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Study On Parameters Related To Linear Equation-Solving

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Abstract: Proportionality was the main concept applied by ancient scholars when solving linear equations, particularly in the method of false position. Variations on the method of false position were employed for more complex linear equations. One such method is that of "double false position." This method will give solutions for problems that can be represented by the (modern) equation Mx + B = N, and solutions for problems that can be represented by systems of two equations and two unknowns. This method was so effective, that mathematicians continued to use it even after the advent of the algebraic notation that provided the means to efficiently write equations (Berlinghoff & Gouvea, 2004). In addition, one of the current benefits to using the method of "double false position" is that many students have trouble writing an algebraic equation from a word problem. However, most can substitute values in to see if they work. This method ties in to what is currently called "guess and check," which can be an intermediate step in going from a word problem to an equation.
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Introduction

Humans have been solving linear equations for centuries. Linear equations arise naturally when applying mathematics to the real world (Berlinghoff & Gouvea, 2004). This guess was usually some convenient value to work with and need not be anywhere near the correct solution. He would then determine the result yielded by his guess. If he did not guess the correct solution, he would calculate the ratio he would need to multiply his incorrect result by in order to attain the correct result. He would then multiply the original guess by that ratio.

Proportional reasoning played a key role in the method of false position. Problem 26 from the *Rhind Papyrus* illustrates this idea well. "Find a quantity such that when it is added to one quarter of itself, the result is 15." The solution using typical modern algorithms would begin by defining a variable to represent the unknown quantity. A common choice for this variable is *x*. Then, the problem may be represented algebraically by the equation

$$x+\frac{1}{4}x=15\ .$$

So, by combining like terms, the equation becomes

$$\frac{5}{4}x=15$$

Multiplying both sides of the equation by four fifths yields

$$x=15\cdot\frac{4}{5}$$

Thus, the unknown quantity is 12.

Compare this with the solution using the method of "false position." Make a convenient guess. A convenient guess for this example would be some multiple of 4. Let the guess, G, be 16. Calculate the result using this guess in the problem statement: When the quantity of 16 is added to one quarter of itself (i.e. 4) the result is 20. The symbolic representation of this statement is

$$16+\frac{1}{4}(16)=16+4=20$$

The proportion by which this result should be multiplied in order to get the correct solution of 15 is fifteen twentieths:

$$20\cdot\frac{15}{20}=15\ .$$

Now multiply the original guess by this proportion:

$$16\cdot\frac{15}{20}=12\ .$$

Thus, the unknown quantity is 12.

In general, the algorithm for the solution to a linear equation using false position can be demonstrated as follows. Note that this method works only if the variables directly related to the result. Therefore, for illustrative purposes, first let the word problem be represented by the linear equation

$Mx = N.$

Remember that this algebraic shorthand would not have been employed at the time.

1. Make a guess, G. Typically, M could be represented by a ratio and this guess would be a multiple of the denominator of M. However, any guess will do.
2. Calculate the result with the guess: M-G
3. If MG is not equal to the desired result N, then G is not the correct solution. The proportion by which MG should be multiplied to achieve the N desired result N is given by $\frac{N}{MG}$ Indeed, we see

$$MG \cdot \frac{N}{MG} = N \text{ .}$$

4. Multiply the original guess by this proportion to find the correct solution:

$$x = G \cdot \frac{N}{MG} = \frac{N}{M} \text{ .}$$

Using modern notation, it is clear that this is the correct solution to the equation

$Mx = N.$

The calculations provided above should indicate why this method will work for all linear equations where there is a direct proportional relationship between the input and the output.

We can illustrate the underlying concept of proportions geometrically with similar triangles.

The *Rhind Papyrus* was written by the scribe Ahmes in approximately 1650 BC. This document gives evidence of Ancient Egyptian linear word problems and their solutions.

The solutions to these problems are not derived in a manner that most mathematics students would recognize today. The following algorithm is often taught in a one year algebra course:

1) label a variable

2) write an equation

3) perform the "order of operations" in reverse in order to isolate the variable. However, in Ancient Egypt, scribes used the method of "false position." First, the scribe would "posit" (guess) a possible solution to the word problem.

Review of literature:

Boyer's (1968) A History of Mathematics is almost entirely about Greek mathematics. It covers ancient Greek mathematics to a degree that none of the other mentioned texts do. Perhaps one of the most valuable tools for a secondary teacher available is Historical Topics for the Mathematics Classroom (National Council for Teachers of Mathematics, 1989).

This text consists of a series of "capsules" (short chapters). Each capsule gives a brief historical overview of a particular topic (e.g. Napier's Rods). The capsules are grouped by general topic (algebra, geometry, trigonometry, etc.). Specifically, this text provides a historical context to graphical approaches to equation solving. In addition, it provides a concise overview of the methods employed to solve quadratics and cubics.

Various researchers (Vaiyavutjamai & Clements, 2006) have illustrated that very little attention has been paid to quadratic equations in mathematics education literature, and there is scarce research regarding the teaching and learning of quadratic equations.

A limited number of research studies focusing on quadratic equations have documented the techniques students engage in while solving quadratic equations (Bossé & Nandakumar, 2005), geometric approaches used by students for solving quadratic equations (Allaire & Bradley, 2001), students' understanding of and difficulties with solving quadratic equations (Kotsopoulos, 2007; Lima, 2008; Tall, Lima, & Healy, 2014; Vaiyavutjamai, Ellerton, & Clements, 2005; Zakaria & Maat, 2010), the teaching and learning of quadratic equations in classrooms (Olteanu & Holmqvist, 2012; Vaiyavutjamai & Clements, 2006), comparing how quadratic equations are handled in mathematics textbooks in different countries (Saglam & Alacaci, 2012), and the application of the history of quadratic equations in teacher preparation programs to highlight prospective teachers' knowledge (Clark, 2012).

In general, for most students, quadratic equations create challenges in various ways such as difficulties in algebraic procedures, (particularly in factoring quadratic equations), and an inability to apply meaning to the quadratics. Kotsopoulos (2007) suggests that recalling main multiplication facts directly influences a student's ability while engaged in factoring quadratics. Furthermore, since solving the quadratic equations by factorization requires students to find factors rapidly, factoring simple quadratics becomes quite a challenge, while non-simple ones (i.e., ax2 + bx + c where a ^ 1) become harder still. Factoring quadratics can be considerably complicated when the leading coefficient or the constant term has many pairs of factors (Bossé & Nandakumar, 2005).

The research of Filloy & Rojano (1989) suggested that an equation such as with an expression on the left and a number on the right is much easier to solve symbolically than an equation such as. This is because the first can be ‘undone’ arithmetically by reversing the operation ‘multiply by 3 and subtract 1 to get 5' by 'adding 1 to 5 to get and then dividing 6 by 3 to get the solution.

Meanwhile the equation cannot be solved by arithmetic undoing and requires algebraic operations to be performed to simplify the equation to give a solution. This phenomenon is called 'the didactic cut'. It relates to the observation that many students see the 'equals' sign as an operation, arising out of experience in arithmetic where an equation of the form is seen as a dynamic operation to perform the calculation, 'three plus four makes 7', so that an equation such as is seen as an operation which may possibly be solved by arithmetic 'undoing' rather than requiring algebraic manipulation (Kieran, 1981).

Linear Equation – Solving:

Consider the line $y=Mx$. We are looking for the x coordinate corresponding to the y-coordinate N. We guess an x-coordinate and find the corresponding y coordinate on the line. The point (G,MG) lies on the line $y = Mx$. Thus, it becomes possible to create similar right triangles, using the proportionality of corresponding legs to obtain

$$\frac{G}{MG} = \frac{x}{N}.$$

So x must be given by

$$G \cdot \frac{N}{MG}$$

The idea of applying ratios to solve mathematical problems was not unique to the ancient Egyptians. Chinese scholars produced the text *The Nine Chapters on the Mathematical Art.* This text was edited by Liu Hui in 236

A.D., though the time of its origin is still in question (Berlinghoff & Gouvea, 2004). It seems to have originated sometime between 1100 B.C. and 100 B.C. "Proportionality seems to have been a central idea for these early Chinese mathematicians, both in geometry (e.g. similar triangles) and in algebra (e.g. solving problems by using proportions)" (Berlinghoff & Gouvea, 2004). The original text contains problems and solutions, as did the *Rhind Papyrus,* but Liu Hui added commentary and justifications for the solutions.

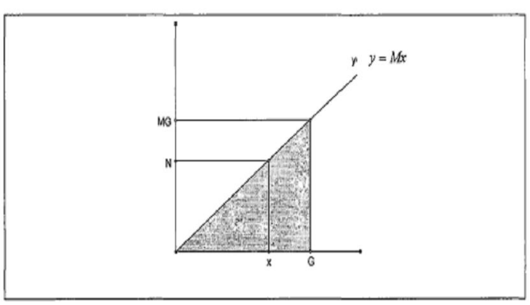

Figure 1. False Position Using Similar Triangles.

The method of double false positions follows a format similar to that of false position, however, the method requires two guesses. Given a problem that can be represented in the form $Mx + B = N$. Begin by making a guess, Gx, for the solution. Calculate the result and compare it to N. If it is not the correct solution, calculate the magnitude of the difference, E_1, between the result and N.1 Now, make a second guess, G2. If it is not the correct solution, calculate the magnitude of difference, between the result and N. In order to find the correct solution, use both guesses and errors in the following way:

1 Mathematicians did not generally acknowledge the use of negative numbers until the 17th century AD, hence only positive errors would be considered.

If both guesses yield either underestimates (less than the result desired) or overestimates (greater than the result desired), then the formula to find the solution is given by:

$$x = \frac{E_1 \cdot G_2 - E_2 \cdot G_1}{E_1 - E_2}.$$

If one guess yields an underestimate (less than the result desired) and the other guess yields an overestimate, (greater than the result desired), then the formula to find the solution is given by:

$$x = \frac{E_1 \cdot G_2 + E_2 \cdot G_1}{E_1 + E_2}.$$

The latter formula is used as a means to avoid dealing with negative numbers (Berlinghoff, 2005, p 123).

It is possible to display these general solutions geometrically with similar triangles. In the figure below, the correct solution, x, yields the correct result in the linear relationship y=mx+b. Both guesses are overestimates (and could similarly have been underestimates).

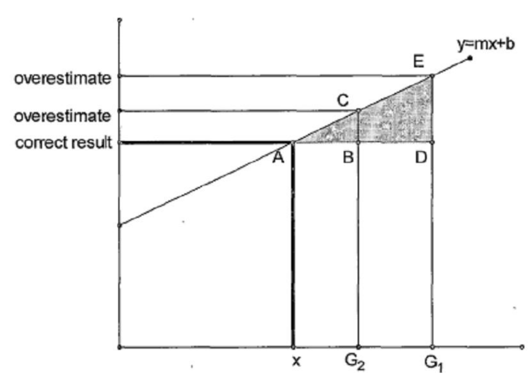

Figure 2. Double False Position Using Similar Triangles (Different Types).

Triangles ABC and ADE are similar because all of their angles are congruent. Each of the following values can be derived from this diagram:

DE = the difference between the correct result and the
first guess = error 1= Ex
BC = the difference between the correct result and the
second guess = error 2= E_2
AD = Q-x
AB = G_2-x
Thus, the proportion

$$\frac{E_1}{G_1 - x} = \frac{E_2}{G_2 - x}$$

can be created.

Simplification of this proportion yields the familiar equation

$$x = \frac{E_2G_1 - E_1G_2}{E_1 - E_2}$$

In Figure 3 the correct solution, x, yields the correct result in the linear relationship y=mx+b. The first guess is an overestimate, while the second is an underestimate.

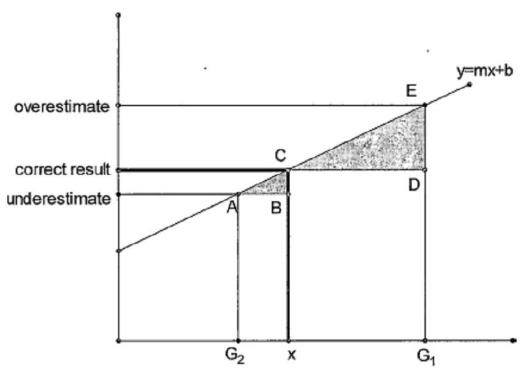

Figure 3. Double False Position Using Similar Triangles

(Same Types)

Triangles ABC and CDE are similar because all of their angles are congruent. Each of the following values can be derived from this diagram:

DE = the difference between the correct result and the
first guess = error 1 = £j
BC = the difference between the correct result and the
second guess = error $2 = E_2$
BD = G_1-x
AB = x-G_2
Thus,

$$\frac{E_2}{x - G_2} = \frac{E_1}{G_1 - x}.$$

Simplification of this proportion yields

$$x = \frac{E_2G_1 + E_1G_2}{E_1 + E_2}.$$

Analytic geometry provides another way of interpreting this solution algorithm in modern terms. If these pieces of data were viewed as points, the correct solution could be found using the concept of slope. Let (x_1y_1) be the first guess and its result. Let (x_2, y_2) be the second guess and its result. Finally, let (x,y) be the correct solution and its result. These three points are collinear, as the solutions are found by performing the same linear operations on each of the guesses. In particular, they each lie on the line with slope M and y-intercept B. Since the first guess (x_1y_1) and the correct solution (x,y) lie on the same line,

$$M = \frac{y - y_1}{x - x_1}.$$

Since the second guess (x_2,y_2) and the correct solution (x,y) lie on the same line,

$$M = \frac{y - y_2}{x - x_2}.$$

Since each of these ratios is equal to the same constant, then

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2}.$$

This equation can be simplified in an effort to solve for x, the correct solution.

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2}$$

Multiplying both sides by (x-x_2) (x-x_1) yields
$(y-y_1)(x-x_2) = (y-y_2)(x-x_1)$.
Distributing gives
$x(y-y_1)-x_2(y-y_1)=x(y-y_2)-x_1(y-y_2)$.
By regrouping the terms it follows that
$x_1(y-y_2)-x_2(y-y_1) = x(y-y_2)-x(y-y_1)$.
Isolating x provides

$$\frac{x_1(y-y_2)-x_2(y-y_1)}{((y-y_2)-(y-y_1))} = x$$

Finally, rewrite the equation to find

$$\frac{(y-y_1)x_2-(y-y_2)x_1}{((y-y_1)-(y-y_2))} = x.$$

In terms of the original guesses and their errors, the final result can be represented as follows

$$x = \frac{E_1 \cdot G_2 - E_2 \cdot G_1}{E_1 - E_2}.$$

The ancient scholars that developed this method used the concept of proportionality to derive their solutions, since in linear equations the change in the output is proportional to the change in the input (Berlinghoff & Gouvea, 2004, pl24). This method is similar to the method of "surplus and deficiency" found in the ancient Chinese texts. However, this Chinese method employed the use of one overestimate (surplus) and one underestimate (deficiency). Babylonian sources illustrate another variation of false position in the solution of linear equations. In some ways the method provides a connection between "false position" and "double false position." The method involves making one guess (as in false position), but also calculating the result if the guess were increased by one unit (tin essence, making a second fixed guess). In this Babylonian variation of false position the solver makes a guess, finds the result, and then calculates the error. Then, the guess is increased by one unit, and the difference in the amount of error is observed. Finally, the proportion by which the change in the error would need to be multiplied in order to decrease the original error to zero is calculated. The solution is now obtained by adding this proportion to the original guess.

Figure *4* depicts a line *(y = mx + b)*. The unknown is the correct x-coordinate that yields the desired result (To). Each increase by one on the x axis results in an increase by the amount of the slope on the y axis. Remember that slope is calculated by "rise over run." In this variation of false position, the "run" will always be one. Thus, to reach the desired result, the question is to find out how many "slopes" need to be added to go from the original guess to the correct solution.

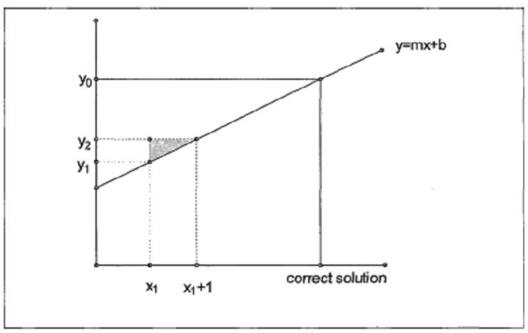

Figure 4. The Babylonian Variation on False Position

These methods of solving linear equations would not be familiar to most secondary school students today. However, with a little time and effort, students would learn to appreciate where the algorithms that are used today came from. These methods would reinforce and give a deeper understanding of proportional reasoning concepts underlying modern algorithms. These methods and their geometrical interpretations might also be introduced into the curriculum for students that are struggling with the modern algorithms, as alternative ways to solve and visualize the solutions of linear equations.

Applications to the Classroom Here is an example of a problem that the Ancient Egyptians solved using the method of false position in approximately 1650 BC. Each of the problems below will contain both the historical and modern approaches to the solution.

Problem 1: From *The Rhind Papyrus.*

A quantity; its half and its third are added to it. It becomes 10.

The Solution: Using current algorithms.

Let x be the unknown quantity.

Writing the' word problem as an equation using the variable x would yield

$$x+\frac{1}{2}x+\frac{1}{3}x=10\ .$$

Combining like terms gives

$$\frac{11}{6}x=10$$

Multiplying both sides by the reciprocal of $\frac{11}{6}$ isolates x in

$$x=10\cdot\frac{6}{11}\ .$$

Simplifying leads to

$$x=\frac{60}{11}=5\frac{5}{11}\ .$$

Thus the unknown quantity is $5\frac{5}{11}$.

The Solution: Using "False Position."

Make a guess (using a number that will work easily with the denominators 2 and 3):

G = 6

Calculate the result using the guess. Substituting the guess into the problem yields

$$6+\frac{1}{2}(6)+\frac{1}{3}(6)\ .$$

This simplifies to

6 + 3 + 2 = 11.

Calculate the ratio by which 11 would be multiplied to get the correct result of 10

$$11\cdot\frac{10}{11}=10\ .$$

Multiplying the original guess by this ratio leads to

$$6\cdot\frac{10}{11}\ .$$

Thus, the correct solution is $\frac{60}{11}=5\frac{5}{11}$.

Here is another example that can be solved using the method of "false position."

Problem 2: From *The Rhind Papyrus (Problem 26).*

When a quantity is added to one-fourth of itself the result is 15.

The Solution: Using current algorithms.

Let x be the quantity. Then, writing an equation to represent the word problem gives

$$x+\frac{1}{4}x=15\ .$$

Combining like terms leads to

$$\frac{5}{4}x = 15 .$$

Multiplying both sides by the reciprocal $\frac{5}{4}$ of results in

$$x = 15 \cdot \frac{4}{5} .$$

Thus, the unknown quantity is 12.

The Solution: Using "false position."

Make a convenient guess (using a number that is a multiple of the denominator 4):

G = 8.

Calculate the result using the guess. Substituting the guess into the problem yields a problem statement:

When the quantity eight is added to one-fourth of itself (i.e. 2) the result is 10. This statement can be represented as

$$8 + \frac{1}{4}(8) .$$

This simplifies to

8 + 2 = 10.

Calculate the ratio by which 10 would be multiplied to get the correct result of 15

$$10 \cdot \frac{15}{10} = 15 .$$

Multiplying the original guess by this ratio leads to

$$8 \cdot \frac{15}{10} .$$

Thus, the correct solution is $\frac{120}{10} = 12$.

Here is an example of "double false position" from the early 1800's.

Problem 3: From *Daboil's Schoolmaster's Assistant.*

A purse of 100 dollars is to be divided among four men A,B,C, and D, so that B may have four more dollars than A, and C eight more dollars than B, and D twice as many as C; what is each one's share of the money?

The solution: Using current algorithms.

Let A receive x dollars. Then B receives x+4, C receives x+4+8, and D receives 2(x+4+8). Then, writing an equation to represent the word problem gives

x + (x + 4) + (x + 4 + 8) + (2 (x + 4 + 8)) = 100.

Combining like terms leads to

$5x + 40 = 100$.

Subtracting 40 from both sides yields

5x - 60.

Multiplying both sides by the reciprocal of 5 results in

$$x = 60 \cdot \frac{1}{5} .$$

Thus A received \$12, B received \$16, C received \$24, and D received \$48.

The Solution: Using "Double False Position."

Make a guess of how much money A receives:

$G_1=6$

Calculate the result using this guess:

6 + (6 + 4) + (6 + 4+8) + (2(6+4 + 8)) = 70.

This is an underestimate by 30. So, the error *(El)* is 30.

Make a second guess: $G_2=8$

Calculate the result using this guess:

8 + (8 + 4) + (8 + 4 + 8) + (2(8 + 4 + 8)) = 80.

This is an underestimate by 20. So, the error (E_2) is 20.

Since the two errors are the same type (both underestimates), use the formula for double false position that is appropriate:

The solution= $\frac{E_1 G_2 - E_2 G_1}{E_1 - E_2}$

Substitution yields $\frac{30.8 - 20.6}{30 - 20}$

Simplifying leads to

The Solution = $\frac{120}{10}$

So, the solution is 12.

Thus A received \$12, B received \$16, C received \$24, and D received \$48.

An example using the Chinese method of "surplus and deficiency."

Problem 4: From *Jiuzhang (Problem 17).*

The price of 1 acre of good land is 300 pieces of gold; the price of 7 acres of bad land is 500. One has purchased altogether 100 acres; the price was 10,000. How much good land was bought and how much bad?

The Solution: Using current algorithms.

Let the amount of good land be x acres. Let the amount of bad land be y acres. The price of x acres of good land is:

x acres $\frac{300}{1}$ gold pieces per acre = 300xgold pieces.

The price of acres of bad land is:

y acres $\frac{500}{7}$ gold pieces per acre = y $\frac{500}{7}$ y gold pieces.

The total cost would then be

$$300x + 500 \cdot \frac{y}{7} .$$

Thus, the following system of equations can be developed

$$x+y=100$$
$$300x+500\cdot\frac{y}{7}=10000.$$

Solving the first equation for one variable leads to

x = 100-y.

Substituting this equation into the second equation gives

$300(100-y) + 500.\frac{y}{7} = 10000.$

Distributing achieves

$30000-300y + \frac{500}{7}y = 10000.$

Combining like terms leads to

$$20000=\left(300-\frac{500}{7}\right)y.$$

This can be simplified to

$$20000=\frac{1600}{7}\cdot y.$$

Finally, multiplying both sides by the reciprocal of $\frac{1600}{7}$ provides

$$20000\cdot\frac{7}{1600}=y.$$

Thus, the solution for y is

$$y=\frac{140000}{1600}=87.5.$$

Substituting this value back into the first equation (after having solved it for x) yields

x = 100-y = 100-87.5 = 12.5.

Thus, the amount of good land is 12.5 acres and the amount of bad land is 87.5 acres.

The Solution: Using the Chinese method of "surplus and deficiency."

Begin by making a guess for the amount of good land:

$G_1 = 5$.

Calculate the amount of bad land:

y = 100-5 = 95.

Now calculate the yield based on the amounts of land:

$$300(5)+500\cdot\frac{95}{7}=\frac{5800}{7}.$$

This is an underestimate by $\frac{12000}{7}$, a "deficiency." Now make a guess that might give an overestimate, a "surplus."

$G_2 = 20.$

Calculate the amount of bad land:

y = 100-20 = 80.

Now calculate the yield based on the amounts of land

$$300(20)+500\cdot\frac{80}{7}=\frac{82000}{7}.$$

This is an overestimate by $\frac{12000}{7}$ a "surplus."

Thus, to solve the problem, use the formula

$$\frac{E_1G_2+E_2G_1}{E_1+E_2}.$$

Substitution yields

$$\frac{E_1G_2+E_2G_1}{E_1+E_2}=\frac{\left(\frac{1200}{7}\right)(5)+\left(\frac{1200}{7}\right)(20)}{\left(\frac{1200}{7}\right)+\left(\frac{1200}{7}\right)}=12.5.$$

Thus, there are 12.5 acres of bad land and 87.5 acres of bad land

Here is an example of a Problem that was solved using the Babylonian variation of "false position.''

Problem 5: From the *VAT 8389 (Problem 76).*

One of two fields yields $\frac{2}{3}$ sila per sar, the second yields $\frac{1}{2}$ sila per sar (sila and sar are measures for are measures for capacity and area, respectively). The yield of the first field was 500 sila more than that of the second; the areas of the two fields were together 1800 sar.

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