

The New Prime theorem (18)**Hardy-Littlewood Conjecture** $E : x^2 + 1$

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jiangchunxuan@vip.sohu.com**Abstract:** Using Jiang function we prove Hardy-Littlewood conjecture $E : x^2 + 1$ [2].[Chun-Xuan Jiang. **The New Prime theorem (18) Hardy-Littlewood Conjecture** $E : x^2 + 1$. *Academ Arena* 2018;10(11):129-130]. (ISSN 1553-992X). <http://www.sciencepub.net/academia>. 8. doi:[10.7537/marsaaj101118.08](https://doi.org/10.7537/marsaaj101118.08).**Keywords:** prime; theorem; function; number; new**Theorem.** The prime equation

$$P_n = (2P_1P_2 \cdots P_{n-1})^2 + 1 \quad (1)$$

has infinitely many prime solutions.

Proof. We have Jiang function[1]

$$J_n(\omega) = \prod_P [(P-1)^{n-1} - \chi(P)] \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$(2q_1q_2 \cdots q_{n-1})^2 + 1 \equiv 0 \pmod{P}, \quad q_i = 1, \dots, P-1, i = 1, \dots, n-1 \quad (3)$$

From (3) we have

$$\left(\frac{-1}{P}\right) = (-1)^{\frac{P-1}{2}}, \text{ if } \left(\frac{-1}{P}\right) = 1 \text{ then } \chi(P) = 2(P-1)^{n-2}, \text{ if } \left(\frac{-1}{P}\right) = -1 \text{ then } \chi(P) = 0.$$

Substituting it into (2) we have.

$$J_n(\omega) = \prod_{3 \leq P} [(P-1)^{n-2} (P-2 - (-1)^{\frac{P-1}{2}})] \neq 0 \quad (4)$$

We prove that (1) has infinitely many prime solutions. $J_n(\omega) \subset \phi^{n-1}(\omega)$

We have the best asymptotic formula [1]

$$\pi_2(N, n) = \left| \{P_1, \dots, P_{n-1} \leq N : P_n = \text{prime}\} \right| \sim \frac{J_n(\omega)\omega}{2 \times (n-1)! \phi^n(\omega) \log^n N} \quad (5)$$

Example 1. Let $n = 2$. From (1) we have

$$P_2 = (2P_1)^2 + 1 \quad (6)$$

From (4) we have

$$J_2(\omega) = \prod_{3 \leq P} [P-2 - (-1)^{\frac{P-1}{2}}] \neq 0 \quad (7)$$

Example 2. Let $n = 3$. From (1) we have

$$P_3 = (2P_1P_2)^2 + 1 \quad (8)$$

From (4) we have

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)(P-2 - (-1)^{\frac{P-1}{2}})] \neq 0 \quad (9)$$

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series $\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \frac{1 + \chi(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$ [1], which can count the number of prime number. The

prime number is not random. But Hardy singular series $\sigma(H) = \prod_P \left(1 - \frac{\nu(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$ is false [2], which cannot count the number of prime numbers.

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