Websites:
http://www.sciencepub.net/academia
http://www.sciencepub.net

Emails:
aarena@gmail.com
editor@sciencepub.net

Academia Arena

MARSLAND PRESS
Multidisciplinary Academic Journal Publisher

Elzaki-Laplace Transform Of Some Significant Functions

Dinesh Verma

Associate Professor Mathematics, Department of Applied Sciences
Yogananda College of Engineering and Technology (YCET), Jammu
drdinesh.maths@gmail.com

Abstract: The paper inquires the Elzaki- Laplace transform of some significant functions which can be used for solving various differential and integral equations. The purpose of paper is to prove the applicability of obtaining Elzaki-Laplace transform of some significant functions.
[Dinesh Verma. **Elzaki-Laplace Transform Of Some Significant Functions.** *Academ Arena* 2020;12(4):38-41]. ISSN 1553-992X (print); ISSN 2158-771X (online). http://www.sciencepub.net/academia. 4. doi:10.7537/marsaaj120420.04.

Keywords: Elzaki-Laplace Transform, Significant Functions

I. Introduction

Elzaki transform and Laplace Transform approaches play a significant role in solving various problems in science and engineering separately [1, 2, 3, 4,]. The differential and integral equations are generally solved by adopting Laplace transform method or Elzaki method or Fourier Transform [5, 6, 7, 8,]. In this paper, we present a new approach called Elzaki- Laplace transform for obtaining Elzaki-Laplace transform of some significant functions.

II. Basic Definitions

The Laplace Transform [9, 10, 11,] with parameter p of $u(x)$ is

$$L\{u(x)\} = \int_0^\infty e^{-px} u(x)\, dx$$

$$for\ Parameter\ p > 0$$

The Elzaki Transform [1, 2] with parameter q of $v(x)$ is

$$E\{v(y)\} = q\int_0^\infty e^{-\frac{y}{q}} v(y) dy$$

The usual Laplace –Elzaki transform is defined as

$$LE\{f(x,y)\} = \bar{f}(p,q) = q\int_0^\infty \int_0^\infty f(x,y) R(x,y)\, dxdy$$

Where,

$$R(x,y) = e^{-(px+\frac{y}{q})}$$

III. Elzaki-Laplace Transform Of Some Functions:

[A]

$$EL\{1\} = q\int_0^\infty \int_0^\infty 1.e^{-(px+\frac{y}{q})} dxdy$$

$$EL\{1\} = q\int_0^\infty \int_0^\infty e^{-(px+\frac{y}{q})} dxdy$$

$$EL\{1\} = \left[q\int_0^\infty e^{-\frac{y}{q}} dy\right]\left[\int_0^\infty e^{-px} dx\right]$$

$$q\left[-qe^{-\frac{y}{q}}\right]_0^\infty \left[\frac{e^{-px}}{-p}\right]_0^\infty$$

$$LE\{1\} = \frac{q^2}{p}$$

[B]

$$EL\{xy\} = q\int_0^\infty \int_0^\infty xy\, e^{-(px+\frac{y}{q})} dxdy$$

$$EL\{xy\} = \left[q\int_0^\infty ye^{-\frac{y}{q}} dy\right]\left[\int_0^\infty xe^{-px} dx\right]$$

$$= q\left[-q^2e^{-\frac{y}{q}}\right]_0^\infty \left[\frac{e^{-px}}{-p^2}\right]_0^\infty$$

$$EL\{xy\} = \frac{q^3}{p^2}$$

(C)

$$EL\{e^{ax+by}\} = q\int_0^\infty e^{ax+by} e^{-(px+\frac{y}{q})} dxdy$$

=

$$\left[\int_0^\infty e^{ax} e^{-px} dx\right]$$

=

$$\left[\int_0^\infty e^{-(p-a)x}\,dx\right]$$

=

$$q\left[\frac{e^{-y\left(\frac{1}{q}-b\right)}}{-\left(\frac{1}{q}-b\right)}\right]_0^\infty\left[\frac{e^{-(p-a)x}}{-(p-a)}\right]_0^\infty$$

=

$$\left[\frac{q^2}{1-bq}\right]\left[\frac{1}{p-a}\right]$$

$$\boldsymbol{EL\{e^{ax+by}\}=\frac{q^2}{(1-bq)(p-a)}}$$

(D)

$$EL\{sinax\ sinby\}=q\int_0^\infty\{sinax\ sinby\}\,e^{-\left(px+\frac{y}{q}\right)}dxdy$$

$$=\left[q\int_0^\infty e^{-\frac{y}{q}}sinby\ dy\right]\left[\int_0^\infty e^{-px}sinax\ dx\right]$$

$$=q\left[\left\{e^{-\frac{y}{q}}\frac{\left(-\frac{1}{q}sinby-bcosby\right)}{b^2+\frac{1}{q^2}}\right\}\right]_0^\infty$$

$$\left[e^{-px}\frac{(-xsinby-acosby)}{p^2+a^2}\right]_0^\infty$$

=

$$\left[q\left\{\frac{b}{b^2+\frac{1}{q^2}}\right\}\right]\left[\left\{\frac{a}{p^2+a^2}\right\}\right]$$

$$\boldsymbol{EL\{sinax\ sinby\}=\frac{abq^3}{(1+b^2q^2)(p^2+a^2)}}$$

(E)

$$EL\{cosax\ cosby\}=q\int_0^\infty\{cosax\ cosby\}\,e^{-\left(px+\frac{y}{q}\right)}dxdy$$

$$=\left[q\int_0^\infty e^{-\frac{y}{q}}cosy\ dy\right]\left[\int_0^\infty e^{-px}cosax\ dx\right]$$

$$=q\left[\left\{e^{-\frac{y}{q}}\frac{\left(-\frac{1}{q}cosby-bsinby\right)}{b^2+\frac{1}{q^2}}\right\}\right]_0^\infty$$

$$\left[e^{-px}\frac{(-pcosby-asinby)}{p^2+a^2}\right]_0^\infty$$

$$=\left[q\left\{\frac{\frac{1}{q}}{b^2+\frac{1}{q^2}}\right\}\right]\left[\left\{\frac{p}{p^2+a^2}\right\}\right]$$

$$\boldsymbol{EL\{cosax\ cosby\}=\frac{pq^2}{(1+b^2q^2)(p^2+a^2)}}$$

(F)

$$EL\{sinhax\ sinhby\}=q\int_0^\infty\{sinhax\ sinhby\}\,e^{-\left(px+\frac{y}{q}\right)}dxdy$$

$$=\left[q\int_0^\infty e^{-\frac{y}{q}}sinhby\ dy\right]$$

$$\left[\int_0^\infty e^{-px}sinhax\ dx\right]$$

$$=\left[q\int_0^\infty e^{-\frac{y}{q}}\left(\frac{e^{by}-e^{-by}}{2}\right)dy\right]$$

$$\left[\int_0^\infty e^{-px}\left(\frac{e^{ax}-e^{-ax}}{2}\right)dx\right]$$

$$=\left[q\int_0^\infty\frac{1}{2}\left\{e^{-y\left(\frac{1}{q}-b\right)}-e^{-y\left(\frac{1}{q}+b\right)}\right\}dy\right]$$

$$*\left[\int_0^\infty\frac{1}{2}\left\{e^{-x(p-a)}-e^{-x(p+a)}\right\}dy\right]$$

$$=\frac{q}{2}\left[\left\{\frac{e^{-y\left(\frac{1}{q}-b\right)}}{-\left(\frac{1}{q}-b\right)}+\frac{e^{-y\left(\frac{1}{q}+b\right)}}{\left(\frac{1}{q}+b\right)}\right\}\right]_0^\infty$$

$$*\left[\frac{e^{-x(p-a)}}{-(p-a)}+\frac{e^{-x(p+a)}}{(p+a)}\right]_0^\infty$$

On solving, we get,

$$\boldsymbol{EL\{sinhax\ sinhby\}=\frac{abq^3}{(1-b^2q^2)(p^2-a^2)}}$$

(G)

$$EL\{coshax\ coshby\}=q\int_0^\infty\{coshax\ coshby\}\,e^{-\left(px+\frac{y}{q}\right)}dxdy$$

$$=\left[q\int_0^\infty e^{-\frac{y}{q}}coshby\ dy\right]$$

$$*\left[\int_0^\infty e^{-px}coshax\ dx\right]$$

$$=\left[q\int_0^\infty e^{-\frac{y}{q}}\left(\frac{e^{by}+e^{-by}}{2}\right)dy\right]$$

$$*\left[\int_0^\infty e^{-px}\left(\frac{e^{ax}+e^{-ax}}{2}\right)dx\right]$$

$$=\left[\frac{q}{2}\left\{\frac{e^{-y\left(\frac{1}{q}-b\right)}}{-\left(\frac{1}{q}-b\right)}-\frac{e^{-y\left(\frac{1}{q}+b\right)}}{\left(\frac{1}{q}-b\right)}\right\}\right]_0^\infty$$

$$*\left[\frac{1}{2}\left\{\frac{e^{-x(p-a)}}{-(p-a)}-\frac{e^{-x(p+a)}}{(p+a)}\right\}\right]_0^\infty$$

$$=\left[\frac{q}{2}\left\{\frac{q}{(1-qb)}+\frac{q}{(1+qb)}\right\}\right]$$

$$*\left[\frac{1}{2}\left\{\frac{1}{(p-a)}+\frac{1}{(p+a)}\right\}\right]$$

On solving, we get,

$$EL\{coshax\ coshby\} = \frac{pq^2}{(1-b^2q^2)(p^2-a^2)}$$

(H)

$$EL\{x^n y^n\} = q\int_0^\infty\int_0^\infty x^n y^n e^{-\left(px+\frac{y}{q}\right)}dxdy$$

$$EL\{xy\} = \left[q\int_0^\infty y^n e^{-\frac{y}{q}}dy\right]\left[\int_0^\infty x^n e^{-px}dx\right]$$

$$= \left[q\left\{nq\int_0^\infty y^{n-1}e^{-\frac{y}{q}}dy\right\}\right]$$

$$*\left[\frac{n}{p}\int_0^\infty x^{n-1}e^{-px}dx\right]$$

$$= \left[q\left\{nq(n-1)q\int_0^\infty y^{n-2}e^{-\frac{y}{q}}dy\right\}\right]$$

$$*\left[\frac{n}{p}.\frac{n-1}{p}\int_0^\infty x^{n-2}e^{-px}dx\right]$$

Expand up to n terms

$$= \left[q\left\{[n(n-1)(n-2)\ldots\ldots 2.1]q^n\int_0^\infty e^{-\frac{y}{q}}dy\right\}\right]$$

$$*\left[[n(n-1)(n-2)\ldots\ldots 2.1]\frac{1}{p^n}\int_0^\infty e^{-px}dx\right]$$

$$= [n!\,q^{n+1}q]\left[n!\frac{1}{p^n}\frac{1}{p}\right]$$

$$= [n!\,q^{n+2}]\left[n!\frac{1}{p^{n+1}}\right]$$

$$EL\{x^n y^n\} = \frac{(n!)^2 q^{n+2}}{p^{n+1}}$$

IV. Conclusion

In this paper, we present a new approach called Elzaki- Laplace transform for obtaining Elzaki-Laplace transform of some significant functions. It may be finished that the technique is accomplished for obtaining Elzaki-Laplace transform of some significant functions and are tabulated as follows:

S. No.	$EL\{f(x,y)\}$	$\bar{f}(q,p)$
1.	$EL\{1\}$	$\frac{q^2}{p}$
2.	$EL\{xy\}$	$\frac{q^3}{p^2}$
3.	$EL\{x^n y^n\}$	$\frac{(n!)^2 q^{n+2}}{p^{n+1}}$
4.	$EL\{e^{ax+by}\}$	$\frac{q^2}{(1-bq)(p-a)}$
5.	$EL\{sinax\ sinby\}$	$\frac{abq^3}{(1+b^2q^2)(p^2+a^2)}$
6.	$EL\{cosax\ cosby\}$	$\frac{pq^2}{(1+b^2q^2)(p^2+a^2)}$
7.	$EL\{sinhax\ sinhby\}$	$\frac{abq^3}{(1-b^2q^2)(p^2-a^2)}$
8.	$EL\{coshax\ coshby\}$	$\frac{pq^2}{(1-b^2q^2)(p^2-a^2)}$

References

1 Tarig M. Elzaki, Salih M. Elzaki and Elsayed Elnour, On the new integral transform Elzaki transform fundamental properties investigations and applications, global journal of mathematical sciences: Theory and Practical, volume 4, number 1(2012).

2 Dinesh Verma, Aftab Alam, Analysis of Simultaneous Differential Equations By Elzaki Transform Approach, Science, Technology And Development Volume Ix Issue I January 2020.

3 Rohit Gupta, Rahul Gupta, Dinesh Verma, Eigen Energy Values and Eigen Functions of a Particle in an Infinite Square Well Potential by Laplace Transforms, International Journal of Innovative Technology and Exploring Engineering, Volume-8 Issue-3, January 2019.

4 Rahul Gupta, Rohit Gupta, Dinesh Verma, Application of Convolution Method to the Impulsive Response of A Lightly Damped Harmonic Oscillator, International Journal of Scientific Research in Physics and Applied Sciences, Vol.7, Issue.3, pp.173-175, June (2019).

5 Dr. Dinesh Verma, Applications of Laplace Transformation for solving Various Differential Equations with Variable Coefficients, International Journal for Innovative Research in Science & Technology, Volume 4, Issue 11, April 2018.

6 V. D. Sharma and A. N. Rangari, Fourier-Laplace Transforms of Some Special Functions, International Journal of Engineering Research

and General Science Volume 3, Issue 6, November-December, 2015.
7 Dr. Dinesh Verma, Applications of Laplace Transformation for solving Various Differential Equations with Variable Coefficients, International Journal for Innovative Research in Science & Technology, Volume 4, Issue 11, April 2018.
8 Dinesh Verma, Rohit Gupta, Amit Pal Singh, Analysis of integral Equations of convolution type via Residue Theorem Approach, The International journal of analytical and experimental modal analysis, Volume XII, Issue I, January 2020. Researcher, 10(7), 2018.
9 Dr. Dinesh Verma, A Laplace Transformation approach to Simultaneous Linear Differential Equations, New York Science Journal, 12 (7), 2019.
10 Dr. Dinesh Verma, An overview of some special functions, International Journal Of Innovative Research In Technology, Volume 5, Issue 1 June 2018.
11 Dr. Dinesh Verma, Solving Fourier integral problem by using Laplace transformation, International journal of innovative research in technology, volume 4, issue 11, April 2018.

4/21/2020