Gaps Among Products of *m* Primes

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Abstract: Using Jiang function $J_2(\omega)$ we prove gaps among products of m prime: $d(x)=d(x+1)=d(x+5-3)=d(x+7-3)=\cdots=d(x+P_n-3)=m>1$ infinitely-often, where P_n denotes the $n-$th prime.

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Theorem 1. Let $P_3=5$, gaps among products of two primes

$$d(x)=d(x+1)=d(x+2)=2 \text{ infinitely-often.} \tag{1}$$

where $d(x)$ represents the number of distinct prime factors of x, $d(x)=\sum_{P|x}1,\ d(3)=1$, $d(15)=2$, $d(105)=3$.

Proof (see[1] p.146 theorem 3.1.154). Prime equations are

$$\beta_1=10\alpha+1,\quad \beta_2=15\alpha+2,\quad \beta_3=6\alpha+1 \tag{2}$$

We have Jiang function

$$J_2(\omega)=3\prod_{7<P}(P-4)\neq 0, \tag{3}$$

where $\omega=\prod_{2\le P}P$

We prove that $J_2(\omega)\neq 0$ there exist infinitely many odd integers α such that β_1, β_2 and β_3 are primes.

We have asymptotic formula

$$\left|\{\alpha\le N:10\alpha+1,15\alpha+2,6\alpha+1\}\right|>\frac{J_2(\omega)\omega}{\varphi^4(\omega)}\frac{N}{\log^4 N}, \tag{4}$$

where $\phi(\omega)=\prod_{2\le P}(P-1)$.

From (2) we have

$$3\beta_1=30\alpha+3,$$

$$3\beta_1+1=30\alpha+4=2(15\alpha+2)=2\beta_2, \tag{5}$$

$$3\beta_1+2=30\alpha+5=5(6\alpha+1)=5\beta_3$$

From (5) we prove

$$d(3\beta_1)=d(3\beta_1+1)=d(3\beta_1+2)=2 \text{ infinitely-often.} \tag{6}$$

We prove that there exist infinitely many triples of consecutive integers, each being the products of two distinct primes.

Theorem 2. Let $P_3 = 5$, gaps among products of m primes.

$$d(x) = d(x+1) = d(x+2) = m > 1 \quad \text{infinitely-often} \tag{7}$$

Proof (see [1] p.148, theorem 3.1.158). Suppose that $u, u+1$ and $u+2$ are three consecutive integers, each being the products of $m-1$ distinct primes. Let $M = u(u+1)(u+2)$. We define the three prime equations

$$\beta_1 = \frac{2M}{u}\alpha + 1, \quad \beta_2 = \frac{2M}{u+1}\alpha + 1, \quad \beta_3 = \frac{2M}{u+2}\alpha + 1 \tag{8}$$

Using Jiang function $J_2(\omega)$ we can prove that there exist infinitely many integers α such that β_1, β_2 and β_3 are primes.

From (8) we have

$$u\beta_1 = 2M\alpha + u$$

$$u\beta_1 + 1 = 2M\alpha + u + 1 = (u+1)\left(\frac{2M}{u+1}\alpha + 1\right) = (u+1)\beta_2$$

$$u\beta_1 + 2 = 2M\alpha + u + 2 = (u+2)\left(\frac{2M}{u+2}\alpha + 1\right) = (u+2)\beta_3 \tag{9}$$

We prove

$$d(x) = d(x+1) = d(x+2) = m > 1 \quad \text{infinitely-often.} \tag{10}$$

Theorem 3. Let $P_4 = 7$, gaps among products of two primes.

$$d(x) = d(x+2) = d(x+4) = 2 \quad \text{infinitely-often.} \tag{11}$$

Proof [1,2,3]. Prime equations are

$$\beta_1 = 70\alpha + 1, \quad \beta_2 = 42\alpha + 1, \quad \beta_3 = 30\alpha + 1 \tag{12}$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many integers α such that β_1, β_2 and β_3 are primes.

Frome (12) we have

$$3\beta_1 = 210\alpha + 3,$$

$$3\beta_1 + 2 = 210\alpha + 5 = 5(42\alpha + 1) = 5\beta_2$$

$$3\beta_1 + 4 = 210\alpha + 7 = 7(30\alpha + 1) = 7\beta_3 \tag{13}$$

We prove

$$d(3\beta_1) = d(3\beta_2 + 2) = d(3\beta_1 + 4) = 2 \quad \text{infinitely-often.} \tag{14}$$

Theorem 4. Let $P_4 = 7$, gaps among products of m primes.

$$d(x) = d(x+2) = d(x+4) = m > 1 \quad \text{infinitely-often.} \tag{15}$$

Proof [1, 2, 3]. Suppose that $u, u+2$ and $u+4$ are three odd integers, each being the products of $m-1$ distinct primes. Let $M = u(u+2)(u+4)$

We define three prime equations

$$\beta_1 = \frac{2M}{u}\alpha + 1, \quad \beta_2 = \frac{2M}{u+2}\alpha + 1, \quad \beta_3 = \frac{2M}{u+4}\alpha + 1 \tag{16}$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many integers α such that β_1, β_2

and β_3 are primes.

From (16) we have

$$u\beta_1 = 2M\alpha + u,$$

$$u\beta_1 + 2 = 2M\alpha + u + 2 = (u+2)\left(\frac{2M}{u+2}\alpha + 1\right) = (u+2)\beta_2,$$

$$u\beta_1 + 4 = 2M\alpha + u + 4 = (u+4)\left(\frac{2M}{u+4}\alpha + 1)\right) = (u+4)\beta_3. \quad (17)$$

We prove

$d(x) = d(x+2) = d(x+4) = m > 1$ infinitely-often. (18)

**Theorem 5. Let $P_4 = 7$, ** gaps among products of m primes.

$d(x) = d(x+1) = d(x+2) = d(x+4) = m > 1$ infinitely-often. (19)

Proof. From (12) we have prime equations

$$\beta_1 = 70\alpha + 1, \quad \beta_2 = 105\alpha + 2, \quad \beta_3 = 42\alpha + 1, \quad \beta_4 = 30\alpha + 1 \quad (20)$$

Using Jiang function $J_2(\omega)$ [1] we can prove there exist infinitely many odd integers α such that β_1, β_2, β_3 and β_4 are primes

From (20) we have

$$3\beta_1 = 210\alpha + 3$$

$$3\beta_1 + 1 = 210\alpha + 4 = 2(105\alpha + 2) = 2\beta_2$$

$$3\beta_1 + 2 = 210\alpha + 5 = 5(42\alpha + 1) = 5\beta_3$$

$$3\beta_1 + 4 = 210\alpha + 7 = 7(30\alpha + 1) = 7\beta_4. \quad (21)$$

We prove

$d(3\beta_1) = d(3\beta_1 + 1) = d(3\beta_1 + 2) = d(3\beta_1 + 4) = 2$ infinitely-often. (22)

Using Jiang function we can prove that

$d(x) = d(x+1) = d(x+2) = d(x+4) = m > 1$ infinitely-often. (23)

Theorem 6. Gaps among products of m primes.

$d(x) = d(x+1) = d(x+5-3) = d(x+7-3) = \cdots = d(x+P_n-3) = m > 1$ infintely-often. (24)

where P_n denotes the $n-$th prime.

Proof. Let $\omega_n = \prod_{2 \le P \le P_n} P$. We define the prime equations

$$\beta_1 = \frac{\omega_n}{3}\alpha + 1, \quad \beta_2 = \frac{\omega_n}{2}\alpha + 2, \quad \beta_3 = \frac{\omega_n}{5}\alpha + 1, \quad \beta_4 = \frac{\omega_n}{7}\alpha + 1, \cdots, \quad \beta_n = \frac{\omega_n}{P_n}\alpha + 1. \quad (25)$$

Using Jiang function $J_2(\omega)$ [1] we can prove that there exist infinitely many odd integers α such that β_1, $\beta_2, \cdots, \beta_n$ are primes.

From (25) we have

$$3\beta_1 = \omega_n\alpha + 3,$$

$$3\beta_1+1=\omega_n\alpha+4=2(\frac{\omega_n}{2}\alpha+2)=2\beta_2,$$

$$3\beta_1+2=\omega_n\alpha+5=5(\frac{\omega_n}{5}\alpha+1)=5\beta_3,$$

$$3\beta_1+4=\omega_n\alpha+7=7(\frac{\omega_n}{7}\alpha+1)=7\beta_4,$$

……

$$3\beta_1+P_n-3=\omega_n\alpha+P_n=P_n(\frac{\omega_n}{P_n}\alpha+1)=P_n\beta_n. \quad (26)$$

From（26）we have

$$d(3\beta_1)=d(3\beta_1+1)=d(3\beta_1+2)=d(3\beta_1+4)=\cdots=d(3\beta_1+P_n-3)=2$$ infinitely-often. （27）

Using Jiang function $J_2(\omega)$ [1] we can prove that

$$d(x)=d(x+1)=d(x+5-3)=d(x+7-3)=\cdots=d(x+P_n-3)=m>1$$ infinitely-often. （28）

Goldston *et. al* proved only

$$d(x)=d(x+n\le 6)=2$$ infinitely-often [4]. （29）

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美国匈牙利土耳其数学家正在研究 x 和 $x+n\le 6$ 两个数。有无限多个 x 使得 x 和 $x\le 6$ 两个数每个都是两个素数相乘[4]。例如 $x=14=2\times 7$，$x+1=15=3\times 5,\cdots$，$x=86=2\times 43$，$x+6=91=7\times 13$,这就是当代国际数学最高水平,受到当代数学界的关注。他们并没有证明这个问题，它比哥德巴赫猜想难一万倍。蒋春暄看到[4] 以后，国外就这么点水平吹到天上去，决定写本文，在国内外散发。蒋春暄 2002 年结果[1]。从定理一得出 $x=93=3\times 31$，$x+1=94=2\times 47$，$x+2=95=5\times 19$，有无限多个 x 使得 x，$x+1$，$x+2$ 三个数每个数都是两个素数相乘。从定理二得出，$x=1727913=3\times 11\times 52361$，$x+1=1727914=2\times 17\times 50821$，$x+2=1727915=5\times 7\times 49369$，有无限多个 x 使得每个数都是 m 个素数相乘。定理六，x，$x+1$，$x+2$，$x+4,\cdots$，$x+P_n-3$ 有 n 个数,有无限多个 x 使得每个数都是 m 个素数相乘，这样成果在过去没有数学家想象过，这是素数分布一个重要规律。将来一定会有广泛的应用。这是数学美！这是人类数学中最伟大成就.

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