

The New Prime theorem (17)

$$P_n = 2P_1P_2 \cdots P_{n-1} \pm 1$$

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Abstract: Using Jiang function we prove that such that $P_n = 2P_1P_2 \cdots P_{n-1} \pm 1$ has infinitely many prime solutions.

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Theorem. The prime equation

$$P_n = 2P_1P_2 \cdots P_{n-1} + 1 \quad (1)$$

has infinitely many prime solutions.

Proof. We have Jiang function[1]

$$J_n(\omega) = \prod_P [(P-1)^{n-1} - \chi(P)] \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$2q_1q_2 \cdots q_{n-1} + 1 \equiv 0 \pmod{P}, \quad q_i = 1, \cdots, P-1, i = 1, \cdots, n-1 \quad (3)$$

From (3) we have

$$\chi(P) = (P-1)^{n-2} \quad (4)$$

Substituting (4) into (2) we have

$$J_n(\omega) = \prod_{3 \leq P} [(P-1)^{n-2} (P-2)] \neq 0 \quad (5)$$

We prove that (1) has infinitely many prime solutions. $J_n(\omega) \subset \phi^{n-1}(\omega)$.
We have the best asymptotic formula [1]

$$\pi_2(N, n) = \left| \{P_1, \cdots, P_{n-1} \leq N : P_n = \text{prime}\} \right| \sim \frac{J_n(\omega)\omega}{(n-1)!\phi^n(\omega)} \frac{N^{n-1}}{\log^n N} \quad (6)$$

Example 1. Let $n = 2$. From (1) we have

$$P_2 = 2P_1 + 1 \quad (7)$$

From (5) we have

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \neq 0 \quad (8)$$

Example 2. Let $n = 3$. From (1) we have

$$P_3 = 2P_1P_2 + 1 \quad (9)$$

From (5) we have

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)(P-2)] \neq 0 \quad (10)$$

In the same way we are able to prove that

$$P_m = 2P_1P_2 \cdots P_{n-1} - 1 \quad (11)$$

has infinitely many prime solutions.

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series $\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \frac{1 + \chi(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$ [1], which can count the number of prime number. The

prime number is not random. But Hardy singular series $\sigma(H) = \prod_P \left(1 - \frac{\nu(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$ is false [2-5], which cannot count the number of prime numbers.

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