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An Inventory Model For Repairable Items And Demand Depending On Holding

Dr. Rajeev Kumar

Assistant Professor, Department of Mathematics, Devta Mahavidhalaya Morna, Bijnor, Uttar Pradesh (India)
Email: rjv2878@gmail.com; Contact No. +91-8449056040

Abstract: To achieve this task, inventory practitioners try to boost the demand by analyzing the policy of trade credit. Due to this, the purchasing cost for the retailer reduces as well as the cost to hold the inventory of the supplier also reduces. Traditionally, retailer accepts this offer as she/he does not have sufficient money to pay at once to the vendor and finalize the account. Retailer can deposit the revenue in interest generating account which is received by selling the product during this period. On deposited amount retailer can earn interest. As the trade credit period is over, the vendor charges interest on the balance amount. Hence, trade credit period offered by supplier is a important promotional tool to attract the retailer. Retailer can take this as an alternative incentive policy.

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Introduction:

Repairable inventory theory involves designing inventory systems for items which are repaired and returned to use rather than discarded. Such systems are composed of items which are typically less expensive to repair than to replace, and are considerably more complicated than traditional inventory systems. The typical problem is concerned with the optimal stocking of the repairable parts and the location of these stocks, given that there may be multiple locations. An added dimension to the problem is the determination of the size and location (s) of the repair capacity for these parts. Further, different performance measures may be used, such as cost, backorders, and availability.[1] There are many complicating factors in the design of repairable inventory systems, for example, not all failed units can be repaired and put back into service, some will be condemned and have to be replaced by new procurements. Various solution approaches have been developed to solve the problem, few have been implemented in practice, and no single model has addressed all or most of the complicating factors. [2] Recent trends in the repairable inventory environment, environmental trends and regulations, and trends in product design are calling some of the assumptions of earlier models into question. In this paper we discuss the existing body of literature on repairable inventory, examine the various models proposed and the major assumptions made in those models, and classify them according to their solution methodology, single versus multi-echelon, and exact versus approximate solutions. It is intended to aid practitioners and researchers in identifying the sources for existing methods and the suitability of those to their application, as well as identify areas for additional research. [3] Every inventory practitioners while designing inventory control policies tries their best to optimize the total inventory cost which is blocked ideally in the form of inventory. To achieve this task, inventory practitioners try to boost the demand by analyzing the policy of trade credit. Due to this, the purchasing cost for the retailer reduces as well as the cost to hold the inventory of the supplier also reduces. Traditionally, retailer accepts this offer as she/he does not have sufficient money to pay at once to the vendor and finalize the account. Retailer can deposit the revenue in interest generating account which is received by selling the product during this period. On deposited amount retailer can earn interest. [4,5]

As the trade credit period is over, the vendor charges interest on the balance amount. Hence, trade credit period offered by supplier is a important promotional tool to attract the retailer. Retailer can take this as an alternative incentive policy. On acquainted with the benefits of trade credit option, two researchers Haley and Higgins (1973) introduced the inventory model taking permissible delay in payments. They considered that demand of the customer is fixed. Discounted cash flow (DCF) approach was adopted by Huang and Huang (2004) to investigate the EPQ model under the effect of time value of money and inflation by considering defective products. Mahata and Goswami (2006) developed and analyzed the fuzzy production lot-size inventory model. They incorporated the concept of permissible delay in payments. They considered that demand rate (□) and production rate (□) as triangular fuzzy numbers (TFN). An inventory model is analyzed by Singh et al. (2008). [6]

They assumed that received lot contains imperfect items. They also assumed that supplier offers trade credit to the retailer. Singh and Jain (2009) developed a deterministic inventory model for deteriorating items. Developed model is analyzed under the effect of inflation. Due to the short life time, they assumed that planning horizon is finite. They further considered that retailer had required amount to settle the vendor‟s dues in the beginning of the planning period, but she/he/ wish to avail the offer of trade credit policy offered by vendor. Proposed inventory models have been solved by Singh and Jain with two different approaches. In first, they optimize the inventory cost and in other method they maximize the net profit function. Integrated economic order quantity model for retailer is investigated by Shastri et al. (2014) by considering two-levels of trade credit. Further, they assumed that items present in stock are perishable. [7] An inventory policy is developed by Yadav et al. (2015) to obtain the payment time to settle the account by the retailer to the vendor under the effect of inflation. An inventory model is investigated by Cárdenas-Barrón et al. (2019) for buyer‟s point of view. They assumed that buyer accept the offer of trade credit which is offered by vendor to boost the demand. From above, it is observed that most of the inventory researchers while modeling inventory models considered that buyer have no money to settle the dues of vendor at the beginning of the planning horizon. Retailers have to take loan to clear the account of the vendor. In the present study, a situation that the retailer has sufficient money to settle the dues of vendor but still retailer avail the offer of trade credit offered by vendor. [8]

Advanced systems, especially military systems such as aircrafts, have expensive complex structures that break down because components are either worn out or damaged during operations. To support high operational readiness (or availability), sufficient quantities of spare components (called Line Replaceable Unit or LRU) and maintenance resources (comprising repair manpower and tools) are required to sustain demands arising from LRU breakdowns (or failures). However, since spares and resources are costly, consume space, and become obsolete over time, there is a trade-off between cost and availability. The goal of the planner is to sustain the life cycle of systems with respect to cost and availability. [9]

Repairable Inventory generally follows the same conventions of Rotable inventory with one important distinction: Repairable inventory has a higher scrap rate than Rotable inventory. For example, a part may be of the same asset value and lifespan as a comparable Rotable; however the repair process may have a 25% scrap rate. Each airline typically defines their break-point between Rotable inventory and Repairable inventory at different levels depending on their own economic analysis. Furthermore, some airlines may not even classify inventory as Repairable, but only maintain the Rotable and Expendable categories. However, the Repairable inventory classification is important to airlines and vendors of aircraft inventory because some of the assumptions about Rotable inventory will not apply to Repairable inventory in certain situations such as leasing, exchange agreements, loaning of parts to other airlines, or entering into pooling arrangements. The main danger in intermixing inventory that is clearly Rotable with inventory that is clearly Repairable in nature is that in agreements with parts vendors, maintenance providers, exchange houses, lessors, and a firm that loans parts or any other parts interchange is that both parties should account for the scrap rate that will certainly have an impact in long-term agreements.

One of the easier methods to deal with a huge variation in scrap rates among various part classifications is to ensure that all parties are aware of whether the inventory in question is Rotable or Repairable, and that scrap rates are clearly delineated between the two asset classes.

The importance of accurately representing scrap rates cannot be overemphasized as it fosters a spirit of cooperation between client and vendor. If the scrap rates are masked or otherwise diluted via inclusion into the overall asset pool (Rotables), agreements may be struck which over long-term are not advantageous for either party. For in the absence of accurate scrap information, a vendor of parts, whether via leasing, exchange, loan or pooling, will include a financial risk premium driving up the cost to the client. If the client unwittingly misrepresents scrap rates, the operational cost will be higher than expected by the vendor, potentially damaging the mutually beneficial long term relationship.

Another important factor for consideration of the Repairable asset class also stems from the higher scrap rate. In the aviation industry, Replenishment Lead Time (RLT) can range from mere hours to months or even more than a year. An example of a few hours RLT might be by acquiring a part via loan, exchange or pool provision. An example of many months to a year might be that the part being sought has no suitable alternative but to order direct from the manufacturer. Consider that for a Repairable item with a 25% scrap rate, 1 in 4 removals on average will result in an order to the surplus market or to the OEM for a replacement part. When RLT could possibly stretch into months, and the removal rates may be high, there can be a large quantity of Repairable inventory on order at any given time and this scenario will require close and careful management to avoid stock-outs and potential for Aircraft On Ground (AOG). [11]

METRIC extensions

Arguably, METRIC is a simplistic model and during its implementation, it was found that the Expected Number of Backorders (EBO) computed was often underestimated due to the use of Poisson distributions. Graves (1985) proposes to model the distribution of the number of items in the base pipelines by a negative binomial distribution, i.e., it uses the variance parameter to reduce the gap. The improvement comes from the observation that the variance-to-mean ratio must be one under Poisson distribution, whereas it is usually greater than one in practice. Under Graves' model, both mean and variance of backorders are calculated and the probability distribution is chosen based on the variance-to-mean ratio. It has also been proved that Graves' model performs equivalently to METRIC when the depot stock level is zero. When the depot stock is not equal to zero, empirical results show that Grave's model produces more than 99% accuracy in spares allocation whereas METRIC achieves around 89% accuracy. [11,12]

Sherbrooke (1986, 1992) proposes VARI-METRIC that captures variance based on Graves' model. This model is interesting in that if the variance-to-mean ratio is of the pipeline greater than one, negative binomial distribution will be adopted. If it is equal to one, Poisson distribution will be adopted. If it is less than one, binomial distribution will be adopted. The OPUS9 (1992) and OPUS10 (1998) are METRIC-based spares optimisation software tools developed commercially by Systecon. Besides adopting the structure and assumptions of METRIC, the tool provides additional features, e.g., the user has the flexibility to specify problem scenarios and order policies. [13]

Limited repair capacity

The models discussed hitherto assume infinite repair capacity, which is often an unrealistic assumption in industrial contexts. More specifically, such an assumption will underestimate the quantity of spare parts needed in systems with high repair facility utilisation. Díaz and Fu (1997) first relax this assumption by considering limited repair facilities at the depot. They consider the setting where all failed LRUs are repaired at the depot and propose results and approximations based on queuing theory for three cases – where the queue at the depot follows *M/M/s* single-class model, *M/G/s* single-class model and *M/G/s* multi-class model. For the *M/M/s* single-class model, the failure follows a Poisson process and repair time follows an exponential distribution with limited repair facilities (servers). Based on single-class, different types of failed LRUs will require different types of servers and only one unit of the required type of server. [14]

The mean and variance of the number of items in the repair facility, both in queue and in repair, are calculated using standard *M/M/s* queuing theory. The model is then extended to *M/G/s* where the repair process follows a general distribution. This is further extended to a multi-class model that allows each type of server to be used to repair multiple types of LRUs. Díaz and Fu (1997) provide an aggregation–disaggregation approach to calculate the first two moments of per-class number in queue and repair. Unfortunately, the variance of per-class number in queue and repair pipeline is derived only for the single-server multi-class queue model due to analytical complexity.

This line of work has been extended recently in several interesting ways. Sleptchenko et al. (2002, 2003) use a more general multi-class multi-server queuing model for the repair shop under steady state when the repair capacity is given. Perlman et al. (2001) use congestion externalities to set expediting repair policy to choose either the repair mode with a normal repair time or the one with an expedited repair time. However, to use exact methods, they restrict themselves to a single repair capacity shop. Kim et al. (2000) extend previous results to the system where spares allocated at the bases as well as the depot.

Time-varying demands

All the above models are steady-state models, which work well when demand follows a stationary Poisson distribution, i.e., the demand rate is constant over time. Unfortunately, many repairable items have long lifecycles and hence the demand rates will inevitably change with time. In a time-varying demand situation, these models will not produce accurate results. Jung (1993) first presents a methodology for a recoverable inventory system with time-varying demand using discrete event simulation. The echelon structure is based on METRIC with limited repair capacity at the depot, except that the demand rate tends to decrease in successive periods. Thus, the repair process at the depot is modelled as a nonstationary *M/M/s* system. The expected number of items in queue and repair at the depot is time-dependent due to nonstationary Poisson process. [15]

Jung (1993) implements the SIMAN system for computing this time-dependent value with the empty queue condition at the beginning. Given a fill rate target, it presents a method to determine the stock level at a certain given time point. Unfortunately, the limitation is that only a single item type is allowed, and the method does *not* perform optimisation.

Slay et al. (1996) propose an aircraft sustainability optimisation model that can handle problems with time-dependent demand rates but under infinite resources. The underlying echelon structure is that a depot only supports a base, and the failure at the base is a nonstationary Poisson process whose mean value varies with time. As an optimisation model, it only considers spares allocation but not resource allocation, and it investigates the objective and spare allocation only at *specific* time points of interest. In this model, the failure rate need not decrease with time (as required by Jung, 1993) – it is high during wartime and low in peacetime. The repair time and shipment time may or may not be time-dependent. The expected number in the pipeline will be calculated first by using integration and then the EBO will be calculated at the certain time point of interest. [16]

RAND Corporation has developed a proprietary system called Dyna-METRIC to serve the US Department of Defense. The Dyna-METRIC series are capability assessment models designed to explore ways to improve wartime logistics support to aircraft. They can solve many problems including nonstationary demands and cannibalisation to assess the effects of wartime dynamics and projects operational performance measures. Version 5 (Isaacson and Boren, 1988) is the latest analytical model to-date in which logistics support system is assumed to be 5-echelon and 3-indenture. Version 5 has its limitations. First, it assumes that the aircrafts deployed at each base are identical, i.e., it does not deal with different item types. Second and unfortunately, like Slay et al. (1996), it provides a steady-state solution to a time-dependent problem.

Simulation models

In this subsection, we briefly discuss three influential simulation models, namely Dyna-METRIC (Version 6), SPAR and Pyke (1990). While Version 5 of Dyna-METRIC is an analytic model based on dynamic form of Palm's theorem, Version 6 is a Monte-Carlo simulation tool as an answer to limitations imposed by analytic models, such as infinite capacity. It is a 3-echelon, 2-indenture model that accommodates interesting features such as lateral supply between bases, lateral repair, information lags and exception reporting. It allows items to have priority to be repaired not only based on FCFS scheduling policy. Although superior to analytical versions in repair process, version 6 has its own limitations. For instance, it does not compute spares requirements because the equations on spares allocations are unavailable in the simulation. It has to draw support from analytical model.

Another limitation is that simulation model is usually very slow when compared with executing the counterpart analytical model. SPAR (http://www.clockwork-group.com) isa commercial Monte-Carlo simulation tool that deals with the problem of time-dependent demands. At the point of writing, SPAR requires external FORTRAN programming to perform the role of repair resource allocation. We also observe that optimisation is slow because it entails running simulation many times. Pyke (1990) presents a simulation study for repairable electronic equipment used by military aircrafts. This model covers a 3-echelon system: a repair depot, a stockpile of repair parts and a set of bases. [17]

It considers priority rules for allocating repaired items to bases and sequencing items at the repair depot. It also considers the importance of the initial allocation of a fixed amount of stock, and the lateral transhipment that occurs only when it is possible to fill all the backorders of a specific base. The optimisation is performed taking into account three decisions: repair rule, distribution rule and where the initial stock of spares is allocated.

Queuing models

A line of work in queuing theory is concerned with approximations under time-dependent arrival and service rates. One of the well-known approximations is the Pointwise Stationary Approximation (PSA), which assumes pointwise stationarity in time and approximates long-run average performance measures (see Green and Kolesar, 1991, 1997; Green et al., 1991). Another approximation is the 'closure approximation', which employs negative binomial distribution to approximate the time-dependent measures (Rothkopf and Oren, 1979). In this paper, as part of the effort in evaluating system performance, we will be studying and adapting the results of Rothkopf and Oren (1979).

Problem formulation

The first problem of evaluating system performance is defined as: given a fixed spares and resource allocation configuration ($s0k$, sjk, rg), compute EBO (t) analytically. The second problem is an optimisation problem driven by cost and system performance. The cost model we consider is a function of the investment cost incurred by spares and resources. The investment cost for spares is straightforward, and is computed by the unit costs multiplied by the number of spare units purchased. The investment cost for resources, however, is a little tricky. As spares are purchased and circulate in the system for a number of years, we should calculate a kind of 'purchasing price' for repair men3 using the net present value of all the cost for a repair man throughout the system lifecycle. These expenditures include wages, taxes and social premiums, housing, education, tools, etc., Sleptchenko et al. (2003). We will use the terminology LSC (standing for *Life Support Cost* in

OPUS9 (1992) and OPUS10 (1998)) as the notation for total cost, where

$$LSC = \sum_{k=1}^{K} C_{s_k} \left(\sum_{j=0}^{J} s_{jk} \right) + \sum_{g=1}^{G} C_{r_g} \times r_g \, . \qquad \text{........1}$$

Let *maxEBO* = max*t*∈[0,*T*]*EBO* (*t*). Given a budget amount B, one problem is to find an allocation of spares and repair resources (i.e., deciding the values of (*s*0*k*, *sjk*, *rg*)) that minimises *maxEBO* while not exceeding the budget (i.e., min *maxEBO* s.t. *LSC* ≤ B). Conversely, we wish to find a minimum-cost spare and repair resource allocation such that the EBO *at any time* within the operating horizon will not exceed a specified target *E*max, (i.e., min *LSC* s.t. *maxEBO* ≤ *E*max).

Note that both these optimisation problems are generalisations of the knapsack problem, which renders them NP-hard.4 The good news, however, is that planners are usually not interested in the optimal allocation point with respect to a specific budget or a target EBO, but rather the problem is to combine the two problems as one by seeking a Cost/Effectiveness (or C/E) curve where each point on the curve is an optimal allocation associated with a cost and EBO value. This is known as the *corrective maintenance problem* in this paper. It is well known that marginal analysis, together with convexification, provides an efficient polynomial-time solution approach to solve this problem (Sherbrooke, 1992). The idea of applying marginal analysis to plot the C/E curve will be adopted in our optimisation algorithm presented.

Corresponding author:
Dr. Rajeev Kumar
Assistant Professor,
Department of Mathematics,
Devta Mahavidhalaya Morna, Bijnor,
Uttar Pradesh (India)
Contact No. +91-8449056040
Email: rjv2878@gmail.com

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