let $\langle S \rangle = P_3$

Let x, y, z be the vertices of P_3 . Since G is connected, x (or equivalently z) is adjacent to u_i for some i in K_{n-3} . Then $\{x, y, u_i\}$ is a Y_{ft} -set of G. so that $Y_{ft}(G)=3$ and hence $n=3$. But $X(G)=n-3=0$, which is a logical inconsistency. In the event that u_i is neighboring y, at that point $\{u_i, y\}$ is a Y_{ft} -set of G, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)=n-3=$ negative value, which is a contradiction.

Case (ii) let $\gamma_{ft}(G)=n-1$ and $\chi(G)=n-2$.

Since $X(G)=n-2$, G contains a clique K on n-2 vertices or G doesn't contain a clique on n-2 vertices.
 Let G contains a clique K on n-2 vertices.

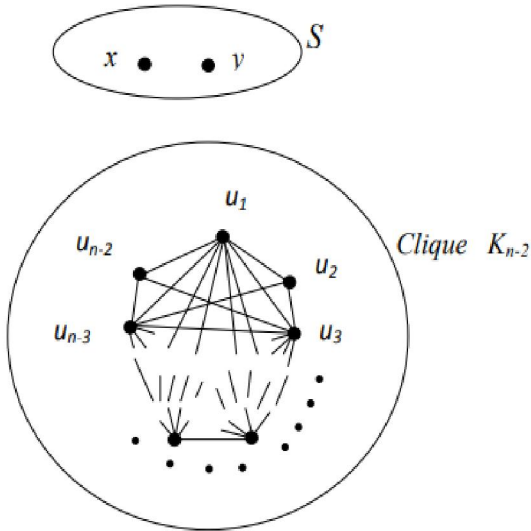


Figure 1.9 clique K on n-2 vertices

Case (iv) Let $\gamma_{ft}(G)=n-3$ and $\chi(G)=n$

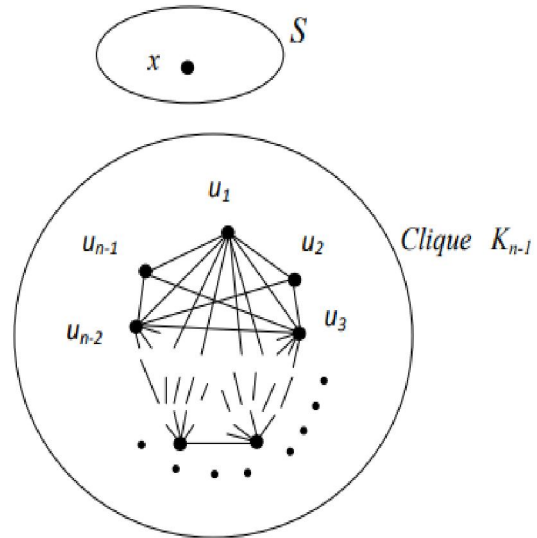


Figure 1.10 clique K on n-1 vertices

Let $S=\{x,y\} \in G-K_{n-2}$. Then $\langle S \rangle = K_2$ or $\overline{K_2}$

Subcase (a) Let $\langle S \rangle = K_2$

Let x,y be the vertices of K_2 . Since G is connected, x (or equivalently y) is adjacent to some u_i of K_{n-2} . Then $\{x,u_i\}$ is Y_{ft} - set, so that $Y_{ft}(G)=2$ and hence $n=3$. But $X(G)=n-2=1$ for which G is completely detached, which is a logical inconsistency.

Subcase (b) Let $\langle S \rangle = \overline{K_2}$

Let x,y be the vertices of $\overline{K_2}$. Since G is associated, x is contiguous a few u_i of K_{n-2} . Then y is adjacent to the same u_i of K_{n-2} . Then $\{u_i\}$ is Y_{ft} - set, so that $Y_{ft}(G)=1$ and hence $n=1$. But $X(G)=n-2=0$, which is a logical inconsistency. In any case x is contiguous u_i of K_{n-2} for some i and y is adjacent to u_j of K_{n-2} for $i \neq j$. In this $\{u_i, u_j\}$ is Y_{ft} - set, so that $Y_{ft}(G)=2$ and hence $n=3$. But $X(G)=1$ for which G is completely detached, which is a logical inconsistency.

On the off chance that G doesn't contain a clique K on n-2 vertices, at that point it very well may be checked that no new fluffy diagram exists.

Case (iii) Let $\gamma_{ft}(G)=n-2$ and $\chi(G)=n-1$

Since $X(G)=n-1$, G contains a clique K on n-1 vertices or doesn't contain a clique K on n-1 vertices. Let G contains a clique K on n-1 vertices.

Leave x alone a vertex of S. Since G is associated the vertex x is neighboring one vertex u_i for some i in K_{n-1} so that $Y_{ft}(G)=1$, we have $n=3$ and $x=3$. Then $K=K_2=uv$. If x is adjacent to u_i , then $G \cong P_3$.

On the off chance that G doesn't contain a clique K on n-1 vertices, at that point it very well may be confirmed that no new fluffy diagram exists.

Since $\chi(G)=n$, $G=K_n$, But for K_n , $\gamma_{ft}(G)=1$, so that $n=4$, $\chi=4$ Hence $G \cong K_4$.

Theorem 1.9 For any associated solid fluffy diagram G , $\gamma_{ft}(G)+\chi(G)=2n-4$ if and only if $G \cong P_4, K_5, K_3(P_2)$.

Proof: If $G \cong P_4, K_5, K_3(P_2)$, then clearly $\gamma_{ft}(G)+\chi(G)=2n-4$. conversely assume that $\gamma_{ft}(G)+\chi(G)=2n-4$. this is possible only if $\gamma_{ft}(G)=n$ and $\chi(G)=n-4$ (or) $\gamma_{ft}(G)=n-1$ and $\chi(G)=n-3$ (or) $\gamma_{ft}(G)=n-2$ and $\chi(G)=n-2$ (or) $\gamma_{ft}(G)=n-3$ and $\chi(G)=n-1$ (or) $\gamma_{ft}(G)=n-4$ and $\chi(G)=n$.

Case (i) Let $\gamma_{ft}(G)=n$ and $\chi(G)=n-4$.

Since $X(G)=n-4$, G contains a clique K on n-4 vertices or doesn't contain a clique K on n-4 vertices. Let G contains a clique K on n-4 vertices.

Let $S = \{v_1, v_2, v_3, v_4\} \in G-K_{n-4}$. Then the induced sub graph $\langle S \rangle$ has the following possible cases

$K_4, \overline{K_4}, K_4-\{e\}, K_3UK_1, K_2UK_2, K_2U\overline{K_2}, K_{1,3}, P_4, P_3UK_1, C_4, C_3(1,0,0)$.

Subcase (a) Let $\langle S \rangle = K_4$

Let be v_1, v_2, v_3, v_4 the vertices of K_4 . Since G is connected, without loss of generality v_1 is adjacent to some u_i of K_{n-4} . Then $\{v_1, u_i\}$ is Y_{ft} - set, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)=n-4$ =negative value, which is a contradiction.

Subcase (b) Let $\langle S \rangle = \overline{K_4}$

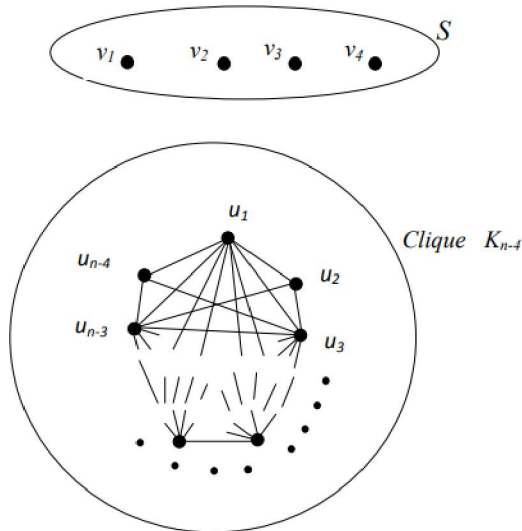


Figure 1.11 clique K on n-4 vertices

Let v_1, v_2, v_3, v_4 be the vertices of $\overline{K_4}$. Since G is associated, one of the vertices of K_{n-4} say u_i is adjacent to all the vertices of S. Then $\{u_i\}$ is Y_{ft} -set, so that $Y_{ft}(G)=1$ and hence $n=1$. But $X(G)=-n-4$ =negative worth, which is a logical inconsistency.

Since G is associated. One of the vertices of K_{n-4} say u_i is adjacent to three vertices of S and u_j for some $i \neq j$ of K_{n-4} K is neighboring fourth vertex of S. Right now $\{u_i, u_j\}$ is Y_{ft} -set, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)$ =negative value, which is a contradiction.

If two vertices say v_1, v_2 are nearby the vertex u_i and the staying two vertices are contiguous u_j . In this case $\{u_i, u_j\}$ is Y_{ft} -set, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)$ =negative worth, which is a logical inconsistency. On the off chance that two vertices state v_1 and v_2 are adjoining the vertex u_i and in the staying two vertices, one vertex is nearby u_i for $i \neq j$ and another one is adjacent to u_k for $i \neq j \neq k$. In this $\{u_i, u_j, u_k\}$ is Y_{ft} -set, so that $Y_{ft}(G)=3$ and hence $n=3$. But $X(G)$ =negative value, which is a contradiction.

Let the four vertices of $\overline{K_4}$ are adjacent to distinct vertices of K_{n-4} . In this $\{u_i, u_j, u_k, u_l\}$ is Y_{ft} -set, so that $Y_{ft}(G)=4$ and hence $n=4$. But $X(G)=0$, which is a contradiction.

Subcase (c) Let $\langle S \rangle = K_4 - \{e\}$

Let v_1, v_2, v_3, v_4 be the vertices of $K_4 - \{e\}$. Leave e alone any of the edges in the cycle C_4 . Since G is associated, without loss of sweeping statement V_4 is adjacent to some u_i of K_{n-4} . Then $\{v_4, u_i\}$ is Y_{ft} -set,

so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)=-n-4$ =negative worth, which is a logical inconsistency. Leave e alone any of the edges inside the cycle C_4 Since G is associated, without loss of all inclusive statement V_4 is adjacent to some u_i of K_{n-4} . Then $\{v_4, u_i\}$ is Y_{ft} -set, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)=-n-4$ =negative worth, which is an inconsistency.

Since G is associated, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 and another vertex u_j for some $i \neq j$ is adjacent to v_4 . Then $\{v_1, u_i, u_j\}$ is Y_{ft} -set of G, so that $Y_{ft}(G)=3$ and hence $n=3$. But $X(G)=-n-4$ =negative value, which is a contradiction

Subcase (e) Let $\langle S \rangle = K_2 \cup K_2$

Let v_1, v_2 be the vertices of K_2 and v_3 and v_4 be the vertices of another K_2 . Since G is connected, there exists a u_i in K_{n-4} is adjacent to v_1 and v_3 . Then $\{v_1, v_3, u_i\}$ is Y_{ft} -set of G, so that $Y_{ft}(G)=3$ and hence $n=3$. But $X(G)=-n-4$ =negative value, which is a contradiction. If v_3 is adjacent to u_j for some $i \neq j$ then $\{v_1, v_3, u_i, u_j\}$ for $i \neq j$ is Y_{ft} -set of G, so that $Y_{ft}(G)=4$ and hence $n=4$. But $X(G)=-n-4=0$, which is a contradiction.

Subcase (f) Let $\langle S \rangle = K_2 \cup \overline{K_2}$

Let v_1, v_2 be the vertices of K_2 and v_3 and v_4 be the vertices of $\overline{K_2}$. Since G is connected, there exists a u_i in K_{n-4} is adjacent to v_1, v_3 and v_4 . Then $\{v_1, u_i\}$ is Y_{ft} -set of G, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)=-n-4$ =negative value, which is a contradiction.

Since G is connected there exists a u_i in K_{n-4} is adjacent to v_1, v_3 and another vertex of K_{n-4} u_j for some $i \neq j$ is adjacent to v_4 , then $\{v_1, u_i, u_j\}$ for $i \neq j$ is Y_{ft} -set of G, so that $Y_{ft}(G)=3$ and hence $n=3$. But $X(G)=-n-4$ =negative value, which is a contradiction.

If v_1 is adjacent to u_i and v_3 is adjacent to u_j for $i \neq j$ and another one vertex v_4 is adjacent to u_k for $i \neq j \neq k$. In this $\{v_1, u_i, u_j, u_k\}$ is Y_{ft} -set, so that $Y_{ft}(G)=4$ and hence $n=4$. But $X(G)=0$, which is a contradiction.

Subcase (g) Let $\langle S \rangle = K_{1,3}$

Let v_1, v_2, v_3, v_4 be the vertices of $K_{1,3}$. Without loss of consensus let us expect that v_1 is a root vertex. Since G is associated, v_1 is adjacent to some u_i of K_{n-4} . Then $\{v_1, u_i\}$ is Y_{ft} -set, so that $Y_{ft}(G)=2$ and hence $n=1$. But $X(G)=-n-4$ =negative worth, which is an inconsistency. In any case let u_i be contiguous any of the swinging vertices state v_2 In this $\{v_1, v_2, u_i\}$ is Y_{ft} -set, so that $Y_{ft}(G)=3$ and hence $n=3$. But $X(G)=-n-4$ =negative worth, which is an inconsistency.

Subcase (h) Let $\langle S \rangle = P_4$

Let v_1, v_2, v_3, v_4 be the vertices of P_4 . Since G is connected, v_1 (or equivalently v_4) is adjacent to u_i for some $i \in K_{n-4}$. Then $\{v_1, v_2, v_3, u_i\}$ is Y_{ft} -set of G , so that $Y_{ft}(G) = 4$ and hence $n = 4$. But $X(G) = n - 4 = 0$, which is a contradiction. If u_i is adjacent to v_2 (or equivalently v_3) then $\{u_i, v_2, v_3\}$ is Y_{ft} -set of G , so that $Y_{ft}(G) = 3$ and hence $n = 3$. But $X(G) = n - 4 = \text{negative value}$, which is a contradiction.

Subcase (i) Let $\langle S \rangle = P_3 \cup K_1$

Let v_1, v_2, v_3 be the vertices of P_3 and P_4 be the isolated vertex of $P_3 \cup K_1$. Since G is connected, there exists a u_i in K_{n-4} is adjacent to v_1 (or equivalently v_3) and v_4 . Then $\{v_1, v_2, u_i\}$ is Y_{ft} -set of G , so that $Y_{ft}(G) = 3$ and hence $n = 3$. But $X(G) = n - 4 = \text{negative value}$, which is a contradiction. If there exists a vertex u_i in K_{n-4} is adjacent to v_2 and v_4 . Then $\{v_2, u_i\}$ is Y_{ft} -set of G , so that $Y_{ft}(G) = 2$ and hence $n = 1$. But $Y_{ft}(G) = n - 4 = \text{negative value}$, which is a contradiction.

Since G is associated, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 (or equivalently v_3) and another vertex u_j for some $i \neq j$ is adjacent to v_4 . Then $\{v_1, v_2, u_i, u_j\}$ is Y_{ft} -set of G , so that $Y_{ft}(G) = 4$ and hence $n = 4$. But $X(G) = n - 4 = 0$, which is a contradiction. If there exists a vertex u_i in K_{n-4} is adjacent to v_2 and u_j for some $i \neq j$ is adjacent to v_4 . Then $\{v_2, u_i, u_j\}$ is Y_{ft} -set of G , so that $Y_{ft}(G) = 3$ and hence $n = 3$. But $X(G) = n - 4 = \text{negative value}$, which is a contradiction.

Subcase (j) Let $\langle S \rangle = C_4$

Let v_1, v_2, v_3, v_4 be the vertices of C_4 . Since G is connected, v_1 is adjacent to some u_i of K_{n-4} . Then $\{v_1, v_2, u_i\}$ is Y_{ft} -set, so that $Y_{ft}(G) = 3$ and hence $n = 3$. But $X(G) = n - 4 = \text{negative value}$, which is a contradiction.

Subcase (k) Let $\langle S \rangle = C_3(1,0,0)$

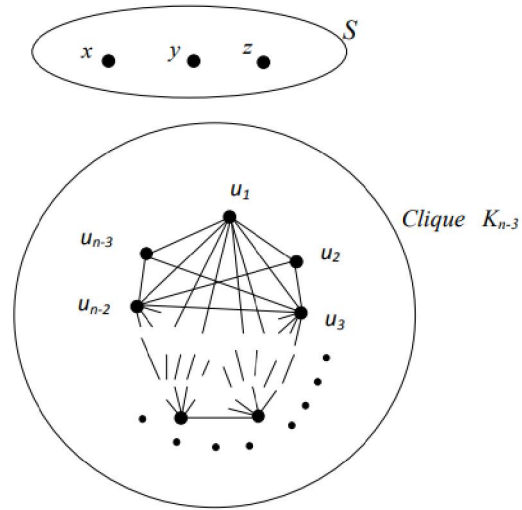
Let v_1, v_2, v_3, v_4 be the vertices of $C_3(1,0,0)$. Since G is connected, v_1 (equivalently v_2) is adjacent to some u_i of K_{n-4} . Then $\{v_1, v_3, u_i\}$ is Y_{ft} -set, so that $Y_{ft}(G) = 3$ and hence $n = 3$. But $X(G) = n - 4 = \text{negative value}$, which is a contradiction.

Since G is associated, v_3 is neighboring a few u_i of K_{n-4} . Then $\{v_3, u_i\}$ is Y_{ft} -set, so that $Y_{ft}(G) = 2$ and hence $n = 1$. But $X(G) = n - 4 = \text{negative value}$, which is a contradiction.

On the off chance that G doesn't contain a clique K on $n - 4$ vertices, at that point it very well may be checked that no new fluffy chart exists

Case (ii) Let $Y_{ft}(G) = n - 1$ and $\chi(G) = n - 3$.

Since $X(G) = n - 3$, G contains a clique K on $n - 3$ vertices or doesn't contain a clique K on $n - 3$ vertices. Let G contains a clique K on $n - 3$ vertices.



Let

$S = \{x, y, z\} \in G - K_{n-3}$. Then $\langle S \rangle = K_3, K_3, K_2 \cup K_1, P_3$

Subcase (a) Let $\langle S \rangle = K_3$

Let x, y, z be the vertices of K_3 . Since G is associated, x is nearby a few u_i of K_{n-3} . Then $\{x, u_i\}$ is Y_{ft} -set, so that $Y_{ft}(G) = 2$ and hence $n = 3$. But $X(G) = n - 3 = 0$, which is a contradiction.

Subcase (b) Let $\langle S \rangle = K_3$

Let x, y, z be the vertices of. Since G is associated, one of the vertices of K_{n-3} say u_i is neighboring all the vertices of S (or) there exists u_i in K_{n-3} which is adjoining x and y and u_j is adjacent to z (or) each vertices of S are adjacent to different vertices of K_{n-3} .

If u_i for some i is nearby all the vertices of S , at that point $\{u_i\}$ in K_{n-3} is a Y_{ft} -set of G , so that $Y_{ft}(G) = 1$ and hence $n = 1$. But $X(G) = n - 3 = \text{negative worth}$, which is a logical inconsistency. Since G is associated u_i for some i is contiguous two vertices of S state x and y and z is adjacent to u_j for $i \neq j$ in K_{n-3} , then $\{u_i, u_j\}$ in K_{n-3} is Y_{ft} -set of G , so that $Y_{ft}(G) = 2$ and hence $n = 3$. But $X(G) = n - 3 = 0$, which is a contradiction. If u_i for some i is adjacent to x and u_i is nearby y and u_k is adjoining z , at that point $\{u_i, u_j, u_k\}$ for $i \neq j \neq k$ in K_{n-3} is a Y_{ft} for $i \neq j \neq k$ in K_{n-3} is a Y_{ft} -set of G , so that $Y_{ft}(G) = 3$ and hence $n = 4$. But $X(G) = n - 3 = 1$, which is for completely disengaged chart, which is a logical inconsistency.

Subcase (c) Let $\langle S \rangle = K_2 \cup K_1$

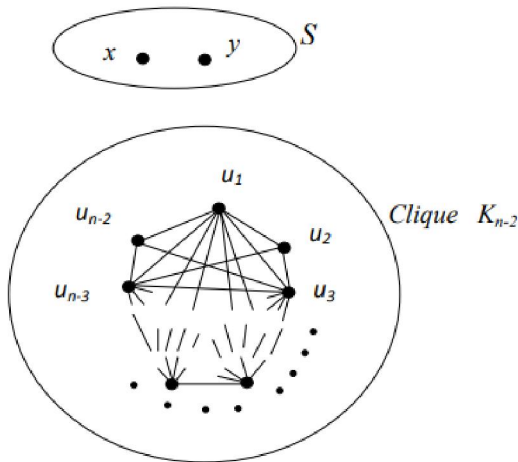
Let xy be the edge of $K_2 \cup K_1$ and z be the isolated vertex of $K_2 \cup K_1$. Since G is connected, there exists a u_i in K_{n-3} is adjacent to x and z . Then $\{x, u_i\}$ is Y_{ft} -set of G , so that $Y_{ft}(G)=2$ and hence $n=3$. But $X(G)=n-3=0$, which is a contradiction. If z is adjacent to u_j for some $i \neq j$ then $\{x, u_i, u_j\}$ for $i \neq j$ is Y_{ft} -set of G , so that $Y_{ft}(G)=3$ and hence $n=4$. But $X(G)=n-3=1$, which is for completely detached diagram, which is a logical inconsistency.

Subcase (d) Let $\langle S \rangle = P_3$

Let x, y, z be the vertices of P_3 . Since G is associated, x (or identically z) is contiguous to u_i for some i in K_{n-3} . Then $\{x, y, u_i\}$ is a Y_{ft} -set of G so that $Y_{ft}(G)=3$ and hence $n=4$. But $X(G)=n-3=1$, which is for completely detached chart, which is a logical inconsistency. If u_i is adjacent to y then $\{u_i, y\}$ is a Y_{ft} -set of G . so that $Y_{ft}(G)=2$ and hence $n=3$. But $X(G)=n-3=0$, which is an inconsistency.

Case (iii) Let $(G)=n-2$ and $\chi(G)=n-2$.

Since $X(G)=n-2$, G contains a clique K on $n-2$ vertices or G does not contain a clique on $n-2$ vertices. Let G contains a clique K on $n-2$ vertices.



Let $S = \{x, y\} \in G - K_{n-2}$. Then $\langle S \rangle = K_2$ or $\overline{K_2}$

Corresponding author:

Mr. Sonu Kumar
 Research Scholar, Department of Mathematics,
 OPJS University, Churu,
 Rajasthan (India)
 Contact No. +91-9812227126
 Email- sonukumar1402@gmail.com

References:

1. Amirthavalli. M., *Fuzzy Logic and Neural Networks*, SCITECH Publications private Ltd, India.
2. Aravamudhan. B. and Rajendran, B., "On Antipodal Graphs", Disc Math 49 (1984), 193-195.
3. Balakrishnan. R., Ranganathan. K., *A Text Book of Graph Theory*, Springer, 1991.
4. Balakrishnan. V.K., *Graph Theory*, Schaum's Outlines Tata McGraw – Hill Edition, 2004.
5. Bela Bollobas, *Modern Graph Theory*, Springer International Edition 1998.
6. Bezdek. C., and Pal. S.K., *Fuzzy Models for Pattern Recognition*, IEEE Press, 1991.
7. Bhattacharya. P., "Some Remarks on fuzzy graphs", Pattern Recognition Lett6: (1987) 297-301.
8. Bhutani. K.R., "On Automorphism of Fuzzy graphs", Pattern Recognition Lett 9: (1989) 159-161.
9. Bhutani. K.R., and Azriel Rosenfeld, "Strong Arcs in Fuzzy Graphs", Information Science 152(2003)319-321.
10. Bhutani. K.R., "On M-Strong Fuzzy graphs", Information Sciences, 155(2):103–109, 2003.
11. Bondy. J.A. and Murty, U.S.R., *Graph Theory with Applications*, Macmillan London, 1976.
12. Brigham. C. and Dutton. D., J. Combinatorics, Inf & Syst Science 12(1987) 75-85.
13. Buckley. F. and Harary. F., *Distances in Graphs*, Addison-Wesley, 1990.
14. Choudam. S.A., *A first course in Graph Theory*, Macmillan India Ltd., 2006.
15. Craine. W.L., "Characterization of Fuzzy Interval Graphs", Fuzzy Sets and Systems 68: 183, 1994.
16. Douglas B. West, *Introduction to Graph Theory*, Prentice Hall of India, Private Ltd., New Delhi, 1999.
17. Foulds. L.R., *Graph Theory Applications* Narosa Publishing House, 1993.
18. Frank Harary, *Graph Theory*, Narosa / Addison Wesley, Indian Student Edition, 1988.
19. Garry Johns and Karen Sleno, "Antipodal graphs and Digraphs" International Journal Math and Math Sci. vol. 16, No. 3 (1993) 579-586.
20. Gary Chartrand, Ping Zhang, *Introduction to Graph Theory*, Tata McGraw–Hill Publishing Company Ltd., New Delhi, 2006.
21. George J. Klir, and Bo Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
22. Jin AKIYAMA, Kiyoshi ANDO, David AVIS, "Eccentric Graphs", Discrete Mathematics 56(1985)1-6.

23. Kaufmann. A., *Introduction to the theory of Fuzzy Subsets*, Vol. 1, Academic Press, New York 1975.
24. Kim. J.B., *Determinant Theory for fuzzy matrices*, Fuzzy Sets and systems 29: 349-356(1989).
25. Kulli. V.R. and Janakiraman. B., "*The common minimal Dominating Graph*", J. Pure Appl. Math., 27(2):193-196, February 1996.
26. Lootsma. F.A., Fuzzy Logic for planning and decision making, Kluwer, 1997.
27. Luo. C.S., "*The theorems of Decomposition and Representation for Fuzzy Graphs*", Fuzzy Sets and Systems 42:237-243, 1991.

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