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Elzaki Transform Approach To Differential Equations

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Abstract: The differential equations with generally solved by adopting Laplace transform method. The paper inquires the differential equations by Elzaki transform. The purpose of paper is to prove the applicability of Elzaki transform to analyze differential equations.

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1. Introduction

Elzaki Transform approach has been applied in solving boundary value problems in most of the science and engineering disciplines [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. It also comes out to be very effective tool to analyze differential equations method [11, 12, 13, 14, 15, 16, 17, 18]. The differential equations are generally solved by adopting Laplace transform method or convolution method of residue theorem [19, 20, 21, 22, 23, 24, 25]. In this paper, we present a new technique called Elzaki transform to analyze differential equations.

2. Basic Definitions

2.1 Elzaki Transform

If the function $\hbar(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of $\hbar(y)$ is given by

$$E\{\hbar(y)\} = \bar{\hbar}(p) = p\int_0^{\infty} e^{-\frac{y}{p}}\hbar(y)dy.$$

The Elzaki Transform [1, 2, 3] of some of the functions are given by

- $E\ \{y^n\} = \ n!\,p^{n+2}, where\ n = 0,1,2,..$
- $E\ \{e^{ay}\} = \frac{p^2}{1-ap},$
- $E\ \{sinay\} = \frac{ap^3}{1+a^2p^2},$
- $E\ \{cosay\} = \frac{ap^2}{1+a^2p^2},$
- $E\ \{sinhay\} = \frac{ap^3}{1-a^2p^2},$
- $E\ \{coshay\} = \frac{ap^2}{1-a^2p^2}.$

2.2 Inverse Elzaki Transform

The Inverse Elzaki Transform of some of the functions are given by

- $E^{-1}\{p^n\} = \frac{y^{n-2}}{(n-2)!}, n = 2, 3, 4 \dots$
- $E^{-1}\{\frac{p^2}{1-ap}\} = e^{ay}$
- $E^{-1}\{\frac{p^3}{1+a^2p^2}\} = \frac{1}{a}\sin ay$
- $E^{-1}\{\frac{p^2}{1+a^2p^2}\} = \frac{1}{a}\cos ay$
- $E^{-1}\{\frac{p^3}{1-a^2p^2}\} = \frac{1}{a}\sin hay$
- $E^{-1}\{\frac{p^2}{1-a^2p^2}\} = \frac{1}{a}\cos hay$

2.3 Elzaki Transform of Derivatives

The Elzaki Transform [1, 2, 3] of some of the Derivatives of h(y) are given by

- $E\{h^{'}(y)\} = \frac{1}{p}E\{h(y)\} - \mathrm{p}\ h(0)$

$or\ E\{\hbar^{'}(y)\} = \frac{1}{p}\bar{\hbar}(p) - \mathrm{p}\ \hbar(0),$

- $E\{\hbar^{''}(y)\} = \frac{1}{p^2}\bar{\hbar}(p) - \hbar(0) - \mathrm{p}\hbar^{'}(0),$

and so on

MATERIAL AND METHOD

(A)

The equations of motion a particle under certain conditions are

$$\boldsymbol{m\ddot{x} + eh\dot{y} = eE \dots\dots\dots (1)}$$

$$\boldsymbol{m\ddot{y} - eh\dot{x} = 0 \dots\dots\dots\dots (2)}$$

$$\boldsymbol{with\ conditions}$$

$$\boldsymbol{x(0) = 0, x'(0) = 0, y(0) = 0, y'(0) = 0}$$

We will find the path of the particle at any instant.

Solution:

Taking Elzaki Transform of (1) on both sides

$$mE\{\ddot{x}\} + \mathrm{eh}E\{\dot{y}\} = E\{eE\}$$

Or

$$m\frac{\bar{x}(p)}{p^2} - mx(0) - mpx'(0) + \mathrm{eh}\frac{\bar{y}(p)}{p} - eh\ \mathrm{py}\ (0) = eEp^2$$

Or

$$m\frac{\bar{x}(p)}{p^2} + \text{eh}\frac{\bar{y}(p)}{p} = eEp^2 \dots\dots\dots (3)$$

Taking Elzaki Transform of (2) on both sides

$$mE\{\ddot{y}\} - \text{eh}E\{\dot{x}\} = 0$$

Or

$$m\frac{\bar{y}(p)}{p^2} - my(0) - mpy'(0) + eh\frac{\bar{x}(p)}{p} - \text{eh px}(0) = 0$$

Or

$$m\frac{\bar{y}(p)}{p^2} - \text{eh}\frac{\bar{x}(p)}{p} = 0 \dots\dots\dots (4)$$

Solving (3) & (4), we get,

$$\bar{x}(p) = meE\left\{\frac{p^4}{m^2 + e^2h^2p^2}\right\}$$

Or

$$\bar{x}(p) = \frac{eE}{m}\left\{\frac{p^4}{1 + w^2p^2}\right\}$$

$$where\ w = \frac{eh}{m}$$

Or

$$\bar{x}(p) = \frac{eE}{m}\left[\frac{p^2}{w^2} - \frac{p^2}{w^2(1 + w^2p^2)}\right]$$

Taking inverse Elzaki transform,

$$x = \frac{eE}{mw^2}[1 - coswt]$$

Or

$$x = \frac{E}{\text{h}\left(\frac{eh}{m}\right)}[1 - coswt]$$

Or

$$x = \frac{E}{\text{hw}}[1 - coswt]$$

And,

$$\bar{y}(p) = \left\{\frac{e^2\text{E}hp^5}{m^2 + e^2h^2p^2}\right\}$$

Or

$$\bar{y}(p) = \frac{e^2\text{Eh}}{m^2}\left\{\frac{p^5}{1 + w^2p^2}\right\}$$

Or

$$\bar{y}(p) = \frac{eE}{mw}\left\{p^3 - \frac{1}{w}.\frac{wp^3}{1 + w^2p^2}\right\}$$

Taking inverse Elzaki transform,

$$y = \frac{eE}{mw^2}\{wt - sinwt\}$$

Or

$$y = \frac{E}{\text{hw}}\{wt - sinwt\}$$

(B)

The differential equation satisfied by a beam uniformly loaded, one end fixed and the second end subjected to tensile force P, is given by

$\boldsymbol{E.I.\ddot{y} = Py - \frac{1}{2}Wt^2 = 0}$**, with conditions**

$\boldsymbol{y(0) = 0, y'(0) = 0.}$

We will find the deflection at any length of the beam.

Solution:

$$E.I.\ddot{y} = Py - \frac{1}{2}Wt^2 = 0$$

This equation can be written as

$$\ddot{y} - \frac{P}{EI}y = \frac{W}{2EI}t^2$$

Taking Elzaki Transform of on both sides

$$E\{\ddot{y}\} - \frac{P}{EI}E\{y\} = -\frac{W}{2EI}E\{t^2\}$$

Or

$$\frac{\bar{y}(p)}{p^2} - y(0) - py'(0) - \frac{P}{EI}\ \bar{y}(p) = -\frac{W}{2EI}2p^4$$

Or

$$\left[\frac{1}{p^2} - \frac{P}{EI}\right]\bar{y}(p) = -\frac{W}{EI}p^4$$

or

$$\bar{y}(p) = -\frac{WEIp^6}{EI(EI - Pp^2)}$$

or

$$\bar{y}(p) = -\frac{Wp^6}{(EI - Pp^2)}$$

or

$$\bar{y}(p) = -W\left[\frac{1}{P}p^4 - \frac{EI}{P^2}p^2 + \frac{E^2I^2}{P^2}\frac{p^2}{(EI - Pp^2)}\right]$$

Taking inverse Elzaki transform,

$$y = -W\left[-\frac{t^2}{2P} - \frac{EI}{P^2} + \frac{EI}{P^2}coshnt\right]$$

or

$$where\ n^2 = \frac{P}{EI}$$

Or

$$y = \left[\frac{Wt^2}{2P} - \frac{EI}{P^2} + \frac{W}{Pn^2}[1 - coshnt]\right]$$

3. Conclusion

In this paper, we have successfully analyzed differential equations by Elzaki Transform technique. It is revealed that the technique is accomplished in analyzing the differential equations.

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