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Review Of Literature Related To Various Statistical Convergence

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Abstract: The investigation of extreme events is extremely relevant for a range of disciplines in mathematical, natural, and social sciences and engineering. Understanding the large fluctuations of the system of interest is of great importance from a theoretical point of view, but also when it comes to assessing the risk associated with low probability and high impact events. In many cases, in order to gauge preparedness and resilience properly, one would like to be able to quantify the return times for events of different intensity and take suitable measures for preventing the expected impacts. Prominent examples are weather and climate extremes, which can have a huge impact on human society and natural ecosystems. The present uncertainty in the future projections of extremes makes their study even more urgent and crucial.

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Introduction:

Statistical Convergence, was published almost fifty years ago, has flatter the domain of recent research. Unlike mathematicians studied characteristics of statistical convergence and applied this notion in numerous extent such as measure theory, trigonometric series, approximation theory, locally compact spaces, and Banach spaces, etc. The present thesis emphasis on certain results studied by Ferenc Mo´ricz in his two research researches i.e., "Statistical Convergence of Sequences and Series of Complex Numbers with applications in Fourier Analysis and summability " and in "Statistical Limit of Lebesgue Measurable functions with ∞ with applications in Fourier Analysis and summability". The perception of conjunction has been generalized in various ways through different methods such as summability and also a method in which one moves from a sequence to functions. In 1932 earlies, Banach coined the first generalization of it and named as "almost convergence". Later it was studied by Lorentz in 1948 [1].

The most recent generalization of the classical convergence i.e., a new type of conjunction named as Statistical Convergence had been originated first via Henry Fast [3] in 1951. He characterizes this hypothesis to Hugo Steinhaus [2]. Actually, it was Antoni Zygmund [20] who evince the results, prepositions and assertion on Statistical Convergence in a Monograph in 1935. Antoni Zygmund in 1935 demonstrated in his book "Trigonometric Series" where instead of Statistical convergence he proposes the term "almost convergence" which was later proved by Steinhaus and Fast ([4] and [3]).

Then, Henry Fast [3] in 1951 developed the notion analogous to Statistical Convergence, Lacunary Statistical Convergence and λ Statistical Convergence and it was reintroduced by Schoenberg [11] in 1959. Since then the several research related to the concept have been published explaining the notion of convergence and is applications. The objective of the study is to discuss the fundamentals and results along with various extensions which have been subsequently formulated [12].

A sequence (xn) in a Banach space X is said to be statistically convergent to a vector L if for any $\varepsilon > 0$ the subset $\{n: kxn - Lk > \varepsilon\}$ has density 0. Statistical convergence is a summability method introduced by Zygmund [4] in the context of Fourier series convergence. Since then, a theory has been developed with deep and beautiful results [5] by different authors, and moreover at the present time this theory does not present any symptoms of abatement. The theory has important applications in several branches of Applied Mathematics (see the recent monograph by Mursaleen [3]). It is well known that there are results that characterize properties of Banach spaces through convergence types. For instance, Kolk [4] was one of the pioneering contributors. Connor, Ganichev and Kadets [5] obtained important results that relate the statistical convergence to classical properties of Banach spaces.

Review of Literature:

The Fibonacci sequence was firstly used in the theory of sequence spaces by Kara and Başarır [5]. Afterward, Kara [6] defined the Fibonacci difference matrix $\hat{F}$ by using the Fibonacci sequence (f_n) for $n \in \{0, 1, \ldots\}$ and introduced the new sequence spaces related to the matrix domain of $\hat{F}$.

Following [7] and [8], high quality papers have been produced on the Fibonacci matrix by many mathematicians [9].

In this paper, by combining the definitions of Fibonacci sequence and statistical convergence, we obtain a new concept of statistical convergence, which will be called Fibonacci type statistical convergence. We examine some basic properties of new statistical convergence defined by Fibonacci sequences. Henceforth, we get an analogue of the classical Korovkin theorem by using the concept of Fibonacci type statistical convergence.

Estimation frequently requires iterative procedures: the more iterations, the more accurate estimates. But when are estimates accurate enough? When can iteration cease? My the rule has become "Convergence is reached when more iterations do not change my interpretation of the estimates".

There is a trade-off between accuracy and speed. Greater accuracy requires more iterations - more time and computer resources. The specification of estimation accuracy is a compromise. Frequently, squeezing that last bit of inaccuracy out of estimates only affects the least significant digits of printed output, has no noticeable effect on model-data fit, and does not alter interpretation. Three numerical convergence rules are often employed:

1) Estimates are pronounced "accurate enough" when a predetermined "maximum" number of iterations have been performed.

2) Estimates are deemed converged when no estimate changes more than a small pre-set "tolerance" value during an iteration.

3) Estimates have converged when there is less residual difference between the observed data and that expected than can actually be observed.

Be wary! In a recent analysis of responses to a set of math tests, linked in block diagonal matrix form, I set these three convergence criteria to reasonable values. The computer program BIGSTEPS ran smoothly. All appeared well. The outcome is shown in Figure 1. As most of us would expected, both the 2995 children and the 1031 math items appear close to normally distributed. The children were from 9 grades, so the spread of 7 logits across the examinees could be right.

A question arose, however, when I went back and inspected the linking design. Children in the lower and higher grades had been deliberately over-sampled in order to get good child measures and item calibrations at the extremes. Yet this bias towards the extremes does not appear in Figure 1!

After eliminating other theories for this unexpected result, suspicion focussed on the analysis itself. Perhaps the familiar values for the convergence criteria were not stringent enough in this case. Accordingly, the criteria were made more stringent, and estimates were again obtained. The initial run used 50 iterations. The revised run, 263 iterations. Now both the child and item distributions are clearly bimodal. The range of child abilities is about 9 logits, an increase of 2 logits. This result makes much better sense.

Establishing convergence is more than a statistical nicety. It can have profound substantive implications.

From this inequality, it seems that the sequence (xk) is statistically convergent of order α to one, and it belongs to the set S α (g) where α > 1 2. We state in advance that from the example that is given above, we obtain the inclusion c (g) ⊂ S α (g) that strictly holds where α = 1. This means that a sequence that is not ordinary convergent in paranormed space can be statistically convergent of order α in this space. Furthermore, from this example, it seems that some sequences that are unbounded divergent can be statistically summable of order α in paranormed spaces.

The concept of statistical convergence has applications in different fields of mathematics such as number theory, statistics and probability theory, approximation theory, optimization, probability theory and fuzzy set theory. In this paper, the concepts of statistical convergence, strongly p-Cesàro summability and statistically Cauchy sequence of order α in paranormed spaces are introduced. Some topological properties of these concepts in paranormed spaces are investigated. Relations between statistical convergence of order α and strongly p-Cesàro summable of order α in paranormed spaces are considered in Theorem 8, Corollary 1, Theorem 9, and examples are given for clear understanding. These definitions and results provide new tools to deal with the convergence problems of sequences occurring in many branches of science which are given above. We state that the concept of paranorm is a generalization of absolute value. Hence, the introduced constructions and obtained results in this paper open new directions for further research. It would be interesting to develop connections between statistical convergence of order α in paranormed spaces and many branches of science.
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The relationships between the various modes of convergence can be summarized in the diagram below. A solid line means that convergence in the mode at the tail of the arrow implies convergence in the mode at the head. A dashed line means that convergence in the mode at the tail of the arrow implies the existence of a subsequence that converges in the mode at the head of the arrow. (The idea for this kind of diagram came from Elements of Integration by Robert Bartle, 1966).

First, consider the functions [0, 1], [0, 1/2], [1/2, 1], [0, 1/3], [1/3, 2/3], etc. The sequence f_n converges to 0 in measure and in L^p. However, there is no x for which $f_n(x)$ converges to 0 ($f_n(x) = 1$ infinitely often) and so f_n converges neither almost everywhere nor almost uniformly. Note that in this case $\Omega = [0, 1]$ is a finite measure space, and the constant function 1 is an L^p bound on the sequence.

Next, consider the functions $f_n = n\,[1/n, 2/n]$. The sequence f_n converges pointwise to 0 everywhere. It converges almost uniformly and converges in measure. However, the L^p norm of f_n is 1 for all n and so no subsequence converges to 0 in L^p norm. Note again $\Omega = [0, 1]$ is a finite measure space in this example.

Let A be a subset of positive integers. We consider the interval $[1, n]$ and select an integer in this interval, randomly. Then the ratio of the number of elements of A in $[1, n]$ to the total number of elements in $[1, n]$ belongs to A, probably. For $n \to \infty$, if this probability exists, that is, this probability tends to some limit, then this limit is used as the asymptotic density of the set A. Let us mention that the asymptotic density is a kind of probability of choosing a number from the set A. Now, we give some definitions and properties of asymptotic density. The set of positive integers will be denoted by $\mathbb{Z}^+$. Let A and B be subsets of $\mathbb{Z}^+$. If the symmetric difference $A\Delta B$ is finite, then we can say A is asymptotically equal to B and denote $A \sim B$. Freedman and Sember introduced the concept of a lower asymptotic density and defined the concept of convergence in density, in [1].

The study of statistical convergence was initiated by Fast [2]. Schoenberg [3] studied statistical convergence as a summability method and listed some of the elementary properties of statistical convergence. Both of these mathematicians mentioned that if a bounded sequence is statistically convergent to L, then it is Cesàro summable to L. Statistical convergence also arises as an example of 'convergence in density' as introduced by Buck [4]. In [5], Zygmund called this concept 'almost convergence' and established the relation between statistical convergence and strong summability. The idea of statistical convergence has been studied in different branches of mathematics such as number theory [6], trigonometric series [5], summability theory [1], measure theory [7] and Hausdorff locally convex topological vector spaces [8]. The concept of $\alpha\beta$-statistical convergence was introduced and studied by Aktuğlu [9]. In [10], Karakaya and Karaisa extended the concept of $\alpha\beta$-statistical convergence. Also, they introduced the concept of weighted $\alpha\beta$-statistical convergence of order γ, weighted $\alpha\beta$-summability of order γ and strongly weighted $\alpha\beta$-summable sequences of order γ in [10]. In [11], Braha gave a new weighted equi-statistical convergence and proved the Korovkin type theorems using the new definition.

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