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Review Of Literature Related To The Evolution Of Solving Equation (Linear, Quadratic, And Cubic)

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Abstract: Various studies (e.g. Stacey, 1988; Vinner, 1991; Kieran, 1992; Esty, 1992; Sfard & Linchevski, 1994; Bell, 1995; Linchevski & Herscovics, 1996; McDowell, 1996; Souviney, 1996; Dreyfus, 1999; Lithner, 2000; Mason, 2000, Maharaj, 2005) have focused on the teaching and learning of school mathematics. These studies have indicated some important sources of students' difficulties in mathematics. A limited number of research studies focusing on quadratic equations have documented the techniques students engage in while solving quadratic equations (Bossé & Nandakumar, 2005), geometric approaches used by students for solving quadratic equations (Allaire & Bradley, 2001), students' understanding of and difficulties with solving quadratic equations (Kotsopoulos, 2007; Lima, 2008; Tall, Lima, & Healy, 2014; Vaiyavutjamai, Ellerton, & Clements, 2005; Zakaria & Maat, 2010), the teaching and learning of quadratic equations in classrooms (Olteanu & Holmqvist, 2012; Vaiyavutjamai & Clements, 2006), comparing how quadratic equations are handled in mathematics textbooks in different countries (Saglam & Alacaci, 2012), and the application of the history of quadratic equations in teacher preparation programs to highlight prospective teachers' knowledge (Clark, 2012).
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Introduction

The lobes of the alga Micrasterias (Lacalli and Harrison, 1987) and some microtubule arrays near the cell surfaces of ciliates such as Paramecium and Tetrahymena (Frankel, 1989) are further unicellular examples. Such phenomena show the ability of a living organism to establish a quantitative measure of spacing between adjacent repeats of similar structures and to use it repeatedly in the same direction. Here, "repeatedly" is not intended to imply a time sequence in which structures are formed one by one. In some cases, very numerous parts of the overall pattern are expressed precisely simultaneously, e.g., up to 80 striations in the reproductive cap primordium of Acetabularia. In a large taxonomic group wherein one may expect the mechanism for formation of a particular kind of pattern to have been conserved, the pattern formation process can be essentially simultaneous in some species and time sequential in others (e.g., the onset of seg mentation in the class Insecta). In this research, we consider the problem of generating the parts of a pattern simultaneously. This approach does not lack generality.

Kieran (1992) considered a student's inability to acquire an in-depth sense of the structural aspects of algebra to be the main obstacle. Sfard and Linchevski (1994) have analysed the nature and growth of algebraic thinking from an epistemological perspective supported by historical observations. They indicated that the development of algebraic thinking was a sequence of ever more advanced transitions from operational (procedural) to structural outlooks. Mason (2000:97) has argued that "... the style and the nature of questions encountered by students strongly influences the sense that they make of the subject matter". The questions that come to the mind of an educator are influenced by the perspective and disposition that he/she has towards mathematics and pedagogy (Mason, 2000).

These questions in turn influence the sense learners make of the subject matter. In this article I focus on the outcomes and implications of research on (a) use of symbols in mathematics, (b) algebraic/trigonometric expressions, (c) solving equations, and (d) functions and calculus.

In seeking to explain the complex phenomena of biological pattern formation, one must start with an a priori concept of where the complexity lies. Wortis et al. (in press) have drawn the contrast between complex machines and simple machines with complex behaviour. Molecular biologists seek the former: the complex machine as a multiplicity of genes and gene products, mutually governed by many regulative processes which are complex by their sheer number but each rather simple in character. Physical scientists tend to seek the latter: dynamic processes which can be described by a few simple terms in two or three equations, but which display complex behaviour.

This research describes work within the latter paradigm. Our results rely on mathematical analysis to an extent; however, for the study of wide ranges of parameter values, our method has been, as in most work of this kind, to put the model into a computer and take its workings as a topic for experimental study in much the 0 1993 WILEY-LISS, INC. same spirit as an experimental biologist's study of a developmental phenomenon. Such studies are undertaken as contributions to the theory of natural phenomena, in this case biological pattern formation. Nevertheless, while it is in progress, the work resembles an experimental study in its own right, with the model as the subject. An important class of biological phenomena is the generation of two-dimensional structures periodically repeated in space. Such patterns occur in diverse biological contexts, for example (1) a multicellular epithelial sheet, as for the striped and spotted coat patterns of many mammals; (2) a multinucleate syncytium having a single layer of some thousands of nuclei, such as the Drosophila blastoderm in which striped patterns of pair-rule gene-expression arise; (3) the surface of a large single cell, for instance the growing tip of the alga Acetabularia which generates both vegetative and reproductive whorls at a site which may be centimeters from the only nucleus (Harrison et al., 1988; Berger et al., 1987).

Review of Literature

Boyer's (1968) A History of Mathematics is almost entirely about Greek mathematics. It covers ancient Greek mathematics to a degree that none of the other mentioned texts do. Perhaps one of the most valuable tools for a secondary teacher available is Historical Topics for the Mathematics Classroom (National Council for Teachers of Mathematics, 1989).

This text consists of a series of "capsules" (short chapters). Each capsule gives a brief historical overview of a particular topic (e.g. Napier's Rods). The capsules are grouped by general topic (algebra, geometry, trigonometry, etc.). Specifically, this text provides a historical context to graphical approaches to equation solving. In addition, it provides a concise overview of the methods employed to solve quadratics and cubics.

Various researchers (Vaiyavutjamai & Clements, 2006) have illustrated that very little attention has been paid to quadratic equations in mathematics education literature, and there is scarce research regarding the teaching and learning of quadratic equations.

In general, for most students, quadratic equations create challenges in various ways such as difficulties in algebraic procedures, (particularly in factoring quadratic equations), and an inability to apply meaning to the quadratics. Kotsopoulos (2007) suggests that recalling main multiplication facts directly influences a student's ability while engaged in factoring quadratics. Furthermore, since solving the quadratic equations by factorization requires students to find factors rapidly, factoring simple quadratics becomes quite a challenge, while non-simple ones (i.e., ax2 + bx + c where a ^ 1) become harder still. Factoring quadratics can be considerably complicated when the leading coefficient or the constant term has many pairs of factors (Bossé & Nandakumar, 2005).

The research of Filloy & Rojano (1989) suggested that an equation such as with an expression on the left and a number on the right is much easier to solve symbolically than an equation such as. This is because the first can be 'undone' arithmetically by reversing the operation 'multiply by 3 and subtract 1 to get 5' by 'adding 1 to 5 to get and then dividing 6 by 3 to get the solution.

Meanwhile the equation cannot be solved by arithmetic undoing and requires algebraic operations to be performed to simplify the equation to give a solution. This phenomenon is called 'the didactic cut'. It relates to the observation that many students see the 'equals' sign as an operation, arising out of experience in arithmetic where an equation of the form is seen as a dynamic operation to perform the calculation, 'three plus four makes 7', so that an equation such as is seen as an operation which may possibly be solved by arithmetic 'undoing' rather than requiring algebraic manipulation (Kieran, 1981).

Lima & Healy (2010) classified an equation of the form 'expression = number' as an evaluation equation, because it involved the numerical evaluation of an algebraic expression where the input value of x could be found by numerical 'undoing', and more general linear equations as manipulation equations, because they required algebraic manipulation for their solution.

The data of Lima & Tall (2008) presented an analysis of Brazilian students' work with linear equations that did not fit either the didactic cut or the balance model. Their teachers had used an 'expert-novice' view of teaching and had introduced the students to the methodology that they, as experts, found appropriate for solving equations, using the general principle of 'doing the same thing to both sides' to simplify the equation and move towards a solution. However, when interviewed after the course, students rarely used the general principle. They did not treat the equation as a balance to 'do the same thing to both sides', nor did they show any evidence of the didactic cut.

According to Matz, (1980) and Payne & Squibb (1990), Our purpose is not simply to find and catalogue errors. Instead we seek to evolve a single theoretical framework that covers all three aspects: the didactic cut, the balance model and the problem with 'doing the same thing to both sides'. Such a theoretical framework should relate to both cognitive development and the emotional effects of the learning experience. To integrate these different aspects into a single framework, we begin with a theoretical construct that relates current learning to previous experience.

This offers a refined formulation of the original research into the didactic cut by Filloy & Rojano (1989), where many of the students were able to solve simple evaluation equations before being taught to solve equations using algebraic manipulation. The notion of an equation as a process of evaluation is supportive for solving evaluation equations but problematic for manipulation equations. Another observation made at the time is that the introduction of the algebraic technique in solving linear equations caused a loss in ability for some students to solve simple equations using arithmetic undoing. This loss in facility when faced with a new technique is common in mathematics learning.

For instance, Gray (1991) noted that some children introduced to column subtraction may make errors that did not occur when they performed the same operation using simple mental arithmetic. This is consistent with the absence of the didactic cut in the data.

According to Lima & Tall (2008). The students had been presented with a new formal principle for solving equations by 'doing the same thing to both sides'. This new principle was not generally implemented as intended, instead the students focused on shifting symbols with additional rules as procedural embodiments that treated both evaluation and manipulation equations in the same way. Thus the students performed the same type of operation in both cases and made the same sort of error.

Tall (2011) formulated a working definition of a crystalline concept as 'a concept that has a structure of relationships that are a necessary consequence of its context'. Such a concept has strong internal bonds that hold it together so that it can be considered as a single entity.

Just as Sfard (1991) spoke of 'condensing' a process from a sequence of distinct steps which we may interpret as a metaphor for transforming a gas that is diffuse to a liquid that can be poured in a single flow, we can think of 'crystallizing' as the transition that turns the flowing liquid into a solid object that can be manipulated in the hand, or, in mathematics, manipulated in the mind as an entity. This metaphor does not mean that a crystalline concept has uniform faces like a chemical crystal, but that it has strong internal bonds that cause it to have a predictable structure.

Van Merrienboer and Jeroen (2013) investigated the perspectives on problem solving and instruction. It was found that problem solving should not be limited to well structured problem solving but be extended to real life problem solving.

Tsai et al. (2012) analyzed visual attention for solving multiple choice science problems. Studies showed that successful problem solvers focused more on relevant factors while unsuccessful problem solvers experienced difficulties in decoding the problem, in recognizing the relevant factors and in self regulating concentration. Kuo et al. (2012) experimented a hybrid approach to promoting students web based problem solving competence and learning attitude. Results show that middle and low achievement students in the experimental group gained significant.

Kaye (1915) argued that it was natural to seek for traces of Greek influence in later works of art and mathematics. There is evidence now to suggest Greek philosophy may be linked to or of a possible Indian origin. The difficulty in translating of scripts in Sanskrit was another reason in that Sanskrit scripts had to be translated before mathematicians could appreciate their actual mathematical value.

Joseph's (1991) study outlined the possibilities that may have led to the development of the vast knowledge of mathematics that we enjoy today this includes the mathematical achievement of all major civilizations such as Mayan, Babylonian, and Chinese among others. In this study, the author explores Indian mathematics by briefly investigating early India and the Vedic period-during which much of the foundations were laid for religious, philosophical and mathematical views of the world. The Vedic achievements are analysed to highlight the abstract and symbolic nature of their mathematics that ultimately led to the development of symbolic algebra (AD 500).

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When introducing algebra the use of letters should be withheld until it is evident that learners are ready for their use, and teaching should recognise and prepare learners for the various uses of letters in algebra as the need arises (Harper, 1987; Stols, 1996).

Pyke (2003) has shown that the learners' use of "... symbols, words, and diagrams to communicate about their ideas each contribute in different ways to solving tasks". The structurality of geometry and the visual overview that it provides facilitate thinking and effective investigation (Sfard, 1995). For example, the formulae for determining the areas of squares and rectangles can be used to introduce algebraic expressions. Such an approach could help learners to make links between arithmetic and algebra. A teaching sequence which allowed students to develop a procedural (operational) meaning for algebraic expressions such as $4x + 4y$ was designed by Chahouh and Herscovics (Kieran, 1992).

Math through the Ages (Berlinghoff & Gouvea, 2004) is an excellent book from which to learn the history of some key mathematical ideas. The text focuses on a few main ideas, and expands upon them. Specifically, it provides interesting stories and histories on people. However, it does not show most of the actual work that was needed to derive the formulae and ideas presented.

On the other hand, Journey through Genius (Dunham, 1990) provides many of the proofs and derivations of formulae in addition to interesting background information. However in this book, the focus of each chapter is a specific theorem, rather than the evolution of a mathematical idea.

Hattie (2009) noted that fluency with prerequisite knowledge, even at a very early stage, was highly predictive of latter success. The key prerequisite concepts and processes necessary to engage meaningfully with quadratics include basic whole number fluency, fraction computation, linear algebraic procudures, and coordinate geometry. A key process in working with quadratics is solving or finding the intercepts, should there be any.

In most curricula this has involved factorisation, the square root method, completing the square, and the use of the quadratic formula. Each of these techniques has its own advantages and disadvantages when it comes to teaching, learning, and applying. Research has shown that students and teachers shy away from some techniques and favour factorisation, generally using coefficients that are easy to factorise since students' ability to perform fractional and radical arithmetic has been reported as low (Bosse & Nandakumar, 2005).

Conclusions

The conclusion of the mathematical problems undertaken in this research work is discussed. The study of the solution STUDY ON THE PARAMETERS RELATED TO THE EVOLUTION OF SOLVING EQUATION (LINEAR, QUADRATIC, AND CUBIC) on the literature of the research findings in this area are reported. The motivation and the main objectives of this thesis are given. In this section, the essence of the present research work is briefed. Organization of the thesis shows the arrangements of the outcome of the present research work.

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